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**THE CONTINUOUS MULTIVARIATE
LOCATION-SCALE MODEL REVISITED:
A TALE OF ROBUSTNESS**

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The Continuous Multivariate Location-Scale Model Revisited: A Tale of Robustness

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Summary

We consider the location-scale family of all multivariate continuous sampling distributions. Any model from the subclass induced by fixing a particular distribution on the unit sphere, combined with a standard improper prior on the scale parameter, will lead to the same inference on location and observables. This finding sheds light on many robustness results in less general models and its implications are illustrated in a number of examples.

Some key words: Bayesian inference; Cardioid distribution; Model robustness; Multivariate distribution theory; Nuisance parameter.

1. Introduction

In the location-scale model we are often interested in the location parameter, whereas the scale may just be a nuisance parameter. Inference on missing values of the observables could also be a reason for studying these models. In a Bayesian context, these considerations translate to focussing our attention on the joint density of the observables, x , and the location parameter, μ . Bayesian models that lead to the same $p(x, \mu)$ are called marginally equivalent, following Osiewalski and Steel (1993c).

The main result of this paper is that all sampling models from the general continuous multivariate location-scale family, combined with a standard improper reference prior on the scale parameter, are marginally equivalent, provided we fix the distribution over the unit sphere. In other words, the conditional distribution of the radius given the point on the sphere does not affect $p(x, \mu)$ and thus the ensuing posterior density of μ or the predictive density. Some examples based on the cardioid circular distribution clearly illustrate that many very different sampling densities can still lead to marginally equivalent models, i.e. the same inference on μ and unobserved elements of x .

We consider general continuous multivariate distributions which constitute the final stage of a series of subsequent generalizations. In decreasing order of generality, we mention v -spherical distributions, as introduced in an, as yet, unpublished paper by the present authors (CentER Discussion Paper 9374, Tilburg University), henceforth denoted by FOS,

l_q -spherical distributions, defined in Osiewalski and Steel (1993b), and spherical distributions, as discussed in Kelker (1970), Cambanis *et al.* (1981), Dickey and Chen (1985) and Fang *et al.* (1990, Chapters 2-4). Similar robustness results in these special cases are generalized and put in a broader perspective here.

2. Marginal Equivalence

In this paper we shall analyze the general location-scale family of continuous multivariate distributions. In particular, we consider the sampling density $p(x|\mu, \tau)$, where $x = \mu + \tau^{-1}y$, $\mu \in \mathbb{R}^n$, $\tau \in \mathbb{R}_+$ and $p(y|\mu, \tau) = f(y)$ is a proper density in $\mathbb{R}^n$ ($n \geq 2$). Thus

$$p(x|\mu, \tau) = \tau^n f\{\tau(x - \mu)\}, \quad (2.1)$$

with $x = (x_1, \dots, x_n) \in \mathbb{R}^n$, μ a location parameter and τ^{-1} a scale parameter. Throughout the paper, all densities are taken with respect to Lebesgue measure in the corresponding space.

A full Bayesian model will consist of a sampling density from (2.1) combined with a prior density on the parameters (μ, τ) , either proper or improper. In practice, our interest will focus on the location parameter, μ , which may be reparameterized in terms of a lower dimensional vector, whereas τ will typically be a nuisance parameter. In addition, we may be interested in prediction of unobserved elements of x .

These considerations naturally lead to the concept of marginal equivalence, as introduced in Osiewalski and Steel (1993c). In particular, two Bayesian models with different sampling densities from the family (2.1) and potentially different priors are marginally equivalent for x and μ if both models lead to the same density $p(x, \mu)$. Note that $p(x, \mu)$ simply corresponds to the joint density of $\tau^{-1}y$ and μ , evaluated at $(x - \mu, \mu)$. Therefore, our attention will be focussed on $p(\tau^{-1}y, \mu)$.

We now express $y \in \mathbb{R}^n$ as $y = u h(\omega)$, where $u = \|y\| \in \mathbb{R}_+$ is the Euclidean norm of y and $h(\omega) = y/\|y\| \in S^{n-1}$, the unit sphere. In order to facilitate variable transformations, we represent the sphere in terms of $\omega \in \Omega \subset \mathbb{R}^{n-1}$ through a one-to-one function $h(\cdot)$ with continuous first order partial derivatives. For example, we shall often characterize points in S^{n-1} through angular polar coordinates.

