Tilburg University

Joint Tests for Regularity and Autocorrelation in Allocation Systems

Deschamps, P.J.

Publication date:
1990

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Deschamps, P. J. (1990). *Joint Tests for Regularity and Autocorrelation in Allocation Systems*. (CentER Discussion Paper; Vol. 1990-42). CentER.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

ECO

Center

for
mic Research

Discussion paper

CBM
R

8414
1990

42

No. 9042

**JOINT TESTS FOR REGULARITY AND AUTOCORRELATION
IN ALLOCATION SYSTEMS**

by Philippe J. Deschamps

R46
518.92

July 1990

ISSN 0924-7815

JOINT TESTS FOR REGULARITY AND AUTOCORRELATION IN ALLOCATION SYSTEMS

by Philippe J. Deschamps*

When the disturbances in a system of demand equations satisfy a vector autoregressive process, the standard tests for regularity (e.g. homogeneity and symmetry) do not have the correct size since they are based on an incorrect maintained hypothesis of spherical disturbances. The maximum likelihood estimation of the autocorrelated system is usually very costly; this has probably led most empirical researchers to ignore the problem. We wish to illustrate its potential seriousness with a number of applications, based on the Rotterdam system and the AIDS system. To this end, we propose a generic procedure, based on the Lagrange multiplier principle, for testing any possible combination of absence of serial correlation, homogeneity, and symmetry against any possible alternative which specifies autocorrelation of an arbitrary given order. In most cases, the statistic is shown to simplify to an easily computed trace. We also present a general, and explicit, estimation procedure for the maximum likelihood estimation of an allocation system under any possible combination of serial correlation, homogeneity, and symmetry. This procedure contains, as a special case, the estimation of the system without the regularity constraints. Our numerical results suggest that taking autocorrelation into account generally decreases the test statistics for regularity.

* Institute for Automation and Operations Research, University of Fribourg, CH-1700 Fribourg, Switzerland and Center for Economic Research, Tilburg University, POB 90153, 5000 LE Tilburg, The Netherlands.

Section 1. Introduction¹

The pioneering work of Berndt and Savin (1975) and Lau (1978) made the profession aware that the adding-up condition has important consequences for the specification of dynamic error processes in allocation systems (multivariate linear regression models with singular error covariance matrix). Berndt and Savin recognized that the only vector autoregressive processes compatible with adding-up are those in which the matrices R_j^* of autocorrelation coefficients satisfy $\iota' R_j^* = k_j \iota'$, where ι is a column vector of ones and k_j is an unknown constant. This condition was later proved by Lau (1978) to be necessary and sufficient for adding-up to hold in a dynamic allocation system.

Berndt and Savin (1975) also present a procedure, based on the work of Hendry (1971), for the maximum likelihood (ML) estimation of dynamic allocation systems. As usual, this procedure involves the deletion of an equation; the estimates are invariant with respect to the index of the deleted equation. They also show that the matrices R_j^* of autocorrelation coefficients are not identifiable without further restrictions. This does not present a problem insofar as the primary parameters of interest are the coefficients of the observable variables (e.g. price and income coefficients) rather than the elements of R_j^* .

One implication of the Berndt-Savin results is the inadequacy of those tests for autocorrelation that are based on the residuals of a single equation, such as the Durbin-Watson (DW) statistic (the single equation DW test has, however, been used in the empirical literature on allocation systems; see, e.g., Deaton and Muellbauer (1980)). The correct procedure involves jointly testing for autocorrelation in the full system, taking into account the restrictions on R_j^* implied by adding-up. If one is willing to assume that the matrices R_j^* are diagonal, adding-up implies $R_j^* = \rho_j I$; since the autocorrelation coefficients are then identical across all equations, it would appear reasonable (though not rigorous) to compute a single DW statistic from the pooled residuals of the entire system. Usually, however, the model attempts to explain the demand for heterogeneous commodities (e.g. food and housing); in this instance an assumption of equal correlation coefficients across commodities is clearly unattractive. It is then much more appealing to test a model with spherical disturbances against a model where the matrices R_j^* are not restricted in any way. One may for this purpose use the likelihood ratio test, in the fashion of Berndt and Savin; or the more recent Lagrange multiplier (LM) test statistic initially proposed by Aitchison and Silvey (1960) and later investigated by Godfrey (1978) and Breusch and Pagan (1980) in the context of dynamic error processes. As has often been pointed out, an important advantage of the latter test is that it only requires estimation under the null hypothesis: this is of importance in our case since the

autocorrelated system is costly to estimate, as will become apparent in the sequel of this paper.

