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**CORPORATE TAX RATE POLICY AND
PUBLIC AND PRIVATE EMPLOYMENT**

by Raymond Gradus and
Aart de Zeeuw

February, 1989

CORPORATE TAX RATE POLICY AND PUBLIC AND PRIVATE EMPLOYMENT ^{*)}

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ABSTRACT

High tax revenues give the government the opportunity to create public employment. However, if the government tries to increase total tax revenues by increasing the corporate tax rate, two negative effects are invoked. Investment decreases, so that generally future tax revenues and private employment decrease. With the purpose to analyse this trade-off a dynamic game between the government and a representative firm is formulated. The government's objective is maximal total employment and the government's instrument is a corporate tax rate policy. The firm's objective is maximal total dividends and the firm's instrument is an investment rate policy. In general it is optimal for the government to start with a low corporate tax rate and to end with a high corporate tax rate. However, the switching time depends on the credibility and reputation of the government. If the government is committed to an announced policy, even if this announced policy becomes suboptimal over time, and if the firm is expected to believe so, the open-loop Stackelberg outcome of the game results. If the government is not committed to an announced policy and if the firm expects rational behaviour of the government at all times, the feedback Stackelberg outcome results. It is shown that in the open-loop outcome the switch in policy occurs later and the results are better for both the government and the firm than in the feedback outcome. Finally, the sensitivity of this switching time with respect to the capital/labour intensiveness is investigated.

Key words: optimal dynamic taxation, employment policy, theory of the firm, Lancaster differential game

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1. Introduction

In many countries nowadays unemployment is considered to be one of the most severe economic problems. The governments have to make a choice, among other things, to rely on the private sector to create employment or to create employment in the public sector. The second possibility is directly successful but has to be financed by money creation or taxation. Since money creation has its limits (Barro, 1974), the government's problem focusses on the question whether or not the creation of extra public employment with high taxation is a good employment policy. For a given level of net wages in the private sector high taxation implies less investment possibilities for the firms and that might imply less employment in the private sector and less future tax revenues.

This paper intends to formalise and solve the basics of this problem. A dynamic game model is set up with the government and a representative firm as players. The government wants to maximise total employment, which is the sum of private and public employment, over some planning period. The firm wants to maximise the total stream of dividends over that same period. It is assumed that prices and wages are fixed and that the firm can sell what it wants and can attract the profit maximising amount of labour at each point of time. It follows that the crucial decision the firm has to make concerns the division of after-tax profits over investment and dividend (Lesourne, 1976; van Loon, 1982). Investment leads to a growth of the capital stock with more profits in the future. The instrument of the government in this model is the corporate tax rate.

The mathematical structure of the model proves to be very similar to the mathematical structure of the Lancaster (1973) model of capitalism. Because it can reasonably be assumed that the government has to decide on the tax policy before the firm makes decisions, the relevant solution concept for the game is the Stackelberg solution concept. The open-loop Stackelberg solution is presented before in the literature for the Lancaster game (Pohjola, 1983), but this result is not fully correct. The correct solution will be derived in this paper and will be used to find the optimal employment policy. As usual the open-loop Stackelberg solution displays

time-inconsistency (Kydlund and Prescott, 1977). A requirement of strong time-consistency leads to the feedback Stackelberg solution, which will be shown to be equal to the open-loop Nash solution of this game. Finally, it will be shown that only in some special cases the open-loop Stackelberg solution for this game is efficient.

In general the solution of the dynamic game leads to a period with a low corporate tax rate and a high investment rate, and a subsequent period with a high corporate tax rate and a low investment rate. Both players are willing to be modest for a while in order to accumulate capital stock, which is beneficial for both of them. Under the requirement of time-consistency the policy switch occurs earlier with lower total employment and lower total dividends. This is the correct model, when the government can not commit itself to an announced policy and when the firm expects rational behaviour of the government at all times. The time-consistent outcome is never efficient and the time-inconsistent outcome is only efficient, when the production technology is labour intensive or when the tax rate cannot become too low. Furthermore, it is shown that the policy switch occurs later for either a very labour-intensive or a very capital-intensive production technology.

