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Management Accounting and Operational Research

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Abstract

We discuss some topics relating management accounting and operational research. Because of the wide area of application of operational research techniques in management accounting we had to make a choice and limited us to budgetting through input-output analysis, cost-volume-profit analysis and transfer prices in decentralized organisations.

Keywords: input-output analysis, cost-volume-profit analysis, transfer prices.

1. Introduction

In a journal devoted to operational research it seems appropriate to start an article on management accounting and operational research with a few remarks on notions used in accounting and on developments in this area. Management accounting has its historical origin in the classical field of cost accounting, which in its turn developed during a long history from the fourteenth and fifteenth centuries' double-entry bookkeeping systems [51].

A well known description of classical cost accounting is from Horngren [20, p. 4]: "Originally, the label cost accounting referred to ways of accumulating historical costs and tracing them to units of output and to departments, primarily for purposes of providing the inventory valuations used in balance sheets and income statements". Defined in this way cost accounting is primarily a subservient to the needs of external financial reporting to e.g. shareholders, lenders, government and public supervisory bodies, though it also served pricing purposes, especially in Western-European countries e.g. [42, 53].

During the last 20 to 30 years there was a shift in emphasis as one more fully realized that the cost accounting system could and should also serve managers in preparing their internal planning and control decisions.

Nowadays cost accounting is usually defined in a very general way. According to Shillinglaw [49, p. 1] "cost accounting is the process of measuring or estimating the costs of specific objectives of individual organizations, together with the entire body of concepts, methods, procedures and record-keeping systems necessary to do this".

In this general context cost accounting is not an aim in itself but it can serve many needs of a firm which are systematically reviewed by Shillinglaw. He distinguishes into serving the internal oriented management accounting and into the external oriented financial accounting, tax accounting and contract accounting.

The first limitation we make in this contribution is to restrict ourselves to management accounting, "the process of measuring, analyzing, estimating, and reporting to management on the costs and benefits of individual activities and segments of the organization".

Roughly speaking management accounting serves two managerial purposes: planning and control. Shillinglaw differentiates planning, the allocation of an organization's resources, into: strategic planning, long-range periodic planning, project and situation planning and short-range-planning. Control consists of management's efforts to prevent undesirable departures from planned actions.

Concerning the activities mentioned, most applications of operational research methods are found in decisions on project and situation planning and short-range periodic planning. Therefore as a second restriction, we confine to these two area's.

Both do not have the same contents for every organization and so an everywhere valid systematic splitting up is not possible.

Generally speaking project and situation planning consists - within the framework of strategic and long-range plans - of decisions to use parts of the organization's resources in specific ways, each decision requiring a choice among competing alternatives. Short-range planning determines in what way the resources will be used, for going-on activities and for project planning together, and predicts the results of the decisions. Central themes are in most cases: cost allocation and budgetting, cost-volume-profit analysis, pricing, inventory management, and in larger organizations, decentralization.

There is no opportunity to review all these themes. In order not to fritter away our discussion on management accounting and operational research, we therefore make a third restriction by confining to cost allocation and budgetting through input-output analysis (section 2), cost-volume-profit analysis (section 3) and transfer prices in decentralized organizations (section 4).

At the end of this section three additional observations should be made.

1. In the most recent literature (cf. among others Kaplan [27]), management accounting embraces besides notions and techniques adopted from cost accounting, also notions like shadow prices of technical constraints, product demands on capacity resources, measures of physical capacities, back-logged orders, and so on. Understood in this way management accounting is no longer just a part of cost accounting, but besides a great overlap, both area's contain notions and techniques not belonging to the other one.
2. After the change in the contents of cost accounting, mentioned in the beginning of this section, the developments have not come to a standstill. Since about the late 1960's one has tried to coin a broader conceptual framework, the so-called information - economics approach [11, 13].

In this context information is a commodity that can be purchased and sold, so it is not correct to speak about the need of information without taking into account the costs of acquiring and producing it. Until now this approach, just as still newer developments [27, ch. 17], has not proven to be operationally useful in developing improved cost accounting systems; therefore we will use the somewhat older decision model approach as a starting-point for this survey.

An implicit limitation we make is to one criterion decision problems, so excluding multi-criteria decision problems, cf. [21].

3. We will not discuss the use in management accounting of statistical methods, of which Markovian models, estimation techniques and regression analysis have many applications in as different area's as selecting a portfolio of credit risks [9] and cost estimation [49, ch. 18].

2. Cost Allocation and Budgetting through Input-Output Analysis

2.1. Introduction

Input-output analysis has been developed in the early thirties by W. Leontief, in order to make operational the theories by Walras, Cassel and Pareto concerning general economic equilibrium in a national economy [35]. The same type of analysis can be carried out for a firm or for a non-profit organization e.g. [12, 44, 50, 56, 58]. At the level of a firm the input-output model embodies the developments from classical cost accounting to managerial accounting [22, 37]. The model can be used for making cost calculations and for planning and controlling activities.

The cost calculations include a.o. the determination of total and variable costs of intermediate and final products, the costs of services, drawing up the estimated profit and loss accounts and stating the consequences for the inputs of changes in output levels of final products. These calculations are especially important for firms with different departments, delivering goods or services to each other. As a second application the input-output analysis at the level of the firm gives the opportunity to trace the course of the costs through the firm. Based on this, the composition of unit costs according to types and quantities can be specified.