Of even greater importance than a test for serial correlation is testing the regularity restrictions implied by economic theory (such as homogeneity and symmetry). Such regularity tests have almost always been conducted under the maintained hypothesis of no autocorrelation. But if this maintained hypothesis is incorrect, it follows from the work of White (1982) that the tests of regularity will generally be of incorrect size. One may then either use the robust statistics proposed by White, or conduct classical tests for regularity in the autocorrelated model. The latter procedure may have better small-sample properties.

The fact that the usual regularity tests should be validated by a preliminary test for autocorrelation was recognized by Berndt and Savin (1975). In their numerical example, the likelihood ratio test statistic for symmetry is reduced by a factor of two when allowance is made for first-order serial correlation with a diagonal R^* . Unfortunately, the authors did not proceed to test for symmetry in the fully correlated model (unrestricted R^*), even though they strongly reject the null hypothesis of zero cross-correlation coefficients.

The previous remarks illustrate the need for a systematic investigation of all possible joint tests, and for the elaboration of a rigorous testing strategy. In particular, one may test for regularity with, or without, the maintained hypothesis of autocorrelation of any given order; and one may test for autocorrelation with, or without, the maintained hypotheses of homogeneity and symmetry (it is, in fact, possible that homogeneity and symmetry introduce autocorrelation into an otherwise spherical model). Use of the LM principle ensures that for most of these tests, it is not necessary to estimate the autocorrelated model; it will be seen in fact that in most cases, the LM test does not require the inversion of inordinately large matrices, even though the regression equation may be nonlinear under the maintained hypothesis.

The aim of this paper is twofold. We will first present a general, and explicit, estimation procedure for the ML estimation of an allocation system with autocorrelated errors, homogeneity, and symmetry. This procedure contains, as a special case, the estimation of the system without the regularity constraints. We will then present a generic procedure, based on the LM principle, for testing any possible combination of absence of serial correlation, homogeneity, and symmetry against any possible alternative which specifies autocorrelation of an arbitrary given order.

When the regularity restrictions are homogeneity and symmetry, a list of all possible tests of H_0 against H_1 is provided in Table 1. In the row and column headings of Table 1, A denotes the *absence* of autocorrelation; H denotes homogeneity; S denotes symmetry; and the bars denote

logical negation. The list involves 12 tests rather than $8 \times 8 = 64$, since symmetry and adding-up imply homogeneity and since H_0 must be nested within H_1 .

TABLE 1: LIST OF POSSIBLE JOINT TESTS

	H_0				
	AHS	$AH\bar{S}$	$A\bar{H}\bar{S}$	$\bar{A}HS$	$\bar{A}\bar{H}\bar{S}$
H_1	$AH\bar{S}$ Test no. 1				
	$A\bar{H}\bar{S}$ Test no. 2	Test no. 3			
	$\bar{A}HS$ Test no. 4				
	$\bar{A}\bar{H}\bar{S}$ Test no. 5	Test no. 6		Test no. 7	
	$\bar{A}\bar{H}\bar{S}$ Test no. 8	Test no. 9	Test no. 10	Test no. 11	Test no. 12

It is important to note that in each column of Table 1, the various tests should be performed in order of increasing restrictiveness under the alternative, i.e. Test 9 should be performed before Test 6 for example (it may also be said that a nonsignificant result in Test 9 *validates* the use of Test 6). Indeed, in this instance, it is only when the joint hypothesis of no autocorrelation and of homogeneity ($AH\bar{S}$) is not rejected against the fully unrestricted alternative ($\bar{A}\bar{H}\bar{S}$) that one can be confident in the proper specification of $\bar{A}\bar{H}\bar{S}$, so that Test 6 has the proper size. If Test 9 does not reject, then Test 6 should have more power than Test 9, since it is based on a smaller (but probably still correct) alternative.