The paper is organised as follows. Section 2 formulates the differential game between government and firm. Section 3 derives the relevant solutions for this differential game. In section 4 the different outcomes are compared and interpreted. In section 5 the effect of variations in the indicator of capital/labour intensiveness is analysed. Section 6 concludes the paper.

2. The model

Suppose that the representative firm is operating under a constant-returns-to-scale production technology of the Cobb-Douglas type

$$Q = K^{\alpha} L^{1-\alpha}, \quad 0 < \alpha < 1, \quad (1)$$

where Q denotes production, K denotes capital stock and L denotes labour in the private sector. It is assumed that the firm is not constrained on both

output and labour markets. This implies that unemployment corresponds to classical unemployment in the sense of Malinvaud (1977). Furthermore, suppose that the real wage $\frac{w}{p}$, $0 < \frac{w}{p} < 1$, is fixed and take for simplicity $p=1$. The assumption of a fixed real wage can be sustained by the theory of implicit contracts or efficiency wages (Stiglitz, 1986) or by trade union behaviour (Van der Ploeg, 1987). The maximisation of profit

$$\Pi = Q - wL \quad (2)$$

leads to the well-known condition that the marginal productivity of labour equals the real wage, which implies that labour is a linear function of the capital stock:

$$L = ((1-\alpha)\frac{1}{w})^{\frac{1}{\alpha}} K. \quad (3)$$

Substitution of (1) and then (3) into (2) gives profit as a linear function of the capital stock:

$$\Pi = \beta K, \quad (4)$$

where the rentability of the capital stock β is given by

$$\beta = \alpha((1-\alpha)\frac{1}{w})^{\frac{1}{\alpha}-1}. \quad (5)$$

Investment I accounts for the growth of the capital stock

$$\dot{K}(t) = I(t). \quad (6)$$

Without affecting the basic results of the paper it is assumed that there is no depreciation. Investment can only be financed by retained earnings. The firm has to decide on the division of after-tax profit $\Pi - TX$, where TX denotes total taxation, over dividend D and investment:

$$\Pi - TX = D + I. \quad (7)$$

The firm's objective is to maximise the total stream of dividends over a planning period $[0, T]$

$$\int_0^T D(t) dt \quad (8)$$

and the firm's instrument is the investment rate i , which is defined as

$$i = \frac{I}{\Pi - TX}, \quad 0 \leq i \leq 1. \quad (9)$$

Since the labour input L does not show up dynamically in the optimisation problem, it is correct to first maximise profit with respect to L and then maximise (8) (see e.g. Feichtinger and Hartl, 1986).

It is assumed that the government can use tax income TX to create public employment for the same wage w as in the private sector. Under the assumption of Ricardian debt neutrality it does not make any difference, whether the government is given the opportunity to borrow money or not. The government's objective is to maximise total employment over a planning period $[0, T]$

$$\int_0^T (L(t) + \frac{TX(t)}{w}) dt \quad (10)$$

and the government's instrument is the corporate tax rate τ , which is defined as

$$\tau = \frac{TX}{\Pi}, \quad 0 < \tau_1 \leq \tau \leq \tau_2 < 1, \quad \tau_1 \neq \tau_2, \quad (11)$$

where τ_1 and τ_2 are the minimal and the maximal tax rate, respectively. It seems reasonable to assume that there are always some taxes and that profits are never taxed away completely. Substitution of (7), (9), (11) and then (4) into (8) leads to the following behavioural model for the firm:

$$\underset{i(.)}{\text{maximise}} \int_0^T (1-\tau(t))(1-i(t))\beta K(t) dt, \quad 0 \leq i(.) \leq 1. \quad (12)$$

Substitution of (5) into (3) leads to $L = \frac{(1-\alpha)\beta}{\alpha w} K$ and substitution of this result, (11) and then (4) into (10) leads to the following behavioural model for the government:

$$\underset{\tau(.)}{\text{maximise}} \int_0^T \left(\frac{1-\alpha}{\alpha} + \tau(t)\right) \frac{\beta}{w} K(t) dt, \quad 0 < \tau_1 \leq \tau(.) \leq \tau_2 < 1. \quad (13)$$

Substitution of (9), (11) and then (4) into (6) leads to the dynamic constraint

$$\dot{K} = (1-\tau(t))i(t)\beta K(t). \quad (14)$$

The strategic dynamic interaction between the government and the firm is described by the differential game (12)-(14). This differential game is in structure very similar to the Lancaster (1973) game of capitalism, which was further investigated by Hoel (1978), Pohjola (1983) and Başar, Haurie and Ricci (1985). The government plays the role of the workers and the firm plays the role of the capitalists. In the next section several relevant equilibria for the differential game (12)-(14) will be derived.