We factorize the joint density function of (u, ω) , into the marginal density of ω , denoted by $f_2(\omega)$, and the conditional density of u given ω , $f_1(u; \omega)$. Observe that then properness of $f(y)$ is equivalent to both $f_2(\omega)$ and $f_1(u; \omega)$ being proper. Clearly, for a given $h(\cdot)$, $f_2(\omega)$ uniquely determines a probability measure over the unit sphere.

We remind the reader that the Bayesian models considered here can differ in the choice of $f(\cdot)$, and therefore $f_1(\cdot)$ and $f_2(\cdot)$, and in the choice of $p(\mu, \tau)$.

3. Inference Robustness under an Improper Prior

In a location-scale model, we often do not possess real prior information regarding the nuisance parameter τ . A common solution to this problem is to specify a prior density which is uniform on $\log \tau$ and has the following product structure:

$$p(\mu, \tau) = p(\mu)p(\tau) = p(\mu)c\tau^{-1}, \quad (c > 0). \quad (3.1)$$

Adopting this reference prior in our framework has an interesting consequence, as is formalized in the following theorem.

Theorem. *If $p(x|\mu, \tau) = \tau^n f\{\tau(x - \mu)\}$ and $p(\mu, \tau) = p(\mu)c\tau^{-1}$ ($c > 0$), then the joint density $p(x, \mu)$ does not depend on $f_1(u; \omega)$.*

Proof: Consider $z = \tau^{-1}u$, so that $p(u, \omega, \mu, z) = f_1(u; \omega)f_2(\omega)p(\mu)cz^{-1}$. Thus, $p(\omega, \mu, z) = f_2(\omega)p(\mu)cz^{-1}$ and does not depend on $f_1(u; \omega)$. Therefore, $p(zh(\omega), \mu)$ and $p(x, \mu)$ do not depend on $f_1(u; \omega)$ either. •

The key to this result is the fact that the density $p(\tau) = c\tau^{-1}$ is invariant under the transformation $z = \tau^{-1}u$ for any value of $u > 0$, so that u disappears once we make the variable transformation from τ to z . No other prior than (3.1) will induce this particular property.

The Theorem implies that if we fix the distribution on the unit sphere by choosing a particular $h(\omega)$ and $f_2(\omega)$, the conditional distribution of the radius, given by $f_1(u; \omega)$, does not influence the resulting density $p(x, \mu)$. This means that all Bayesian models consisting of a sampling model from a subclass in (2.1) induced by a particular $f_2(\omega)$, given $h(\omega)$, and a prior corresponding to (3.1) are marginally equivalent. Choosing a distribution on the unit sphere, S^{n-1} , in combination with (3.1) suffices to obtain perfect robustness of posterior inference on μ with respect to any choice of a proper density $f_1(u; \omega)$. Of course, in order to deduce a proper posterior density $p(\mu|x)$ from $p(x, \mu)$, we need that the predictive density $p(x)$ is finite. In addition, we achieve robustness of predictive inference on unobserved elements of x , say x_f , given the observations, say x_o , where properness of $p(x_f|x_o)$ requires that $p(x_o) < \infty$.

Clearly, by varying $f_1(\cdot)$, we can obtain dramatically different sampling models that will, however, still lead to the same inferences on x and μ . Section 5 will provide some illustrative examples.

4. The Case of General Priors

In order to examine marginal equivalence under other prior structures than (3.1), an explicit expression for $p(x, \mu)$ proves useful.

Denoting the Jacobian of the transformation from (u, ω) to y by $u^{n-1}s(\omega)$, where $s(\cdot)$ depends on the form of $h(\cdot)$ adopted, we can express the density $f(y)$ as the product of $f_1(u; \omega)f_2(\omega)$ and the inverse of this Jacobian. This immediately leads to the following

expression for $p(x, \mu)$:

$$p(x, \mu) = \frac{\|x - \mu\|^{1-n}}{s \left\{ h^{-1} \left(\frac{x - \mu}{\|x - \mu\|} \right) \right\}} f_2 \left\{ h^{-1} \left(\frac{x - \mu}{\|x - \mu\|} \right) \right\} \int_0^\infty f_1 \left\{ \tau \|x - \mu\|; h^{-1} \left(\frac{x - \mu}{\|x - \mu\|} \right) \right\} \tau p(\mu, \tau) d\tau. \quad (4.1)$$

Thus, given $h(\omega)$, any set of combinations of $f_2(\omega)$, $f_1(u; \omega)$ and $p(\mu, \tau)$, the latter either proper or improper, that leads to the same $p(x, \mu)$ in (4.1) defines a class of marginally equivalent Bayesian models. In particular, if we fix the distribution over the unit sphere, marginal equivalence is characterized by pairs of $f_1(u; \omega)$ and $p(\mu, \tau)$ that lead to the same value of the integral in (4.1). Examples of such marginal equivalence can be constructed directly from the univariate integral identities in Appendix B of FOS.