It should also be noted that the LM tests in the first three columns of Table 1 require significantly less computation than the tests in the last two. In fact, it will be shown that the generic formula for the twelve tests reduces to an easily computed trace in cases 1, 2, 3, 5, 6, 8, 9, and 10 of Table 1.

The previous remarks suggest the following strategy for jointly testing the regularity restrictions (HS) of economic theory. First perform sequentially Tests 10, 9, 6, 8, 5, and 4, in that order. If at any stage of this procedure, the null hypothesis is rejected, proceed to estimate the fully autocorrelated model and perform Test 11. If none of the six tests rejects, perform Test 2. It should be clear that this testing strategy would be prohibitively expensive if Wald or likelihood ratio tests, rather than Lagrange multiplier tests, were used.

A brief plan of the paper follows. In Section 2, we will present generic expressions for the

maximum likelihood estimation of an allocation system under the six possible combinations of constraints (AHS , $AH\bar{S}$, $A\bar{H}\bar{S}$, $\bar{A}HS$, $\bar{A}H\bar{S}$, $\bar{A}\bar{H}\bar{S}$). Generic formulas for the gradient of the loglikelihood and for the information matrix are given in Section 3. The generic LM test statistic is stated, and shown to simplify considerably in cases 1, 2, 3, 5, 6, 8, 9, and 10 of Table 1 (this result was previously shown by Berndt and Savin (1977) only in the first three cases, where the model is linear under the alternative and the null). A simplification also occurs when A is interpreted as autocorrelation with diagonal, but not necessarily null, matrices of correlation coefficients. In this case the model is only slightly more difficult to estimate than the fully uncorrelated model (the Aitken transformation of the data matrices can be performed elementwise, in Cochrane-Orcutt fashion). Section 4 illustrates the methodology by estimating the AIDS model in levels (Deaton and Muellbauer (1980)) and the Rotterdam model (Theil (1975); Barten (1969)) with quarterly British data. In the case of the AIDS system, seasonal factors induce a very high degree of serial correlation, which can be adequately modeled by specifying disturbances of the form $u_t = R_1^* u_{t-1} + R_4^* u_{t-4} + \epsilon_t$. Section 5 concludes.

Section 2. Restricted and unrestricted maximum likelihood estimation

As shown by Berndt and Savin (1977), any dynamic allocation system satisfying the maintained (and untestable) restriction of adding-up may be written without loss of generality as:

$$Y = BX + U \quad (1)$$

$$U = \sum_{j=1}^p R_j U_{-j} + E \quad (2)$$

where Y is an $n \times T$ matrix of T observations on n dependent variables, B is an $n \times k$ matrix of coefficients, X is a $k \times T$ matrix of T observations on k regressors, U and E are $n \times T$ matrices of current disturbances, the R_j are $n \times n$ matrices of autocorrelation coefficients, and the U_{-j} are $n \times T$ matrices of lagged disturbances. We will assume that $\text{vec} E \sim N(0, I_T \otimes \Sigma)$, where Σ is a positive definite matrix of order n . It is emphasized that (1) and (2) are interpreted as an *incomplete* allocation system, after the deletion of an equation; so that, typically, n is the total number of commodities in a demand system minus one.² Upon substituting $U_{-j} = Y_{-j} - BX_{-j}$ and Equation (1) into Equation (2), we obtain:

$$Y = \sum_{j=1}^p R_j Y_{-j} + BX - \sum_{j=1}^p R_j BX_{-j} + E. \quad (3)$$

Upon letting:

$$R = (R_1 \ R_2 \ \dots \ R_p) \quad (4)$$

$$Y_1 = \begin{pmatrix} Y_{-1} \\ Y_{-2} \\ \vdots \\ Y_{-p} \end{pmatrix} \quad (5)$$

$$X_1 = \begin{pmatrix} X_{-1} \\ X_{-2} \\ \vdots \\ X_{-p} \end{pmatrix} \quad (6)$$

$$U = Y - BX \quad (7)$$

$$U_1 = \begin{pmatrix} U_{-1} \\ U_{-2} \\ \vdots \\ U_{-p} \end{pmatrix} = \begin{pmatrix} Y_{-1} - BX_{-1} \\ Y_{-2} - BX_{-2} \\ \vdots \\ Y_{-p} - BX_{-p} \end{pmatrix} = Y_1 - (I_p \otimes B)X_1, \quad (8)$$