3. Strategic equilibria

In this section game equilibria for the differential game (12)-(14) are derived. It is essential to establish first, whether the mood of play is cooperative or non-cooperative, whether the players act simultaneously or sequentially, whether the players can commit themselves or not and what information is available to the players (see Başar and Olsder, 1982). It is reasonable to assume here, that the game is non-cooperative and that the government chooses the corporate tax rate policy before the firm chooses the investment rate policy. Therefore the Stackelberg solution concept seems appropriate. The "open-loop" Stackelberg solution requires that the players commit themselves from the beginning to a strategy for the whole period and

assumes that the players have no information on the "state" of the system, which is in this game the level of the capital stock. This solution displays time-inconsistency (see Kydland and Prescott, 1977), which means that the government will have an incentive to deviate from the announced policy at some later point in time. The assumption that each player in principle reconsiders its strategy at each point in time requires that the players are not committed to an announced strategy and that they have information on the state of the system (this is sometimes referred to as the requirement of "strong time-consistency"; see Başar, 1989). The requirement leads to the "feedback" Stackelberg solution (see Simaan and Cruz, 1973). As will be shown below, the feedback Stackelberg solution of the differential game (12)-(14) coincides with both the open-loop and the feedback Nash solution. For reasons of exposition the open-loop Nash solution will be presented first. Finally, the efficient or Pareto optimal equilibria will be derived. In this section it will be assumed that the maximal corporate tax rate τ_2 is at least $\frac{1}{2}$.

Proposition 1

The open-loop Nash solution for the differential game (12)-(14) is given by

$$\begin{aligned}\tau(t) &= \tau_1, \quad i(t) = 1, \quad \text{for } t \in [0, t^N) \\ \tau(t) &= \tau_2, \quad i(t) = 0, \quad \text{for } t \in (t^N, T],\end{aligned}\tag{15}$$

where $t^N = T - \frac{1}{\beta(1-\tau_2)}$.

Proof

The open-loop Nash solution results from the application of Pontryagin's maximum principle for each player and the Nash equilibrium concept.

The Hamiltonian function for the maximisation problem of the government is given by

$$H_G(K, \tau, p_G, t) = \left\{ \left(\frac{1-\alpha}{\alpha} + \tau \right) \frac{1}{w} + p_G(1-\tau)i(t) \right\} \beta K.\tag{16}$$

It follows that the government's rational reaction to the strategy of the firm is given by

$$\begin{aligned}\tau(t) &= \tau_1, \text{ if } \frac{1}{w} - p_G(t)i(t) < 0 \\ \tau(t) &= \tau_2, \text{ if } \frac{1}{w} - p_G(t)i(t) > 0\end{aligned}\quad (17)$$

and that the costate p_G is given by the adjoint system

$$\begin{aligned}\dot{p}_G(t) &= -\beta\left\{\left(\frac{1-\alpha}{\alpha} + \tau(t)\right)\frac{1}{w} + p_G(t)(1-\tau(t))i(t)\right\} \\ p_G(T) &= 0.\end{aligned}\quad (18)$$

The Hamiltonian function for the maximisation problem of the firm is given by

$$H_F(K, i, p_F, t) = \{(1-\tau(t))(1-i) + p_F(1-\tau(t))i\}\beta K. \quad (19)$$

It follows that the firm's rational reaction to the strategy of the government is given by

$$\begin{aligned}i(t) &= 0, \text{ if } p_F(t) - 1 < 0 \\ i(t) &= 1, \text{ if } p_F(t) - 1 > 0\end{aligned}\quad (20)$$

and that the costate p_F is given by the adjoint system

$$\begin{aligned}\dot{p}_F(t) &= -\beta\{(1-\tau(t))(1-i(t)) + p_F(t)(1-\tau(t))i(t)\} \\ p_F(T) &= 0.\end{aligned}\quad (21)$$