A third application which the input-output model as a specification of the (linear) relations between final and intermediate products offers, is in a planning context. The connection between cost calculations and production planning in the input-output model lead Kistner and Luhmer to speak about "Dualität von Produktionsplanung und Kostenrechnung" [30].

However, in classical input-output models, aspects as optimal using available production capacities, the existence of market restrictions with respect to the quantities of final products to be sold or substitution possibilities between inputs, do not come into consideration. If these aspects cannot be ignored the input-output model has to be extended with a criterion function and the relevant restrictions and data, usually giving rise to a linear optimization model, with which the optimal product-mix can be determined. As such the input-output model can be seen as a special case of a linear optimization model: without a criterion function being specified it has a unique solution, the elements of which are all non-negative.

Finally input-output models are a useful tool for analyzing differences between estimated and realized results of business activities, again especially in situations with several interdependent departments. Using the same model for cost calculations, planning and control creates the opportunity for an inte-

grated approach.

2.2. The Input-Output Table

Input-output analysis starts with splitting up a productive system into three kinds of sectors. The primary sectors provide the input, the final sectors absorb the output, whereas the intermediate sectors transform the input of the primary sectors into the output for the final sectors. Input-output analysis concentrates on specifying the relations within and between all these sectors, which can be done either on a nominal basis (values of goods and services delivered in a given period) or on a real basis (quantities of goods and services). As soon as these relations are presented in an input-output table, the input-output model can be formulated.

Inputs are primary if there is, during the period studied, no production phase within the firm, preceding them, like labour, raw materials from suppliers outside the firm and services of capital goods. Outputs are final if there is, during the period studied, no production phase within the firm, following them, like sales to customers outside the firm or mutations in inventory. The contents of the notion intermediate sector depends on the level of aggregation wanted. We take as intermediate sectors cost centres or departments, i.e. units in which all costs and revenues are gathered, related to a particular function in the transformation process. It is essential that every cost centre is assumed to produce just one product or service, using one specific technique. As usual in cost accounting we differentiate for the intermediate sectors between preceding cost centres, which do not produce final output, like central service departments, and final cost centres producing intermediate and final products. On the basis of the assumptions mentioned, table 2.1. presents the standard representation of an input-output table.

Table 2.1. Scheme of an input-output table.

from \ to		preceding cost centre	final cost centre	final outputs	row total
		1.....m	m+1..... m+n	1.....k	
preceding cost centre	1	x_{11}^a	x_{1m}^a	$x_{1,m+1}^c$ $x_{1,m+n}^c$	x_1^a
	m	x_{m1}^a x_{mm}^a	$x_{m,m+1}^c$ $x_{m,m+n}^c$		x_m^a
final cost centre	m+1	$x_{m+1,1}^b$ $x_{m+1,m}^b$	$x_{m+1,m+1}^d$ $x_{m+1,m+n}^d$	$y_{m+1,1}$ $y_{m+1,k}$	x_{m+1}^b
	m+n	$x_{m+n,1}^b$ $x_{m+n,m}^b$	$x_{m+n,m+1}^d$ $x_{m+n,m+n}^d$	$y_{m+n,1}$ $y_{m+n,k}$	x_{m+n}^b
primary inputs	1	v_{11}^a	v_{1m}^a	$v_{1,m+1}^b$ $v_{1,m+n}^b$	v_1
	p	v_{p1}^a v_{pm}^a	$v_{p,m+1}^b$ $v_{p,m+n}^b$		v_p
column total		x_1^a x_m^a	x_{m+1}^b x_{m+n}^b	y_1^c y_k^c	g

The quantities x_{ij} are intermediate transactions, giving the value of goods delivered by cost centre i to cost centre j during the period under consideration. In the same way y_{ij} represents the value of goods from final cost centre i to the final sector j and v_{ij} the value taken by cost centre j from the primary input i. The following relations hold

$$(2.1) \quad \sum_{j=1}^m x_{ij}^a + \sum_{j=m+1}^{m+n} x_{ij}^c = x_i^a \quad (i = 1, \dots, m)$$

$$(2.2) \quad \sum_{j=1}^m x_{ij}^b + \sum_{j=m+1}^{m+n} x_{ij}^d + \sum_{j=1}^k y_{ij} = x_i^b \quad (i = m+1, \dots, m+n).$$

The meaning of the other column and row totals is self-evident and of no importance for the rest of this contribution.

A scheme like table 2.1. specifies for every distinguished department the profit and loss account during the period studied. Every column presents the costs of a department and the corresponding row the returns. By introducing profit or loss as a primary input in the table, the row total will equal the

column total for every department. In this way the traditional double-entry bookkeeping system is transformed into a bookkeeping system with only one cell.