we may rewrite Equation (3) as:

$$U = RU_1 + E. \quad (9)$$

Similarly, if we note that:

$$\text{vec} \left(\sum_{j=1}^p R_j BX_{-j} \right) = \sum_{j=1}^p (X'_{-j} \otimes R_j) \text{vec } B$$

and that $\text{vec}(BX) = (X' \otimes I_n) \text{vec } B$, we may express Equation (3) as:

$$\text{vec} (Y - RY_1) = \left((X' \otimes I_n) - \sum_{j=1}^p (X'_{-j} \otimes R_j) \right) \text{vec } B + \text{vec } E. \quad (10)$$

It is immediately apparent from (9) that the maximum likelihood estimator of R is:

$$\hat{R} = U U_1' (U_1 U_1')^{-1} \quad (11)$$

(see Hendry (1971)). In order to impose homogeneity and symmetry on $\text{vec } B$ in (10), we use the methodology presented in Deschamps (1988). If we let $B = (C \mid S \mid s)$, where $(S \mid s)$ is an $n \times (n+1)$ matrix of price coefficients and where s is the last column of B , homogeneity and symmetry are stated, respectively, as $S \mathbf{1}_n = -s$ and $S = S'$. We define:

$$D_H = \begin{pmatrix} & O_{(k-n-1) \times 1} \\ I_{k-1} & \\ & -\mathbf{1}_n \end{pmatrix}, \quad \text{and:} \quad (12)$$

$$D_S = \begin{pmatrix} I_{n(k-n-1)} & O \\ O & L \end{pmatrix} \quad (13)$$

where L is an $n^2 \times n(n+1)/2$ matrix such that $\text{vec}S$ equals L times the stacked lower triangle of S (for an explicit form of L , see Balestra (1976); and for an algorithm generating a compact computer representation of L , see Deschamps (1988)). In the case where S is 2×2 , L has the following form:

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

With these definitions, we may impose the homogeneity and symmetry restrictions on $\text{vec}\hat{B}$ as:

$$\text{vec } \hat{B} = (D'_H \otimes I_n) D_S \hat{b} \quad (14)$$

where:

$$\hat{b}' = (\text{vec}'\hat{C} \quad \hat{S}_{11} \quad \hat{S}_{21} \quad \hat{S}_{22} \quad \hat{S}_{31} \quad \hat{S}_{32} \quad \hat{S}_{33} \quad \dots \quad \hat{S}_{nn})$$

is a vector of unconstrained parameters.

Upon substituting $\text{vec}B = (D'_H \otimes I_n) D_S b$ into Equation (10), we see immediately that the ML estimator of b is:

$$\hat{b} = \left(Z'(I_T \otimes \hat{\Sigma}^{-1})Z \right)^{-1} Z'(I_T \otimes \hat{\Sigma}^{-1}) \text{vec}(Y - \hat{R}Y_1), \quad \text{with:} \quad (15)$$

$$\begin{aligned} Z &= \left((X' \otimes I_n) - \sum_{j=1}^p (X'_{-j} \otimes \hat{R}_j) \right) (D'_H \otimes I_n) D_S \\ &= \left((X' D'_H \otimes I_n) - \sum_{j=1}^p (X'_{-j} D'_H \otimes \hat{R}_j) \right) D_S \end{aligned} \quad (16)$$

$$\hat{\Sigma} = \hat{E} \hat{E}' / T \quad (17)$$

$$\hat{E} = Y - \hat{R}Y_1 - \hat{B}X + \hat{R}(I_p \otimes \hat{B})X_1 \quad (18)$$

where $\hat{R}$ is given by (11) with U_1 replaced by $\hat{U}_1 = Y_1 - (I_p \otimes \hat{B})X_1$, and where $\hat{R}_j$ is the appropriate block of $\hat{R}$.