In the open-loop Nash solution there is a period $(t^N, T]$ where $i(t) = 0$ and thus $\tau(t) = \tau_2$. The point in time t^N can be found from the adjoint system for the costate p_F . Because it is assumed that $\tau_2 \geq \frac{1}{2}$, so that $\tau_2 > 1 - \frac{1}{2\alpha}$, the value of the costate p_G at t^N is larger than $\frac{1}{w}$. Furthermore, both co-states p_F and p_G are monotonically decreasing. It follows that before the point in time t^N $i(t) = 1$ and $\tau(t) = \tau_1$. Q.E.D.

Proposition 2

i) If $\tau_1 \geq 1 - \frac{1}{2\alpha}$, the open-loop Stackelberg solution for the differential game (12)-(14) with the government as leader and the firm as follower is given by

$$\begin{aligned}\tau(t) &= \tau_1, \quad i(t) = 1, \quad \text{for } t \in [0, t_0^S) \\ \tau(t) &= \tau_1, \quad i(t) = 0, \quad \text{for } t \in (t_0^S, T],\end{aligned}\quad (22)$$

$$\text{where } t_0^S = T - \frac{1}{\beta(1-\tau_1)}.$$

ii) If $\tau_1 < 1 - \frac{1}{2\alpha}$, the open-loop Stackelberg solution for the differential game (12)-(14) with the government as leader and the firm as follower is given by

$$\begin{aligned}\tau(t) &= \tau_1, \quad i(t) = 1, \quad \text{for } t \in [0, t_1^S) \\ \int \tau(t) dt &= \frac{2\alpha-1}{\beta} \text{ over } (t_1^S, T], \quad i(t) = 0, \quad \text{for } t \in (t_1^S, T],\end{aligned}\quad (23)$$

$$\text{where } t_1^S = T - \frac{2\alpha}{\beta}.$$

Proof

The proof resembles Pohjola's (1983) derivation of the open-loop Stackelberg solution for the Lancaster game. However, Pohjola is not correct in his conclusion that the costate q_G (see below), which he denotes by z , changes after the follower's policy switch.

The open-loop Stackelberg solution results from the sequential application of Pontryagin's maximum principle and the Stackelberg equilibrium concept. Wishart and Olsder's (1979) result on discontinuous Stackelberg solutions is used.

The firm is follower. The firm's rational reaction to the strategy of the government is already derived in the proof of proposition 1.

The Hamiltonian function for the maximisation problem of the government is now given by

$$\begin{aligned}H_G(K, p_F, \tau, p_G, q_G, t) &= \left\{ \left(\frac{1-\alpha}{\alpha} + \tau \right) \frac{1}{w} + p_G(1-\tau)i(p_F) \right\} \beta K \\ &\quad - \left\{ (1-\tau)(1-i(p_F)) + p_F(1-\tau)i(p_F) \right\} \beta q_G.\end{aligned}\quad (24)$$

It follows that the government's optimal strategy is given by

$$\begin{aligned}\tau(t) &= \tau_1, \quad \text{if } g(t) < 0 \\ \tau(t) &= \tau_2, \quad \text{if } g(t) > 0,\end{aligned}\quad (25)$$

where

$$g(t) = \left\{ \frac{1}{w} - p_G(t)i(t) \right\} K(t) + \{ (1-i(t)) + p_F(t)i(t) \} q_G(t), \quad (26)$$

and that the costate q_G is given by the adjoint system

$$\begin{aligned} \dot{q}_G(t) &= -\beta(1-\tau(t)) \{ p_G(t)K(t) + (1-p_F(t))q_G(t) \} \frac{di}{dp_F}(t) - i(t)q_G(t) \\ q_G(0) &= 0, \end{aligned} \quad (27)$$

where the derivative $\frac{di}{dp_F}(t)$ should be understood as $\frac{-\delta(\hat{t}-t)}{\beta(1-\tau(t))}$ with δ the Dirac function and $\hat{t}$ the point of time where the firm switches from investment to dividend. It follows that the costate q_G is constant and zero before the switch-point $\hat{t}$, and constant and non-zero after $\hat{t}$. The value of the co-state q_G after $\hat{t}$