2.3. The Input-Output Model

In order to perform the applications of input-output models mentioned in section 2.1 it is assumed that there exists for every cost centre j a fixed ratio between input and output. So every input ($= x_{ij}$) is a linear homogeneous function of the output ($= x_j$)

$$(2.3) \quad x_{ij} = a_{ij} x_j$$

$$(2.4) \quad v_{ij} = b_{ij} x_j.$$

The a_{ij} are called the intermediate coefficients, the b_{ij} the primary input coefficients. Because for every unit the input equals the output, the coefficients a_{ij} and b_{ij} give also the production per unit input of cost centre j . Therefore it is correct to compute these coefficients by dividing the elements of the input-output table by the corresponding column totals, as is customary to do. In making ex ante calculations they are usually assumed to be independent of time, but there may be situations in which time dependency has to be introduced.

Combining (2.1), (2.2) and (2.3), (2.4) leads to

$$(2.5) \quad A^a X^a + A^c X^b = X^a$$

$$(2.6) \quad A^b X^a + A^d X^b + Y = X^b$$

with $X^a = (x_1^a, \dots, x_m^a)'$, $X^b = (x_{m+1}^b, \dots, x_{m+n}^b)'$ and Y the column vector of the final outputs, added over the rows. Traditional bookkeeping methods solve these simultaneous equations in a step by step procedure, where costs of preceding departments are charged to final cost centres, which in their turn roll off to the final outputs. The corresponding formulae can easily be derived. Because they are rather clumsy, we condense (2.5), (2.6) to

$$(2.7) \quad A X + Y^* = X$$

by introducing

$$A = \begin{pmatrix} A^a & A^c \\ A^b & A^d \end{pmatrix}, \quad X = \begin{pmatrix} x^a \\ x^b \end{pmatrix}, \quad Y^* = \begin{pmatrix} \sigma_m \\ y \end{pmatrix}.$$

Thus for given Y the vector of output levels of the cost centres equals

$$(2.8) \quad X = (I - A)^{-1} Y^*.$$

The matrix multiplier, $(I - A)^{-1}$, is called the inverse of the Leontief matrix. Conditions for the existence of this inverse are discussed in [31]. An economic interpretation has been given by Hawkins and Simon [17]. The sum of the elements of a column vector of this inverse represents the total production value which has to be generated in the cost centre in question and the cost centres supplying to it, in order to produce one unit of final output by this cost centre; apply (2.8), taking for Y the corresponding unit vector. For given output levels Y , the necessary primary input levels follow from

$$(2.9) \quad V = BX = B(I - A)^{-1} Y^*$$

with $V = (v_1, \dots, v_p)'$ and $B = (B^a \ B^b)$, the matrix of primary input coefficients. The coefficients of a column of $B(I - A)^{-1}$ specify the build-up of a unit of output into type and quantity of the primary inputs. They are named cumulated primary input coefficients. The sum of these coefficients added per column equals 1: for every dollar final output one needs one dollar for acquiring the necessary inputs. This property can also easily be verified mathematically.

The input-output model discussed so far is a static model for one period. For the dynamic version in which final outputs of a given period are primary inputs for the next period we refer to the literature, e.g. [31].

A derivation analogous to the preceding one can be given if the physical production specifications per unit output of the cost centres and the final output vector are given. Starting from an input-output table on a real basis, both the costs per unit produced as the sensitivity of cost prices if prices of primary inputs change, can be computed. Contrary to the nominal situation the distinction between preceding and final cost centres is now of direct importance because the input levels of preceding cost centres may now be more or less independent of the output levels of the departments they serve. An example are the costs of a service department such as management. In practice the re-

lations between supplying and receiving departments will then be based on allocation coefficients. This method is an input oriented approach as contrary to the earlier given output oriented approach [56, p. 159]. As preceding cost centres do not maintain relations with the consumer market of the firm, a standard has to be developed [27, ch. 11]. If the quantities of the final output levels and the corresponding input levels of final and preceding cost centres are determined, the real input-output table can be converted into a nominal table using known market prices of primary inputs and final outputs and agreed upon standards for preceding cost centres.

2.4. New Developments

As stated in section 2.2 the central assumption in classical input-output analysis runs: every centre produces just one product through a process which combines different factors of production in a unique or complementary way, not depending on the output level. Generally speaking the complementarity condition is the better fulfilled, the shorter the period, the analysis covers. Though there are mostly several production techniques available for a product, often there doesn't exist any choice in the short run, because the lifetime of investments in fixed assets usually determines the production technique in the short run. In some cases uniqueness of production functions can be found by an appropriate definition of cost centres.

More difficulties may arise from the assumption of constant ratio's between output and input, e.g. if a department delivering inputs has fixed costs, or if primary inputs are only to some extent dependent on the level of production. The existence of fixed next to variable costs doesn't mean that the input-output technique cannot be used any longer. However, it has consequences for the quality of the estimates. By adapting the model formulation to the reaction of inputs to changes in the level of output, a higher quality of the estimates can be reached. An important contribution to the yet scarce economic literature on generalizations of input-output models is by Steffens, who proved the existence of a unique non-negative solution for the case of piece-wise linear production functions and also developed a procedure to compute this solution [52]. For other contributions in this direction, cf. [7, 33, 47].

Input-output models in classical and related settings are frequently used at the level of the enterprise, especially in the chemical and in the iron and steel industry. Many applications are mentioned in [16 p. 187-191; 54].