It is straightforward to check that Equation (15) can be simplified as:

$$\hat{b} = \left(Z'(I_T \otimes \hat{\Sigma}^{-1})Z \right)^{-1} D'_S \text{vec} \left(\hat{\Sigma}^{-1} (Y - \hat{R}Y_1) X' D'_H - \sum_{j=1}^p \hat{R}_j' \hat{\Sigma}^{-1} (Y - \hat{R}Y_1) X'_{-j} D'_H \right). \quad (19)$$

In order to impose homogeneity only, it suffices to replace D_S in (16) and (19) by an identity matrix of order $n(k-1)$, which reduces Equation (14) to $\hat{B} = \hat{B}_H D_H$ with $\hat{B}_H = (\hat{C} \mid \hat{S})$. Of course, $\hat{b}$ is then redefined as $\text{vec}\hat{B}_H$. Similarly, Equations (16) and (19) imply unconstrained estimation when D_S and D_H are replaced by identity matrices; $\hat{b}$ is in this case redefined as $\text{vec}\hat{B}$. We summarize our results in the following theorem.

Theorem 1. The maximum likelihood estimation of (1)–(2) under homogeneity and symmetry implies the solution of Equations (11), (17), and (19) with Z , $\hat{B}$ and $\hat{E}$ given by (16), (12)–(14), and (18), respectively. Estimation under homogeneity requires the solution of Equations (11), (17), and (19) with Z and $\hat{E}$ given by (16) and (18), and with $\hat{B}$ given by (12), (14) and by $D_S = I_{n(k-1)}$. Unconstrained estimation requires the solution of (11), (17), and (19) with Z and $\hat{E}$ given by (16) and (18), and with $\hat{b} = \text{vec} \hat{B}$.

If we assume uncorrelated disturbances, Equation (11) is replaced by $\hat{R} = O$, and formulas (18) and (19) are considerably simplified (see Deschamps (1988)). The system also becomes much simpler if we assume that $R_j = \rho_j I_n$, since (9) may be written in this case as:

$$U = \sum_{j=1}^p \rho_j U_{-j} + E$$

with U and U_{-j} given by (7) and (8). The ML estimates of the ρ_j are then the GLS coefficient estimates in the regression of $\text{vec} \hat{U}$ on $(\text{vec} \hat{U}_{-1}, \dots, \text{vec} \hat{U}_{-p})$. Furthermore, it is straightforward to check that Equations (18) and (19) respectively simplify to:

$$\hat{E} = Y_* - \hat{B} X_* \quad (18')$$

$$\hat{b} = \left(D'_S (D_H X_* X_*' D'_H \otimes \hat{\Sigma}^{-1}) D_S \right)^{-1} D'_S \text{vec} \left(\hat{\Sigma}^{-1} Y_* X_*' D'_H \right) \quad (19')$$

with $Y_* = Y - \sum_{j=1}^p \hat{\rho}_j Y_{-j}$ and $X_* = X - \sum_{j=1}^p \hat{\rho}_j X_{-j}$.

Section 3. Joint tests for autocorrelation, homogeneity, and symmetry

It is well-known that for linear models of the form $z = Zb + \text{vec}E$, with $\text{vec}E \sim N(0, I_T \otimes \Sigma)$, the gradient of the (unconcentrated) loglikelihood $L(b, \Sigma)$ with respect to b is given by:

$$\frac{\partial L}{\partial b} = Z'(I_T \otimes \Sigma^{-1})(z - Zb) = Z' \text{vec}(\Sigma^{-1}E).$$

The information matrix is also well-known to be block-diagonal. Under regularity assumptions, the block of this matrix corresponding to b is:

$$\mathcal{I}_{bb} = \mathcal{E} \left(\frac{\partial L}{\partial b} \frac{\partial L}{\partial b'} \right) = Z'(I_T \otimes \Sigma^{-1})Z.$$

When the regression equation is nonlinear, as in (3), it follows immediately that if we may write:

$$z = Zb + \text{vec}E \quad \text{and} \quad z_1 = Z_1c + \text{vec}E$$

where (z, Z) does not involve b and (z_1, Z_1) does not involve c , then the gradient of the loglikelihood with respect to $\alpha = (b, c)$ is equal to:

$$\frac{\partial L}{\partial \alpha} = \begin{pmatrix} Z' \text{vec}(\Sigma^{-1}E) \\ Z_1' \text{vec}(\Sigma^{-1}E) \end{pmatrix}$$

and that the first diagonal block of the information matrix is:

$$\begin{aligned} \mathcal{I}_{\alpha\alpha} &= \begin{pmatrix} Z'(I_T \otimes \Sigma^{-1})Z & Z'(I_T \otimes \Sigma^{-1})Z_1 \\ Z_1'(I_T \otimes \Sigma^{-1})Z & Z_1'(I_T \otimes \Sigma^{-1})Z_1 \end{pmatrix} \\ &= \begin{pmatrix} \mathcal{I}_{bb} & \mathcal{I}_{bc} \\ \mathcal{I}_{bc}' & \mathcal{I}_{cc} \end{pmatrix}. \end{aligned} \quad (20)$$