$$q_G(\hat{t}+h) = \int_{\hat{t}-h}^{\hat{t}+h} \dot{q}_G(t) dt = \int_{\hat{t}-h}^{\hat{t}+h} \{ p_G(t)K(t) + (1-p_F(t))q_G(t) \} \delta(\hat{t}-t) dt,$$

with $h > 0$, can be found by partial integration. The result is

$$q_G(\hat{t}+h) = -p_G(\hat{t})K(\hat{t}). \quad (28)$$

It follows that $g(t) = (\frac{1}{w} - p_G(\hat{t}))K(\hat{t})$ for $t > \hat{t}$.

If $\tau_1 > 1 - \frac{1}{2\alpha}$, then $g(t) < 0$ and $\tau(t) = \tau_1$ for $t > \hat{t}$, and the switch-point for this case can be found from the adjoint system for the costate p_F .

If $\tau_1 < 1 - \frac{1}{2\alpha}$, then $g(t) = 0$ for $t > \hat{t}$, and the switch-point for this case as well as

$$\int \tau(t) dt = \frac{2\alpha-1}{\beta} \text{ over } (\hat{t}, T] \quad (29)$$

can be found from the adjoint systems for the costates p_G and p_F with $p_G(\hat{t}) = \frac{1}{w}$. Q.E.D.

In proposition 2ii the government's strategy after t_1^S is not unique. Due to the error in the derivation this non-uniqueness of the equilibrium was not

found by Pohjola (1983). The government can choose, for example, an average tax rate $1 - \frac{1}{2\alpha}$ or the government can choose to continue for a while with the minimal tax rate τ_1 and then to switch to the maximal tax rate τ_2 at

$$t_2^S = T - \frac{2\alpha(1-\tau_1)-1}{\beta(\tau_2-\tau_1)}. \quad (30)$$

Proposition 3

The feedback Stackelberg solution for the differential game (12)-(14) with the government as leader and the firm as follower coincides with the open-loop Nash solution.

Proof

The feedback Stackelberg solution results from the application of dynamic programming and the Stackelberg equilibrium concept.

The Hamilton-Jacobi-Bellman equations are given by

$$V_{Gt}(t, K) + \max_{\tau} \left\{ \left(\frac{1-\alpha}{\alpha} + \tau \right) \frac{1}{w} + V_{GK}(t, K) (1-\tau)i \right\} \beta K \} = 0 \quad (31)$$

$$V_{Ft}(t, K) + \max_i \left\{ ((1-\tau)(1-i) + V_{FK}(t, K) (1-\tau)i) \beta K \right\} = 0, \quad (32)$$

where V_G and V_F are the value functions for the government and the firm, respectively.

Because the rational reaction of the firm i does not depend on the action τ of the government, the feedback Stackelberg solution with the government as leader and the firm as follower coincides with the feedback Nash solution.

It is easy to check, with value functions $V_G(t, K) = p_G(t)K$ and $V_F(t, K) = p_F(t)K$ and with the proof of proposition 1, that this Stackelberg solution also coincides with the open-loop Nash solution. This last result can also be derived as a consequence of state separability (Dockner, Feichtinger and Jørgensen, 1985). Q.E.D.

Proposition 4

The Pareto optimal or efficient solutions for the differential game (12)-(14) with λ and $(1-\lambda)$, $0 \leq \lambda \leq 1$, denoting the relative weights of the

objective functions of the government and the firm, respectively, are given by

$$\begin{aligned}
 \text{i) if } \lambda > (1-\lambda)w, \tau(t) &= \tau_1, i(t) = 1, \text{ for } t \in [0, t_1^P) \\
 \tau(t) &= \tau_2, i(t) = 1, \text{ for } t \in (t_1^P, t_2^P) \\
 \tau(t) &= \tau_2, i(t) = 0, \text{ for } t \in (t_2^P, T],
 \end{aligned} \tag{33}$$

where $t_2^P = T - \frac{1-\lambda}{\beta\{\lambda(\frac{1-\alpha}{\alpha} + \tau_2)\frac{1}{w} + (1-\lambda)(1-\tau_2)\}}$ and