3. Cost-Volume-Profit Analysis

3.1. Introduction

In section 2 we discussed input-output analysis as a tool for computing the costs of final products. Production structure (technology), production capacities and the product-mix are assumed to be given. The part of management accounting which links up with input-output analysis is cost-volume-profit analysis or C.V.P. analysis. Here production structure and production capacities are given but the product-mix has to be determined. In the conceptual most simple case C.V.P. analysis finds the minimal number of units to be produced and sold in order to cover total costs of production during a period, called the breakeven point. Revenues and costs are then assumed to be known for certain as functions of production volume. In the course of time C.V.P. analysis has developed into an important tool for profit planning and profit control, especially for determining the optimal product-mix. Mathematical programming methods play an important role in the analysis [5].

Because in practice mainly deterministic models are used, section 3.2 is devoted to them; some remarks on other problems will be presented in section 3.3.

3.2. The deterministic C.V.P. model

The general deterministic C.V.P. model is defined as follows. Suppose there are n products with known cost and revenue functions. The production capacities, which are assumed to be fixed by the long-range plan of the firm, impose constraints on the production, just as market conditions do for the amounts of goods that can be sold.

The optimization model for the period $[0, \tau]$ then runs:

maximize

$$(3.2.1) \quad z = \int_0^{\tau} w_t(X(t)) dt$$

subject to the constraints

$$(3.2.2) \quad x(t) \in \mathcal{X}(t), \quad t \in [0, \tau]$$

with

- z = profit in period $[0, \tau]$,
 n = number of different products,
 $x_j(t)$ = production rate of product j at time t ,
 $X(t)' = (x_1(t), \dots, x_n(t))$,
 $w_t(X(t))$ = profit of production $X(t)$ at time t ,
 $\mathcal{X}(t)$ = set of feasible productions at time t .

Let $r_t(X(t))$ be the revenue and $c_t(X(t))$ the cost of the production $X(t)$ at time t , then profit at time t equals

$$(3.2.3) \quad w_t(X(t)) = r_t(X(t)) - c_t(X(t)).$$

Depending on the conditions set to the variables, functions and sets in (3.2.1), (3.2.2) and (3.2.3), there emerge different optimization problems.

In most models the functions $r_t(\cdot)$ and $c_t(\cdot)$ and the $\mathcal{X}(t)$ are assumed to be time-independent for the planning period considered and then the indices t can be omitted. If we define the production x_j of product j in the planning period by

$$(3.2.4) \quad x_j = \int_0^{\tau} x_j(t) dt,$$

the profit planning problem can be reformulated as finding the quantities of production $X' = (x_1, \dots, x_n)$ that maximize the profit subject to conditions related to the whole period $[0, \tau]$. By stating the problem in this way the distribution of the production within the planning period is left out of consideration. The profit maximization problem is now completely time-independent and runs

maximize

$$(3.2.5) \quad z = w(X)$$

subject to

$$(3.2.6) \quad X \in \mathcal{X}$$

with $w(X)$ the profit during the planning period as a function of the whole production in that period and $\mathcal{X}$ the set of feasible production programs.

We continue by presenting some example of C.V.P. analysis, emphasizing the relation to the model (3.2.5), (3.2.6).

The central question in breakeven analysis is: how much should be produced and sold during a period in order that total revenues equal total costs. If there is one product with a linear cost function $cx + f$ and a fixed market price p , the profit is

$$(3.2.7) \quad w(x) = (p-c)x - f$$

with $(p-c)$ being the contribution margin per unit and f the costs which are independent of the level of production. The breakeven point x_b follows from

$$(3.2.8) \quad x_b = \frac{f}{p-c} ;$$

in this simple case the contribution margin for each unit equals the marginal contribution and also the average contribution per unit.

Mathematical programming techniques become useful if several different products can be produced and these products need the same production facilities. A simple situation arises if the profit function for every single product has the shape (3.2.7). Then

$$(3.2.9) \quad z = (P-C)'X - d$$

with X the vector of production levels, P the vector of market prices, C the vector of variable costs per unit and d total fixed costs. The conditions, the production levels should satisfy because of limited available capacities, can often be expressed by the linear constraints

$$(3.2.10) \quad AX \leq B$$

$$(3.2.11) \quad X \geq 0 .$$

B is the vector of available production capacities and $A = (A_1, \dots, A_n)$ the matrix of technological coefficients. In the situation where market constraints can be neglected, (3.2.10) and (3.2.11) form the specification of condition (3.2.6). Finding the breakeven point for this model brings down to solving the linear optimization model:

minimize

$$(3.2.12) \quad z$$

subject to (3.2.9), (3.2.10), (3.2.11) and $z \geq 0$. There are two cases in which there doesn't exist a solution to the problem:

- 1) the contribution margins in P-C are all negative,
- 2) there exist positive contribution margins but there is no solution with a non-negative value of z satisfying the constraints.

Both situations imply that the breakeven point doesn't exist.

However, in most planning situations it is not sufficient to find just the breakeven point. The more important question concerns the optimal product-mix, usually defined as the mix which maximizes profit. A simple and very often applied C.V.P. model to compute this mix is the linear optimization model: maximize (3.2.9) subject to the constraints (3.2.10) and (3.2.11).