It is immediately clear, upon examination of (9), (10), and (14), that the preceding developments apply to our model if we define b as in Section 2, $c = \text{vec}R$, Z as in (16), and $Z_1 = (U_1' \otimes I_n)$. It follows that:³

$$\frac{\partial L}{\partial \alpha} = \begin{pmatrix} D_S' \text{vec} \left(\Sigma^{-1} E X' D_H' - \sum_{j=1}^p R_j' \Sigma^{-1} E X_{-j}' D_H' \right) \\ \text{vec} \Sigma^{-1} E U_1' \end{pmatrix} \quad (21)$$

$$\begin{aligned} \mathcal{I}_{bb} &= D_S' \left((D_H X \otimes \Sigma^{-1}) - \sum_{j=1}^p (D_H X_{-j} \otimes R_j' \Sigma^{-1}) \right) \times \\ &\quad \left((X' D_H' \otimes I_n) - \sum_{j=1}^p (X_{-j}' D_H' \otimes R_j) \right) D_S \end{aligned} \quad (22)$$

$$\mathcal{I}_{bc} = D_S' \left((D_H X U_1' \otimes \Sigma^{-1}) - \sum_{j=1}^p (D_H X_{-j} U_1' \otimes R_j' \Sigma^{-1}) \right) \quad (23)$$

$$\mathcal{I}_{cc} = U_1 U_1' \otimes \Sigma^{-1}. \quad (24)$$

Our generic Lagrange multiplier test statistic follows immediately from the above results. Following Breusch and Pagan (1980), it may be written as:

$$LM = \left(\frac{\partial L}{\partial \alpha}(\hat{\alpha}_0) \right)' \mathcal{I}_{\alpha\alpha}^{-1}(\hat{\alpha}_0) \left(\frac{\partial L}{\partial \alpha}(\hat{\alpha}_0) \right) \quad (25)$$

where $\hat{\alpha}_0$ denotes the ML estimate of $(B | R)$ under the null hypothesis. For the 12 tests mentioned in Table 1 of Section 1, the dimension of α will vary according to the alternative H_1 , since $\alpha = b$ when the maintained hypothesis includes A (i.e. $R = O$) and since we have $b = \text{vec}B$, $b = \text{vec}(C | S)$, or $b' = (\text{vec}'C \quad S_{11} \quad S_{12} \quad S_{22} \quad \dots \quad S_{nn})$ when the maintained hypothesis specifies $\bar{H}\bar{S}$, $H\bar{S}$, or HS , respectively. Since, as was seen in Section 2, the specification of $\bar{H}\bar{S}$ and $H\bar{S}$ result from replacing D_S and D_H by I_{nk} and I_k in the first case and D_S by $I_{n(k-1)}$ in the second case, we have the following theorem.

Theorem 2. *The LM test statistic for the twelve cases in Table 1 is given by Equation (25). $\frac{\partial L}{\partial \alpha}(\hat{\alpha}_0)$ and $\mathcal{I}_{\alpha\alpha}(\hat{\alpha}_0)$ are generated according to (8), (11)–(18), (20)–(24), and according to the following rules:*