$$t_1^P = t_2^P - \frac{1}{\beta(1-\tau_2)} \ln\left\{ \frac{\lambda(\frac{1-\alpha}{\alpha} + 1)\frac{1}{w}}{\lambda(\frac{1-\alpha}{\alpha} + \tau_2)\frac{1}{w} + (1-\lambda)(1-\tau_2)} \right\}.$$

$$\begin{aligned}
 \text{ii) if } \lambda < (1-\lambda)w, \tau(t) &= \tau_1, i(t) = 1, \text{ for } t \in [0, t_3^P) \\
 \tau(t) &= \tau_1, i(t) = 0, \text{ for } t \in (t_3^P, T],
 \end{aligned} \tag{34}$$

where $t_3^P = T - \frac{1-\lambda}{\beta\{\lambda(\frac{1-\alpha}{\alpha} + \tau_1)\frac{1}{w} + (1-\lambda)(1-\tau_1)\}}$.

$$\begin{aligned}
 \text{iii) if } \lambda = (1-\lambda)w, \tau(t) &= \tau_1, i(t) = 1, \text{ for } t \in [0, t_4^P) \\
 \tau(t) &\in [\tau_1, \tau_2], i(t) = 0, \text{ for } t \in (t_4^P, T],
 \end{aligned} \tag{35}$$

where $t_4^P = T - \frac{\alpha}{\beta}$.

Proof

The efficient or Pareto optimal solutions result from the application of Pontryagin's maximum principle to the Hamiltonian functions

$$H_p(K, \tau, i, p, t) = \{\lambda(\frac{1-\alpha}{\alpha} + \tau)\frac{1}{w} + (1-\lambda)(1-\tau)(1-i) + p(1-\tau)i\}BK \tag{36}$$

with $0 \leq \lambda \leq 1$.

It follows that the optimal cooperative strategy is given by

$$\begin{aligned}
 i(t) &= 0, \text{ if } p(t) - 1 + \lambda < 0 \\
 i(t) &= 1, \text{ if } p(t) - 1 + \lambda > 0 \\
 \tau(t) &= \tau_1, \text{ if } \frac{\lambda}{w} < \max(p(t), 1-\lambda)
 \end{aligned}$$

$$\tau(t) = \tau_2, \text{ if } \frac{\lambda}{w} > \max(p(t), 1-\lambda) \quad (37)$$

and that the costate p is given by the adjoint system

$$\begin{aligned} \dot{p}(t) = & -\beta\left(\lambda\left(\frac{1-\alpha}{\alpha} + \tau(t)\right)\frac{1}{w} + (1-\lambda)(1-\tau(t))(1-i(t))\right. \\ & \left.+ p(t)(1-\tau(t))i(t)\right) \end{aligned} \quad (38)$$

Again there is a point in time where the firm switches from investment to dividend. The value of the corporate tax rate τ after that point in time depends only on the value of λ , since $p(t) < 1-\lambda$. The switch-points for the different values of λ can then be found from the adjoint system for the costate p . If $\frac{\lambda}{w} \leq 1-\lambda$, then the value of the corporate tax rate τ before that switch-point is τ_1 . If $\frac{\lambda}{w} > 1-\lambda$, then the government switches from the minimal tax rate τ_1 to the maximal tax rate τ_2 at the point in time where the costate p is equal to $\frac{\lambda}{w}$. Q.E.D.

In the next sections these results will be used for an analysis of the model presented in section 2.

4. The "best" employment policy

The best employment result is achieved, when the government and the firm cooperate with the total employment over the planning period as common objective. As can be seen from Proposition 4i with $\lambda = 1$, this implies that the firm only invests and does not pay out dividend, which is to be expected. However, in the situation of decentralised decision making the best employment result is achieved in the open-loop Stackelberg behavioural equilibrium. Under the assumption of Proposition 2i, which means a labour intensive technology or a high minimal corporate tax rate, the open-loop Stackelberg solution is efficient but with full weight on the objective functional of the firm ($\lambda = 0$ in Proposition 4ii). It is to be expected that in this case only the minimal corporate tax rate is levied. Under the assumption of Proposition 2ii, which means a capital intensive technology with a low minimal corporate tax rate, the open-loop Stackelberg solution is not efficient. The structure of the solution resembles the structure of the