The dual problem corresponding to this model is:

minimize

$$(3.2.13) \quad v = B'T$$

subject to

$$(3.2.14) \quad A'T \geq P - C$$

$$(3.2.15) \quad T \geq 0.$$

It is easy to prove that the optimal values $\bar{t}_i$ of t_i are equal to the marginal contributions of the available production capacities. The economic interpretation of the dual model is as follows. Find the prices ($= T$) per unit of capacity such that the total sacrificed productive value ($= B'T$) is minimal subject to the condition that the costs of a unit of product j , valued by these prices

($= A_j' T$) are at least equal to the marginal contribution of this product ($= p_j - c_j$). The total contribution of the optimal product-mix is then attributed to the production capacities in such a way that completely exhausted and thus scarce capacities get a non-negative valuation $\bar{t}_i$, whereas not fully used capacities get a valuation zero. This result based on economic arguments is in agreement with the complementary slackness condition.

The method of allocating profits, just mentioned, reflects the costing problem on the basis of the best alternative, called opportunity costing. Just as every marginal analysis the method has limited practical value because these opportunity costs hold only for the optimal product-mix, given the capacities B . Small changes in these capacities may imply great modifications in the values of the dual variables, e.g. because a scarce production capacity becomes not scarce or the other way around.

If the C.V.P. model is no longer a linear optimization model but still a convex optimization model, the given interpretation of the values of the dual variables remains valid [59, p. 62-68]. The mostly used model in this class has a concave object function because of diminishing returns, and linear capacity constraints.

In both the linear and the non linear model, during the planning period chosen and with fixed production capacities, the values of the dual variables must be used in valuating these capacities. We come back to these values, also called shadow prices, in section 4.2.2.

The general dynamic version of the C.V.P. model as given by (3.2.1) and (3.2.2) can be formulated as an optimal control problem with constraints on the decision variables, by imposing specific conditions on the sets $\mathcal{X}(t)$. The generalized Lagrange multipliers corresponding to these constraints preserve also in this context the interpretation of marginal valuations [3, p. 24-25]. Until now we did not introduce in the C.V.P. model constraints based on market conditions. If we incorporate these constraints into the model, the interpretation of the dual variables remains again the same. A severe limitation in this case is that market constraints are mostly not very hard, implying a very restricted significance of the interpretation as marginal contributions; cf. also section 3.3. and [14, 18].

So far we discussed the allocation of profit contributions to capacities through methods based on the product-mix model, (3.2.9), ..., (3.2.11) which means that only variable costs c_j are taken into consideration and that fixed costs

are neglected. Besides these models there also exist other C.V.P. models which include both variable and fixed costs. We briefly review two problems in this area.

The first problem is known as the fixed charge problem and concerns the specifications of the cost function $c_t(X(t))$. Let the model be time-independent, so $c_t(X(t)) = c(X)$. The fixed charge problem comes into being if besides the already mentioned (unavoidable) fixed costs d , there are costs v_j only if product j is really taken into production, e.g. switching and set-up costs or costs of maintaining an inventory and distribution system for the product. The associated cost function then runs

$$(3.2.16) \quad c(X) = C'X + a + d$$

with

$$(3.2.17) \quad a = \sum_{j=1}^n a_j$$

and

$$(3.2.18) \quad a_j = 0 \text{ if } x_j = 0 \\ = v_j \text{ if } x_j > 0.$$

By introducing binary variables δ_j and assuming the existence of a maximum production level m_j for product j , we get the following mixed integer linear optimization problem [40; 45; 46, ch. 12]:

maximize

$$(3.2.19) \quad z = (P-C)'X - \sum_{j=1}^n v_j \delta_j - d,$$

subject to the constraints (3.2.10), (3.2.11) and

$$(3.2.20) \quad x_j \leq m_j \delta_j \quad (j = 1, \dots, n)$$

$$(3.2.21) \quad \delta_j \in \{0, 1\} \quad (j = 1, \dots, n).$$

Problems of this type arise frequently in practice, often also with non linear profit functions. As long as the constraints are linear, efficient exact and heuristic solution algorithms are available [18].

The second problem concerns the allocation of joint costs, such as the costs of raw materials being used for different products. Indeed the models (3.2.9).. (3.2.11) or (3.2.19), ..., (3.2.21) determine the optimal product-mix but they do not answer the question how to distribute the joint costs over the products. This distribution is of importance for price setting of products if there is no market price, and for valuating inventories. One method to solve this problem, originally suggested by Weil [57], is using the values of the dual variables in the optimal solution of the product-mix model. A simple worked out example is in [27, p. 398-409]; cf. also [28, 60].

3.3. Some remarks on other problems

Besides deterministic C.V.P. models there have been developed stochastic C.V.P. models. The stochastic aspects may concern the cost and revenue parameters, the market variables and/or the technological coefficients. In the first instance the object function $\underline{w}(X)$ is a stochastic variable, otherwise the constraints are stochastic, either in the capacities or in the matrix of technological coefficients.

