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**SCALE RETURNS OF A RANDOM MATCHING
MODEL**

By Klaus Kultti

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Scale returns of a random matching model

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Abstract

This note considers a random matching model in which the meeting probabilities can be derived from the basics of the model. We determine conditions for the matching technology to exhibit decreasing, constant, and increasing returns to scale.

Keywords: Random matching, scale returns

JEL classification: C78, J41

1. Introduction

Random matching and search models are popular ways to model the economy when Walrasian paradigm of complete markets is too restrictive. There are a multitude of applications ranging from job search and labour markets (eg. Mortensen, 1986), price formation in markets (eg. Rubinstein and Wolinsky, 1985; Wolinsky, 1988; Gale, 1987), endogenous money (eg. Kiyotaki and Wright, 1993; Burdett et al., 1995), to stability analysis of various trading institutions (Lu and McAfee, 1996). Typically it is postulated that there are two types of agents in the economy, and that they are pairwise matched. A recurrent theme with the matching models is speculation about the matching technology. Results are often sensitive to whether the technology exhibits decreasing, constant, or increasing returns to scale; for instance, the results in Mac Namara and Collins (1990), and Bloch and Ryder (1997) hinge on a technology with increasing returns to scale. In this case the decisions to search or to get matched are complementary. However, one can easily think of situations in which increasing the number of agents who search results in congestion that reduces the number of matches.

Since usually the environment where the agents operate, and consequently the manner matches come about, is not specified the modeller is free to choose the technology he wishes. Even though the actual process of matching is often left unspecified it is common to offer a verbal description about the process. For instance, Burdett et al. (1995) who study the decision to wait or search give two ways to picture the origin of matching probabilities. First it can be thought that agents who wait are in fixed locations, and searchers visit these locations. Second, one may imagine that the agents are colliding

to each other like particles, and collisions indicate matches. If one is to model the matching technologies along either of these lines it is not immediately clear what kind of scale returns one should expect.

The aim of this note is to present a well known and well specified random matching model (see Lu and McAfee, 1996) and show that it exhibits constant returns to scale. We also present a variation of the model, and determine the conditions under which it exhibits decreasing, constant, and increasing returns to scale. The matching model itself is attractive since it is amenable to applications, and because the meetings are not restricted to be pairwise; an agent can meet any number of other agents. In applications that focus on price determination this means that both bargaining and auctions can be used.

2. The model

In the basic set-up there are B buyers and S sellers. The sellers are in fixed locations, and the buyers are randomly distributed on the sellers. Consequently, the number of buyers any particular seller expects to meet is binomially distributed with parameters B and $1/S$. Since binomial distributions are awkward to deal with we assume that B and S are large numbers, and approximate the binomial with a Poisson distribution with a parameter $\theta = \frac{B}{S}$. The approximation is exact if the numbers approach infinity while their ratio stays fixed.

To determine the number of matches that result from this matching technology it is enough to focus on one type of agents only, since a match is by definition a pairing of two agents. We study the sellers, and assume that the agents only objective is to get paired with an agent of the opposite type. The probability that a seller meets k buyers is $e^{-\theta} \frac{\theta^k}{k!}$, and as long as a seller meets any buyers a match comes about. We assume that if two or more buyers are distributed on any particular seller the seller chooses any one of them with equal probabilities. The probability that the seller is matched is then $1 - e^{-\theta}$, i.e. one minus the probability of not meeting any buyers. As there are S sellers the expected number of matches is $S(1 - e^{-\theta})$.

Let f be the function that determines the expected number of matches $f: R_+ \times R_+ \rightarrow R_+$ such that $(S, B) \mapsto S(1 - e^{-\theta})$. The following result follows almost immediately.

Claim 1. Function f is homogenous of degree one.

Proof. Consider $\alpha > 0$. $f(\alpha S, \alpha B) = \alpha S \left(1 - e^{-\frac{\alpha B}{\alpha S}} \right) = \alpha S (1 - e^{-\theta}) = \alpha f(S, B)$ ■

Thus, the matching technology exhibits constant returns to scale. The sellers and buyers are treated asymmetrically in this set-up, and it is not immediately clear whether agents would prefer to stay in fixed positions or to be distributed on the positions. Next we consider a slightly more complicated structure in which buyers and sellers can decide whether to stay or search. It is probably useful to think that there are two separate markets or locations; in the sellers' market sellers are in fixed positions and buyers are distributed on them, and in the buyers' market buyers are in fixed locations and sellers are distributed on them. In equilibrium both types of agents have to be indifferent between the markets.

We need to calculate the probability that a buyer is matched in the market where they are distributed on waiting sellers. A buyer always meets a seller for certain because buyers are distributed on the sellers. Thus, the probability that he is the only buyer is the same as the probability that no other buyers arrive at the same location, which is the same as the probability of a seller meeting no buyers, i.e. $e^{-\theta}$. If k other buyers arrive at the location, an event that takes place with probability $e^{-\theta} \frac{\theta^k}{k!}$, any buyer is selected by the seller with probability $\frac{1}{k+1}$. Thus, a buyer is matched with probability

$$e^{-\theta} \left(1 + \frac{1}{2} \theta + \frac{1}{3} \frac{\theta^2}{2!} + \frac{1}{4} \frac{\theta^3}{3!} + \dots \right) = \frac{e^{-\theta}}{\theta} \sum_{i=1}^{\infty} \frac{\theta^i}{i!} = \frac{e^{-\theta}}{\theta} (e^{\theta} - 1) = \frac{1 - e^{-\theta}}{\theta} \quad (1)$$

From (1) we immediately see that when there are more buyers than sellers the buyers are better-off if they are in fixed positions and wait, and if there are less buyers than sellers the buyers prefer to be distributed on sellers.