efficient solutions of Proposition 4iii, but the switch-point differs. However, the open-loop Stackelberg solution dominates the feedback Stackelberg or open-loop Nash solution in the sense that both players are better off. The fact that the government is better off is not surprising, since the leader in an open-loop Stackelberg game is always better off than in the corresponding open-loop Nash game. The firm is also better off. It is easy to show that in both behavioural models the equilibrium value of the firm's objective functional is equal to the level of the capital stock at the switch-point (which is equal to the final level of the capital stock). Furthermore, the firm invests longer in the open-loop Stackelberg equilibrium than in the open-loop Nash equilibrium: $t_0^S > t^N$, $t_1^S > t^N$. Figure 1 illustrates what happens. As open-loop Stackelberg equilibrium the one from proposition 2ii with investment switch t_1^S and with tax switch t_2^S , according to equation (30), is chosen.

[Figure 1]

Two numerical examples might clarify the results. In example 1 the efficient open-loop Stackelberg equilibrium from proposition 2i appears. In example 2 again the open-loop Stackelberg equilibrium with investment switch t_1^S and with tax switch t_2^S is chosen.

Example 1

Suppose $w = \frac{1}{2}$, $K(0) = 1$, $T = 8$, $\tau_1 = 0.25$, $\tau_2 = 0.75$ and $\alpha = 0.25$.

[Table 1]

Example 2

Suppose $w = \frac{1}{2}$, $K(0) = 1$, $T = 8$, $\tau_1 = 0.25$, $\tau_2 = 0.75$ and $\alpha = 0.75$.

[Table 2]

It is interesting to note that private employment as a percentage of total employment increases for the more efficient outcomes. The reason is that the more efficient outcomes have a longer period of investment.

The by now well-known drawback of the open-loop Stackelberg equilibrium is that the leader's strategy is time-inconsistent. It will be immediately clear, that the government's optimal strategy after the firm has stopped investing, at that point in time, is to levy the maximal corporate tax rate, which is not prescribed by the open-loop Stackelberg equilibrium. It follows that the essential questions are, whether the government will deviate from the announced tax policy or not and whether the firm will believe the government's announcement or not. If the government can commit itself to an announced policy or if the government has a strong reputation, the open-loop Stackelberg behavioural model is appropriate. If the government cannot commit itself or has a bad reputation, the feedback Stackelberg or open-loop Nash equilibrium results, which leads to worse outcomes for both employment and the value of the firm. In this way benchmarks are set for the analysis of the trade-off between commitments or reputation, on the one hand, and the effectiveness of an employment policy, on the other hand.

5. Sensitivity for capital/labour intensiveness

Suppose that wages w and the bounds τ_1 and τ_2 on the corporate tax rate τ are fixed. Because wages w are fixed the elasticity parameter α of the Cobb-Douglas production function (1) represents the capital/labour intensiveness of the production technology in the model. For α close to zero the production technology is very labour intensive and for α close to one the production technology is very capital intensive. For the open-loop Nash or feedback Stackelberg strategic equilibrium the point in time where the switch occurs to a higher tax rate and the pay-out of dividends is given by t^N in proposition 1. For the open-loop Stackelberg strategic equilibrium this switch point is given by t_0^S in proposition 2i for $\alpha \leq \frac{1}{2(1-\tau_1)}$ and by t_1^S in proposition 2ii for $\alpha > \frac{1}{2(1-\tau_1)}$. With equation (5) these switch points are a function of α , the indicator of capital/labour intensiveness.

It is easy to show that the rentability of the capital stock β , given by equation (5), is minimal for $\alpha = 1-w$ with value $1-w$ and that $\lim_{\alpha \downarrow 0} \beta = \infty$ and $\lim_{\alpha \uparrow 1} \beta = 1$. It is also easy to show that $\frac{\alpha}{\beta}$ is maximal for α satisfying

$$\alpha + \ln\left\{(1-\alpha)\frac{1}{w}\right\} = 0, \quad (39)$$

which implies $\alpha > 1-w$, and that $\lim_{\alpha \downarrow 0} \frac{\alpha}{\beta} = 0$ and $\lim_{\alpha \uparrow 1} \frac{\alpha}{\beta} = 1$.