1. If H_0 includes A , set R and $\hat{R}$ equal to O in (15), (16), (18), (21), (22), and (23). Otherwise set $R = \hat{R}$ and use Equation (11).
2. If H_1 includes A , replace $\mathcal{I}_{bc}$ and $\mathcal{I}_{cc}$ in (20) by empty matrices and let $\partial L/\partial \alpha = \partial L/\partial b$. Otherwise let $\partial L/\partial \alpha$, $\mathcal{I}_{bc}$ and $\mathcal{I}_{cc}$ be as in (21), (23), and (24).
3. If H_0 includes H , define D_H by Equation (12) in (14) to (16). Otherwise define $D_H = I_k$ in (14) to (16).
4. If H_1 includes H , define D_H by Equation (12) in (21) to (23). Otherwise define $D_H = I_k$ in (21) to (23).
5. If H_0 includes S , define D_S by Equation (13) in (14) to (16). Otherwise let, in (14) to (16), $D_S = I_{nk}$ if H_0 includes $\bar{H}$ and $D_S = I_{n(k-1)}$ if H_0 includes H .
6. If H_1 includes S , define D_S by Equation (13) in (21) to (23). Otherwise let, in (21) to (23), $D_S = I_{nk}$ if H_1 includes $\bar{H}$ and $D_S = I_{n(k-1)}$ if H_1 includes H .

Since in the first three cases of Table 1, the model is linear both under the null and under the alternative, we have the following theorem, due to Berndt and Savin (1977).

Theorem 3. *If both H_0 and H_1 include A , then:*

$$LM = \text{Tr} \left(\hat{\Sigma}_0^{-1}(\hat{\Sigma}_0 - \hat{\Sigma}_1) \right) \quad (26)$$

where $\hat{\Sigma}_0$ and $\hat{\Sigma}_1$ are the ML estimators of Σ under H_0 and H_1 , respectively.

Equation (26) obviously presents a definite advantage over Equation (25) since it only requires the inversion of an $n \times n$ matrix. We will now show that similar simplifications occur in cases 5, 6, 8, 9, and 10 of Table 1 (the largest matrix to be inverted is of order $\max(k, np)$).

Theorem 4. Define:

$$\begin{aligned} Q_X &= X'(XX')^{-1}X \\ Q_X^H &= X'D_H'(D_HXX'D_H')^{-1}D_HX \\ \hat{U} &= Y - \hat{B}_0X \\ \hat{U}_1 &= Y_1 - (I_p \otimes \hat{B}_0)X_1 \\ \hat{\Sigma} &= \hat{U}\hat{U}'/T \\ Q &= \hat{U}'\hat{\Sigma}^{-1}\hat{U} \\ Q_1 &= \hat{U}_1' \left(\hat{U}_1(I_T - Q_X)\hat{U}_1' \right)^{-1} \hat{U}_1 \\ Q_1^H &= \hat{U}_1' \left(\hat{U}_1(I_T - Q_X^H)\hat{U}_1' \right)^{-1} \hat{U}_1 \end{aligned}$$

where $\hat{B}_0$ is the ML estimate of B under H_0 . The Lagrange multiplier test statistic (25) is given, for the tests numbered 8, 9, and 10 in Table 1, by:

$$LM = \text{tr} \left(Q \left(Q_X(I_T + Q_1Q_X - 2Q_1) + Q_1 \right) \right) \quad (27)$$

and for the tests numbered 5 and 6 in Table 1 by:

$$LM = \text{tr} \left(Q \left(Q_X^H(I_T + Q_1^HQ_X^H - 2Q_1^H) + Q_1^H \right) \right). \quad (28)$$

Furthermore, for the tests numbered 6 and 10, the statistic simplifies respectively to $LM = \text{tr}(QQ_1^H)$ and to $LM = \text{tr}(QQ_1)$.

Proof: Since (28) results from replacing X by D_HX in Q_X and Q_1 , it suffices to prove that the LM statistic (25) reduces to Equation (27) for the tests numbered 8, 9, and 10. In these cases H_1 specifies unconstrained estimation, so that Rules 4 and 6 in Theorem 2 prescribe replacing D_S by I_{nk} and D_H by I_k in (21) to (23). Furthermore H_0 specifies no serial correlation, so that Rule 1 prescribes replacing R by O in (21) to (23). We may then write (20) and (21) as:

$$\frac{\partial L}{\partial \alpha} = \begin{pmatrix} \text{vec } \Sigma^{-1}EX' \\ \text{vec } \Sigma^{-1}EU_1' \end{pmatrix} \quad (29)$$

$$\mathcal{I}_{\alpha\alpha} = \begin{pmatrix} XX' & XU_1' \\ U_1X' & U_1U_1' \end{pmatrix} \otimes \Sigma^{-1}. \quad (30)$$