Stochastic object functions are usually treated by studying moments of $\underline{w}(X)$ [14, 24]. Stochastic constraints are replaced by probability statements; so the stochastic equivalent of the deterministic constraint

$$(3.2.22) \quad h_i(X) \leq b_i ,$$

runs

$$(3.2.23) \quad P[\underline{h}_i(X) \leq \underline{b}_i] \geq 1 - \epsilon_i .$$

Assuming normality of the stochastic variables in (3.2.23), it can be transposed into a convex constraint. For the most frequently applied C.V.P. model with linear constraints this transposition was published by v.d. Panne and Popp as early as 1963 [43]. A more general treatment of stochastic aspects in mathematical optimization can be found in [32].

Another way of studying stochastic C.V.P. models is by simulation; this has especially been done for evaluating distribution functions of profits [4, 36, 39].

So far in this section, we discussed volume decisions, assuming prices or at least price-demand functions to be given. This emphasis is in agreement with existing literature. Most firms are not so-called price setting firms and have therefore a small decision region between costs plus a reasonable profit margin and the market prices. Violating the borders of this region implies sharp changes in sales. Only for market leaders or monopolists who can set market prices the region may be less restricted [8, 48] .

From the preceding text it will be clear that the application of mathematical optimization models in C.V.P. analysis is mainly restricted to product-mix decisions and corresponding cost aspects. In this area there are a great many applications running from blending models in the process industry [55] to cutting programs in the meat industry. New developments may be expected in building allocation models for overhead and joint costs, especially by making the models more dynamic [23, 29, 48] and by including more stochastic aspects, using methods developed in decision theory [6, 23] . These improvements, which will make the models better suited for practical applications, will be supported by new developments in mathematical optimization methods, e.g. in the fields of stochastic and discrete optimization [34; 46, p. 501-504] .

4. Transfer Prices in Decentralized Organizations

4.1. Introduction

In the preceding sections we assumed the existence of just one decision centre. However, large enterprises produce many products, often at different locations. The organization and the corresponding decision structure are geared to this: the organization has a stratified structure and decisions are taken decentralized. Divisionalized enterprises are an example of this phenomenon. Under the central management more or less independent units (divisions) are grouped. Decentralization of decisions has as a consequence that decisions are taken at different levels, e.g. by central management and by local management teams. Central management decides about common resources which can be used for several divisions and so have to be allocated to them. Local managers decide about the use of the allocated production capacities and their own means, like the local building and local staff.

Besides these vertical relations between central management and divisions, there also may exist horizontal relations between divisions because the output of one division is the input of another division.

If decisions are decentralized the divisions will try to realize their own objectives (e.g. maximizing profit of the division) as far as they have freedom to do this. But these efforts must not come into conflict with the objective of the organization as a whole (e.g. maximizing profit of the organization). One way to reach agreement between the actions of more or less independent units and the organization as a whole is in the valuation of the common resources and of the intermediate goods. In this section we discuss this valuation, known as transfer pricing. We restrict ourselves to the static analysis for one period.

In agreement with the historical development of the theory, we first discuss transfer prices of intermediate products (section 4.2.1) and next transfer prices of common resources (section 4.2.2). Finally in section 4.2.3 we make some remarks about the similarity of both transfer prices. The setting of these prices influences the profits of the divisions. In section 4.3 comes into question how far these profits can be a basis for evaluating divisional performance.

4.2. The Transfer Price as a Tool for Decision Making

4.2.1. The Transfer Price for Intermediate Products

The classical contribution by Hirschleifer [19] concerns the values, the transfer prices for intermediate products should have. The prices of these products must be chosen in such a way that every division by maximizing its own profit, maximizes the profit of the whole enterprise.

For illustration we start, after Jennergren [26], from an enterprise with two divisions; division 1 produces an intermediate product for division 2 which may also be saleable on the market, division 2 produces a final product. All produced units are directly sold or delivered. Division 2 needs one unit of the intermediate product for one unit of the final product. As opposed to the models in section 2 we assume that the divisions do not deliver goods to themselves and in contradiction to section 3 we neglect fixed costs in the object functions to be maximized.

We define:

x_{10} = number of units of the intermediate product sold by division 1 to the market,

x_{12} = number of units of the intermediate product delivered by division 1 to division 2,

x_{02} = number of units of the intermediate product bought on the market by division 2,

x_{20} = number of units of the final product produced by division 2 (and sold to the market),

$c^{(i)}(.)$ = variable costs per unit of division i ,

p_j = market price per unit of product j ,

z = transfer price per unit delivered by division 1 to division 2.

The model for the enterprise as a whole runs

maximize

$$(4.1) \quad \{p_1 x_{10} + p_2 x_{20} - c^{(1)}(x_{10} + x_{12}) - c^{(2)}(x_{20}) - p_1 x_{02}\}$$

as a function of x_{10} , x_{20} , x_{12} and x_{02} , subject to

$$(4.2) \quad x_{12} + x_{02} = x_{20} ; x_{10}, x_{12}, x_{02}, x_{20} \geq 0.$$

In a divisionalized enterprise every division has its own model and as x_{12} implies different decision variables for the two divisions we differentiate between:

x_{12}^d = number of units of the intermediate product to deliver by
division 1 to division 2,

and

x_{12}^r = number of units of the intermediate product to receive by
division 2 from division 1.