In equilibrium proportion x of buyers and proportion y of sellers are in the sellers' market. The rest of the agents are in the buyers' market. Let us denote the rate in the sellers' market by $\phi = \frac{x B}{y S} = \frac{x}{y} \theta$ and the rate in the buyers' market by

$$\varphi = \frac{(1-y)S}{(1-x)B} = \frac{(1-y)}{(1-x)\theta}. \text{ The buyers are indifferent between markets if}$$

$1 - e^{-\phi} = \frac{1 - e^{-\theta}}{\phi}$, and the sellers are indifferent if $1 - e^{-\theta} = \frac{1 - e^{-\phi}}{\phi}$. The equations are valid only if $\phi = \frac{1}{\theta}$ which is equivalent to $x = y$. This means that $\phi = \theta$ and $\phi = \frac{1}{\theta}$.

Inserting this to either equilibrium condition yields $1 - e^{-\theta} - \theta + \theta e^{-\frac{1}{\theta}} = 0$. This equation has two solutions $\theta_1 = 1$ and $\theta_0 \in (0, 1)$. If the proportion of buyers to sellers happens to be either one of these, then there is an infinite number of equilibria with $x = y$. Otherwise both markets cannot exist simultaneously in equilibrium. Exactly as above we get the following result.

Claim2. When there are two active markets in equilibrium the matching technology exhibits constant returns to scale.

Two active markets can exist only for a set of parameters that is of measure zero. Thus, we ignore this case in the sequel. Either market by itself constitutes an equilibrium in the sense that no agent finds it profitable to go to the other market where he would be alone. As we are not that interested in equilibrium selection, we shall consider the variant where sellers are in fixed locations.

Unlike in many random matching models the buyers always meet a seller; they do not always end up in a match though. One may want to consider a situation where both buyers and sellers may end up without meeting anybody. The obvious way to accomplish this is to postulate that there may be more locations than there are sellers. If the buyers are distributed on the locations then some of them may end up in an empty location. Let us denote the number of locations by $L = L(S, B) \geq S$. Notice that since the important parameters of this model are the numbers of buyers and sellers we allow for the possibility that the number of locations depends on the number of both of them. However, not all cases about the number of locations turn out to be easy to interpret economically.

We denote the rate of the Poisson distribution by $\omega = \frac{B}{L}$. The expected number of matches is $f(S, B) = S(1 - e^{-\omega})$, and for $\alpha > 0$ $f(\alpha S, \alpha B) = \alpha S \left(1 - e^{-\frac{\alpha B}{L(\alpha S, \alpha B)}} \right)$. Now the matching technology exhibits constant returns to scale if $\alpha S \left(1 - e^{-\frac{\alpha B}{L(\alpha S, \alpha B)}} \right) = \alpha S (1 - e^{-\omega})$ which is equivalent to $\frac{\alpha}{L(\alpha S, \alpha B)} = \frac{1}{L(S, B)}$ or $L(\alpha S, \alpha B) = \alpha L(S, B)$. But this is the

condition for L to be a function homogenous of degree one. Similarly, the matching technology exhibits decreasing returns to scale if $\alpha S \left(1 - e^{-\frac{\alpha B}{L(\alpha S, \alpha B)}} \right) < \alpha S (1 - e^{-\omega})$ which is equivalent to $L(\alpha S, \alpha B) > \alpha L(S, B)$. Increasing returns to scale requires $L(\alpha S, \alpha B) < \alpha L(S, B)$. This proves

Claim3. Let $L(S, B) \geq S$ be the number of locations. The matching technology has decreasing, constant, or increasing returns to scale if function L is homogenous of degree greater than one, one, or less than one, respectively.

The simplest choice in this framework is to equal the number of locations and the number of sellers so that the matching technology has constant returns to scale. If something else is wanted one has to say something about the function L . Both decreasing and increasing returns to scale are formally equally easy to implement, but we find only for the latter case an economically interesting interpretation.

Assume that L is constant, and the waiters in our model are firms, and the searchers are workers. Assume further that L is the total number of firms while S is the number of vacancies and B is the number of unemployed workers. If workers search firms, i.e. are distributed on firms, rather than vacancies then the matching technology exhibits increasing returns to scale. Of course, this happens only for a certain range of values since there cannot be more vacancies than there are firms. The job search interpretation seems to be in agreement with the observation that when both the number of vacancies and unemployed increase there are more matches, i.e. it is easier to find a job or an employee.

Motivating L to exhibit increasing returns to scale, and the matching technology decreasing returns to scale, requires a story for the number of locations to increase proportionately more than the number of agents. We do not know any plausible ones.

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