From the Theorem we know that under the invariant prior (3.1) any choice of a proper $f_1(u; \omega)$ will give us a marginally equivalent model. The latter class will be characterized by

$$p(x, \mu) = \frac{c \|x - \mu\|^{-n}}{s \left\{ h^{-1} \left(\frac{x - \mu}{\|x - \mu\|} \right) \right\}} f_2 \left\{ h^{-1} \left(\frac{x - \mu}{\|x - \mu\|} \right) \right\} p(\mu). \quad (4.2)$$

5. Examples

The theory will now be illustrated by some examples in the bivariate case ($n = 2$).

A well-known circular distribution on the unit sphere, S^1 , is the cardioid distribution, which corresponds to the following density function in polar coordinates

$$f_2(\omega) = \frac{1 + 2\rho \cos \omega}{2\pi} I_{[0, 2\pi)}(\omega), \quad (5.1)$$

where $|\rho| < 1/2$. Nachtsheim and Johnson (1988) combine (5.1) with an independent $\text{Gamma}(n/2, 1/2)$ distribution on u^2 , leading to

$$f_1^a(u; \omega) = u \exp(-u^2/2) I_{(0, \infty)}(u). \quad (5.2)$$

They use many other nonuniform distributions on the sphere, keeping $f_1(u; \omega)$ fixed at (5.2), and call the resulting distributions anisotropic. In this Section, however, we fix $f_2(\omega)$ to be the cardioid density in (5.1) with $\rho = 0.3$ and vary the choice of $f_1(u; \omega)$.

Thus, following our Theorem, we generate sampling distributions that all lead to marginally equivalent Bayesian models, provided we use the improper prior in (3.1) for $p(\mu, \tau)$.

Panel a of Figure 1 shows the density function of $y = \tau(x - \mu)$ corresponding to (5.2) in Cartesian coordinates, whereas panel b gives its isodensity contours. The latter

convention will be followed throughout this Section. Figures 2 through 4 correspond to Gamma, Inverted Beta and Beta specifications for $f_1(u; \omega)$. In particular, we have chosen

$$\begin{aligned} f_1^b(u; \omega) &= f_G(u|20, 1) = \{\Gamma(20)\}^{-1} u^{19} \exp(-u) I_{(0, \infty)}(u) \\ f_1^c(u; \omega) &= f_{IB}(u|1, 2, \omega^2 + 1) = \frac{2u}{(1 + \omega^2)^2} \left(1 + \frac{u}{1 + \omega^2}\right)^{-3} I_{(0, \infty)}(u) \\ f_1^d(u; \omega) &= f_B(u|(\omega - \pi)^2 + 1/2, 1/2, 10) \\ &= \frac{\Gamma\{(\omega - \pi)^2 + 1\}}{10 \pi^{1/2} \Gamma\{(\omega - \pi)^2 + \frac{1}{2}\}} \left(\frac{u}{10}\right)^{(\omega - \pi)^2 - 1/2} \left(1 - \frac{u}{10}\right)^{-1/2} I_{(0, 10)}(u). \end{aligned}$$

Insert Figures 1 to 4

Note that $f_1^b(u; \omega)$ retains the independence between u and ω found in (5.2), yet leads to very different isodensity contours. This forcefully illustrates that $f_1(u; \omega)$ is not the labelling function, even under independence. Equivalently, choosing $f_2(\omega)$ generally does not define the isodensity sets. In the special case of v -spherical distributions as introduced in FOS, where $f(\cdot) = g\{v(\cdot)\}$ and $v(\cdot)$ and $g(\cdot)$ are subject to certain restrictions, choosing $f_2(\omega)$ is equivalent to choosing $v(\cdot)$, which determines the isodensity sets, whereas $f_1(u; \omega)$ uniquely identifies the labelling function $g(\cdot)$ given $v(\cdot)$. In particular, we then obtain $f_2(\omega) \propto s(\omega)[v\{h(\omega)\}]^{-n}$ and $f_1(u; \omega) \propto u^{n-1}g[uv\{h(\omega)\}]$. If we specialize further to l_q -spherical distributions, introduced in Osiewalski and Steel (1993b), we choose $v(\cdot)$ to be the l_q -norm, where $q = 2$ corresponds to the usual definition of sphericity.