The model for division 1 is

maximize

$$(4.3) \quad \{p_1 x_{10} + r x_{12}^d - c^{(1)}(x_{10} + x_{12}^d)\}$$

as a function of x_{10} and x_{12}^d , subject to

$$(4.4) \quad x_{10}, x_{12}^d \geq 0 ;$$

for division 2 the model is

maximize

$$(4.5) \quad \{p_2 x_{20} - c^{(2)}(x_{20}) - r x_{12}^r - p_1 x_{02}\}$$

as a function of x_{20} , x_{12}^r and x_{02} , subject to

$$(4.6) \quad x_{12}^r + x_{02} = x_{20} ; x_{12}^r, x_{02}, x_{20} \geq 0.$$

The transfer price r has now to be determined in such a manner that every division can maximize profit within its own environment, whereas the following conditions must be fulfilled:

- (i) the enterprise as a whole maximizes profit,
- (ii) there is equilibrium between deliveries and receipts of the intermediate product, so $x_{12}^d = x_{12}^r = x_{12}$.

We distinguish between two situations:

- a) there is no market for the intermediate product,
- b) there is a market for the intermediate product.

a) In this case $x_{10} = x_{02} = 0$. The enterprise reaches maximum profit if

$$(4.7) \quad p_2 = c^{(1)'}(x_{12}) + c^{(2)'}(x_{20}),$$

which means that production volume is put at the level where the market price of the final product equals the sum of the marginal costs. This result can also be derived by the separate divisions. Maximizing profit, division 1 concludes $r = c^{(1)'}(x_{12}^d)$: it fixes the transfer price equal to the marginal costs and passes the curve $r = c^{(1)'}(x_{12}^d)$ on to division 2. As $x_{12}^d = x_{12}^r$, it holds $c^{(1)'}(x_{12}^d) = c^{(1)'}(x_{12}^r)$. By maximizing its profit division 2 then determines the production volume x_{20} at the point with

$$(4.8) \quad p_2 = r + c^{(2)'}(x_{20}) = c^{(1)'}(x_{12}^r) + c^{(2)'}(x_{20}).$$

So one finds x_{12}^r and through $x_{12}^d = x_{12}^r$ also the production level of division 1.

b) The optimization model for the whole enterprise leads to

$$(4.9) \quad p_1 = c^{(1)'}(x_{10} + x_{12})$$

$$(4.10) \quad p_2 = c^{(1)'}(x_{10} + x_{12}) + c^{(2)'}(x_{20})$$

with $c^{(1)'}(.)$ being the partial derivative to x_{10} .

By substitution we get

$$(4.11) \quad p_2 = p_1 + c^{(2)'}(x_{20}).$$

Maximizing per division results for division 1 in $r = p_1$ and the conclusion that division 1 can determine its optimal production volume without information by division 2. The distribution of the units over delivering to division 2 and selling to the market depends on division 2. From the model (4.5), (4.6) it follows that division 2 determines its production level with equation (4.11);

so x_{12}^r is known and also $x_{12}^d = x_{12}^r$.

From the analysis presented it follows that if there is no market the transfer price should equal the marginal costs of the division delivering the intermediate product. If the intermediate product has a market, then the transfer price must equal the market price.

4.2.2. The Transfer Price for Joint Capacities

If central management of a divisionalized enterprise has at its disposal central means needed for the activities of the divisions, like financial funds, storehouses or research and development capacity, a mechanism has to be constructed that distributes these means among the divisions. This allocation can be based on transfer prices: central management determines prices at which the divisions can obtain these means.

As an illustration we present a simple example, based on Baumol and Fabian [1]. An enterprise has two divisions both producing two products, viz. 1 and 2, and, 3 and 4, respectively. The central management has limited capacity of one production factor, needed by both divisions. The divisions have their own limited production capacities, marked in figure 4.1 by $s_1, \dots, s_5$. The shaded area's denote the feasible programs as far as only the capacities of the divisions themselves are concerned. The quotient of the contributions per unit

Fig. 4.1

Feasible production programs of the divisions 1 and 2 and the production programs in the successive iterations.

of the products 1 and 2 is represented by the tangent of α_1 and that of the products 3 and 4 by the tangent of α_4 . If the divisions maximize their profits, not taking into account the limited joint capacity, the optimal programs are represented by B and D. The divisions communicate the required amounts of the joint capacity to the central management. Let us assume that this management draws the conclusion that more capacity is asked for than available. It then determines prices which the divisions have to pay per unit of this capacity. As a consequence the quotient of the contributions of the products 1 and 2 changes into the tangent of α_2 , whereas this quotient remains the same for the products 3 and 4. The resulting optimal production program for division 1 is now A; for division 2 it remains D. It now turns out that the joint capacity is not fully exhausted, which means that central management has asked too high a price for this capacity; so it will lower it ($\text{tg } \alpha_3$). Because the optimal production program for division 2 is in the first just as in the last situation represented by point D, the optimal program will also be in D if the joint capacity is taken into account. For division 1 it is intuitively clear that the optimal program will be a convex combination of the programs A and B, a property which also can be proved mathematically. This convex combination cannot be determined by prices alone, but one needs in addition an allocation of the capacity of the scarce factor to the divisions. In this simple case the solution is easy because the optimal production program for division 2 is known and as a consequence the remaining capacity for division 1; the dotted line in fig. 4.1 represents this new constraint. The optimal production program for division 1 now corresponds with point C.