[Figure 2]

Figure 2 shows the switch points as a function of α , where w , τ_1 and τ_2 take the same values as in examples 1 and 2. Since the graphs of t_0^S and t_1^S intersect for $\alpha = \frac{1}{2(1-\tau_1)}$ ($= \frac{2}{3}$), it follows that the minimum of t_0^S and t_1^S represents the switch point for the open-loop Stackelberg equilibrium as a function of α . As was stated before, the switch point t^N for the open-loop Nash or feedback Stackelberg equilibrium lies uniformly under the switch point $\min(t_0^S, t_1^S)$. Typical for both equilibria is that for α close to zero or a very labour-intensive production technology the switch occurs close to the end of the planning horizon. In this case the firm waits with the pay-out of dividends until the very end of the planning period. The reasons are that the long period of investment leads to a large capital stock and that $\lim_{\alpha \downarrow 0} \beta = \infty$, so that according to equation (4) the profits are very high. The government is also satisfied, because the long period of investment and the very labour-intensive production technology lead to a lot of private employment, and high profits at the end lead to high tax revenues at the end. For α close to one or a very capital-intensive production technology the switch occurs not so late, but later than for a mixed production technology. In this case the government is willing to postpone a higher tax rate, if the firm is willing to postpone the pay-out of dividends, and vice versa, in order to create higher future profits. In the open-loop Nash or feedback Stackelberg equilibrium the switch occurs the earliest for $\alpha = 1-w$ ($= \frac{1}{2}$). For the chosen values of w and τ_1 the same applies in the open-loop Stackelberg equilibrium.

6. Conclusion

A differential game is played between the government and a representative firm. The firm wants to maximise the total stream of dividends and can determine the investment rate. The government wants to maximise total employment, which is the sum of public and private employment, and can determine the corporate tax rate. If the government can commit itself or has

a strong reputation, the open-loop Stackelberg model is the correct behavioural model. Otherwise, the feedback Stackelberg model, which yields the same strategies as the open-loop Nash model for this problem, should be used. The feedback Stackelberg equilibrium leads to less accumulation of the capital stock and gives worse results for both the government and the firm. In the absence of commitments the effectiveness of an employment policy depends on the government's reputation. A very labour-intensive production technology leads to a very short period of dividend payments. For a very capital-intensive production technology the period of dividend payments is also relatively short, but not so extremely short.

A first suggestion for further research is to extend the basic model and to investigate the precise effects of an interest rate, a discount rate and other production technologies. Afterwards the model should be embedded in a more general macro-economic context.

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Table 1. Example 1 ($\alpha=0.25$)

	OLN/FBS	OLS/Pareto ($\lambda=0$)
investment switch	3.26	6.42
tax switch	3.26	8
total employment	295.64	998.75
private	243.84 (82%)	918.85 (92%)
public	51.80 (18%)	79.91 (8%)
total dividends	7.87	58.12
final capital stock	7.87	58.12

OLN: open-loop Nash equilibrium

FBS: feedback Stackelberg equilibrium

OLS: open-loop Stackelberg equilibrium

Table 2. Example 2 ($\alpha=0.75$)

	OLN/FBS	OLS	Pareto ($\lambda=0.33$)
investment switch	1.28	5.49	6.12
tax switch	1.28	7.58	8
total employment	6.31	39.44	42.35
private	2.26 (36%)	20.89 (47%)	24.20 (57%)
public	4.05 (64%)	18.55 (53%)	18.15 (43%)
total dividends	1.77	11.53	12.88
final capital stock	1.77	11.53	15.35

OLN: open-loop Nash equilibrium

FBS: feedback Stackelberg equilibrium

OLS: open-loop Stackelberg equilibrium

tax revenues in open-loop Stackelberg equilibrium
tax revenues in open-loop Nash equilibrium

Figure 1. Capital accumulation in different strategic equilibria

FIGURE 2: THE SWITCH POINT FROM INVESTMENT TO DIVIDEND

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