The last two examples introduce dependence between u and ω . In addition, $f_1^c(u; \omega)$ does not take the same values for $\omega = 0$ as for $\omega = 2\pi$ and thus leads to discontinuities in the density function in Cartesian coordinates. The last example concentrates all the mass within the circle with radius 10 and does not lead to a bounded density function. The density plot in Figure 4a is thus truncated.

In contrast to v -spherical distributions, isodensity sets are not necessarily located around a common point, as is illustrated by Figures 2 and 4, corresponding to $f_1^b(u; \omega)$ and $f_1^d(u; \omega)$.

We reiterate that posterior inference on the location parameter of the corresponding sampling densities of $x = \mu + \tau^{-1}y$, as well as predictive inference on x itself, is exactly the same in all the examples discussed above. Any choice of a proper density $f_1(u; \omega)$ in combination with (5.1) will constitute a model that is marginally equivalent to the cardioid model of Nachtsheim and Johnson (1988), provided we use prior (3.1), however different the shape of the density function and the isodensity sets may be. Of course, the Theorem applies to any other proper density $f_2(\omega)$ as well.

6. Concluding Remarks

In the general class of location-scale models with continuous multivariate distributions (2.1), we find that using a common reference prior on the scale will result in the same

posterior inference on the location parameter and the same predictive inference on x , within a subclass of (2.1) induced by a common distribution over the unit sphere. This very general result implies that we can generate a continuum of potentially very different sampling distributions, by varying the proper conditional density of the radius, keeping the distribution on the sphere fixed, that all lead to marginally equivalent Bayesian models.

So far, we have been interested in the form of $p(x, \mu)$. Let us now consider $p(\tau|x, \mu)$, the remaining factor of our Bayesian model $p(x, \mu, \tau)$. If x is fully observed, $p(\tau|x, \mu)$ is the conditional posterior density of τ given μ . Under the conditions of the Theorem, we obtain

$$p(\tau|x, \mu) = \|x - \mu\| f_1 \left\{ \tau \|x - \mu\|; h^{-1} \left(\frac{x - \mu}{\|x - \mu\|} \right) \right\}. \quad (6.1)$$

The latter density summarizes the entire influence of the choice of $f_1(\cdot)$ and does not depend on the form of $f_2(\cdot)$. Thus, given μ , posterior inference on the scale parameter is unaffected by the distribution on the sphere. However, this conditional robustness is lost when we calculate the marginal posterior $p(\tau|x)$. In addition, τ is updated by the sample information and its conditional posterior in (6.1), which is a proper density, does not replicate the functional form of the prior in (3.1). Therefore, our Theorem does not hold for more than one independent vector observation from (2.1) and achieving inference robustness seems impossible in the case with repeated sampling.

The Theorem can straightforwardly be generalized to the case where we sample from $t = Ax$ with x distributed according to (2.1) and A a nonsingular matrix function of a parameter vector η . In that case, the conditions of the Theorem combined with any prior $p(\mu, \eta)$ will lead to a density $p(t, \mu, \eta)$ which does not depend on $f_1(\cdot)$. Thus, the robustness results extend to the posterior distribution of η as well, if the latter exists.

This paper generalizes and puts in perspective a host of earlier papers in multivariate distribution theory. Special cases of our theorem were discussed in Zellner (1976) for the multivariate Student- t regression model, whereas Jammalamadaka *et al.* (1987), Chib *et al.* (1988, 1991) and Osiewalski (1991) treated the case of scale mixtures of Normals, generalized to ellipticity in l_2 in Osiewalski and Steel (1993a,c). Finally, l_q -spherical and v -spherical distributions were introduced and examined by Osiewalski and Steel (1993b) and FOS, respectively. The result in this paper clarifies the nature of the robustness results found in the papers mentioned above: we achieve robustness with respect to the conditional distribution of the radius, and not, in general, with respect to the labelling function. In the previous papers, both types of robustness were confounded but the general framework considered in this paper allows us to clearly establish the underlying nature of all these marginal equivalence results. In addition, the fundamental cause is seen to reside in invariance of the improper prior on the scale parameter.

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Figure 1: Density and contour plots for $f_1^a(u; \omega)$

Figure 3: Density and contour plots for $f_1^c(u; \omega)$

Figure 2: Density and contour plots for $f_1^b(u; \omega)$

Figure 4: Density and contour plots for $f_1^d(u; \omega)$

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