A more general formulation of the allocation problem for an enterprise with two divisions and full information at the highest level, is as follows. Let

- A_i, B_i = matrices of technological coefficients of division i ,
 A, B_1, B_2 = vectors of available production capacities,
 X_i = the vector of production levels of division i ,
 P_i = the vector of market prices of division i ,
 C_i = the vector of variable production costs of divisions i ,

then the model may be:

maximize

$$z = (P_1 - C_1)'X_1 + (P_2 - C_2)'X_2$$

subject to

$$a_1 X_1 + a_2 X_2 \leq A$$

$$b_1 X_1 \leq B_1$$

$$b_2 X_2 \leq B_2$$

$$X_1, X_2 \geq 0.$$

In the optimal solution of this model the values of the central production capacities are per unit equal to the shadow prices which belong to the corresponding constraints (cf. section 3.2). The model has all characteristics for solving it by a decomposition method, either with a primal method [10], or with a dual method [2].

Also in the case in which not all information is available at all levels, decomposition methods can be important. Suppose a_i, b_i, B_i, P_i and C_i are only known to division i , whereas the central management just knows A . In the decomposition method by Dantzig and Wolfe the optimal solution is determined iteratively by a pricing system for the joint production capacities, as we roughly sketched for the Baumol-Fabian model. For given transfer prices the divisions send their needs and the corresponding profits to central management, which in its turn determines with a linear optimization model the new values of the transfer prices (shadow prices) of the central capacities. This iteration process is repeated until supply and demand for the central capacities are in equilibrium. In this state both the prices and the amounts of the capacities to be allocated to the divisions are established. The allocated capacities impose new constraints on the divisions, which now can compute their final production programs. So the iteration procedure of a primal decomposition method can be viewed as the mathematical translation of the process of negotiation in a stratified planning system, using a pricing approach to find the equilibrium [41, p. 71 ff].

The iteration procedure of the dual decomposition method is the dual process

of negotiation using a quantity or budget approach instead of a pricing approach [41, p. 81 ff].

4.2.3. Similarities in Transfer Prices

The transfer price is either equal to the market price or to the marginal production costs, as in section 4.2.1, or equal to the shadow price, as in section 4.2.2. All three identifications can be traced back to the same underlying notion: the opportunity costs or the costs of the best alternative. If it is also possible to sell an intermediate product on the market the receiving department has to pay the market price; in this case the proceeds on the market are the alternative returns. If there is no market, the supplying department has the alternative not to produce the last unit and therefore the price of this unit equals marginal production costs. The third identification, the shadow price of common capacities, is based on the value of the capacity if it is optimally used in another department.

The conclusion is that modifications in scarcity influence the transfer price, e.g. the transfer price of intermediate products passes from the market price into the marginal costs if the external market stops buying the product.

4.3. Allocating Profit

In decentralized enterprises divisions are often evaluated on the basis of realized profits and additional rewards are coupled to them. But in determining profits of divisions transfer prices play a very important role. Using the values derived in section 4.2 has consequences, which are in many cases not accepted in the process of allocating additional rewards. Two illustrations: if transfer prices are equal to marginal costs, in most cases fixed costs are not covered, so that the supplying division suffers losses and if transfer prices are shadow prices, then all profits are only imputed to scarce production means. "This conflict between decision making and evaluation of performance is the essence of the transfer pricing conundrum" [27, p. 483].

This is the reason that, though marginal cost prices may be the theoretically correct values of transfer prices for decision making, in practice these values are seldom used. Besides allocation mechanisms for rewards based on realized profits of divisions, also other methods are applied, mainly based on profits obtained by the whole enterprise. In the literature much attention is paid to the Russian method based on planned and realized profits [38] and to a

method suggested by Groves [15] . In this last system of remuneration, rewards are a function of realized profit of the own division and planned profits of the other divisions. It gives a good incentive to maximize own profit and to make a good planning in the allocation of limited joint means [25] .

4.4. Conclusions

Recent literature on managerial economics and decision theory pays much attention to transfer prices and to delegation mechanisms.

In the theory the starting point is that in an enterprise that tries to maximize profit, transfer prices have to be chosen in such a way that the divisions serve the main objective. As shown in section 4.2 transfer prices meet that goal if they are equal to the opportunity costs. Problems arise in allocating the rewards. Whether the theoretical conclusions are applied in practice depends on the existence of an external market.

If there is an external market for an intermediate product the transfer price is also in practice usually taken equal to the market price and the supplying department is a final cost centre with final outputs in the sense of section 2.2. The allocation of rewards can then be based on realized profits provided that charging the costs of intermediate cost centres is correctly regulated. If there is no market price, according to the theory the transfer price should be equal to the marginal costs of the supplying department or equal to the shadow price. The allocation of rewards to preceding cost centres can be based on the scheme of Groves. However, in practice both methods are not applied. That's why still much research has to be done in order to develop good rules. Besides delegation through prices also delegation through budgets should be studied [41] .

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