By the partitioned inversion formula, we have:

$$\mathcal{I}_{\alpha\alpha}^{-1} = \begin{pmatrix} \mathcal{I}^{bb} & \mathcal{I}^{bc} \\ \mathcal{I}^{cb} & \mathcal{I}^{cc} \end{pmatrix} \otimes \Sigma, \quad (31)$$

with:

$$\mathcal{I}^{cc} = \left(U_1 \left(I_T - X'(XX')^{-1}X \right) U_1' \right)^{-1} \quad (32)$$

$$\mathcal{I}^{bb} = (XX')^{-1} \left(I_k + XU_1' \mathcal{I}^{cc} U_1 X' (XX')^{-1} \right) \quad (33)$$

$$\mathcal{I}^{bc} = -(XX')^{-1} XU_1' \mathcal{I}^{cc} \quad (34)$$

$$\mathcal{I}^{cb} = (\mathcal{I}^{bc})'. \quad (35)$$

It follows that:

$$\begin{aligned} \left(\frac{\partial L}{\partial \alpha} \right)' \mathcal{I}_{\alpha\alpha}^{-1} \left(\frac{\partial L}{\partial \alpha} \right) &= \begin{pmatrix} \text{vec}' \Sigma^{-1} E X' & \text{vec}' \Sigma^{-1} E U_1' \end{pmatrix} \begin{pmatrix} \text{vec} (E X' \mathcal{I}^{bb} + E U_1' \mathcal{I}^{cb}) \\ \text{vec} (E X' \mathcal{I}^{bc} + E U_1' \mathcal{I}^{cc}) \end{pmatrix} \\ &= \text{tr} (X E' \Sigma^{-1} E X' \mathcal{I}^{bb} + X E' \Sigma^{-1} E U_1' \mathcal{I}^{cb} + U_1 E' \Sigma^{-1} E X' \mathcal{I}^{bc} + U_1 E' \Sigma^{-1} E U_1' \mathcal{I}^{cc}) \\ &= \text{tr} (X E' \Sigma^{-1} E X' \mathcal{I}^{bb} + 2 X E' \Sigma^{-1} E U_1' \mathcal{I}^{cb} + U_1 E' \Sigma^{-1} E U_1' \mathcal{I}^{cc}) \\ &= \text{tr} (E' \Sigma^{-1} E X' \mathcal{I}^{bb} X + 2 E' \Sigma^{-1} E U_1' \mathcal{I}^{cb} X + E' \Sigma^{-1} E U_1' \mathcal{I}^{cc} U_1) \\ &= \text{tr} (E' \Sigma^{-1} E (X' \mathcal{I}^{bb} X + 2 U_1' \mathcal{I}^{cb} X + U_1' \mathcal{I}^{cc} U_1)). \end{aligned} \quad (36)$$

We now use the definitions in the statement of the theorem, replace U_1 by $\hat{U}_1$, and replace, according to Rule 1 in Theorem 2, E by $\hat{E} = \hat{U} = Y - \hat{B}_0 X$ in (36). We see that:

$$X' \mathcal{I}^{bb} X = Q_X + Q_X Q_1 Q_X$$

$$\hat{U}_1' \mathcal{I}^{cb} X = -Q_1 Q_X$$

$$\hat{U}_1' \mathcal{I}^{cc} \hat{U}_1 = Q_1$$

and Equation (27) follows from (36) and from the symmetry of Q , Q_1 , and Q_X , which implies:

$$\text{tr} (Q Q_1 Q_X) = \text{tr} (Q_1 Q_X Q) = \text{tr} (Q Q_X Q_1).$$

Lastly, Equation (27) simplifies to $\text{tr}(Q Q_1)$ for Test 10 in Table 1, since in this case $\hat{B}_0 = Y X' (X X')^{-1}$ and $\hat{U} = Y(I_T - Q_X)$, so that $Q Q_X = O$ in (27). This concludes the proof of Theorem 4. $\diamond$

We conclude this section with two remarks. First, in the case where the system consists of a single equation, it is straightforward to verify that $\text{tr}(Q Q_1)$ reduces to TR^2 , where R^2 is the

coefficient of determination in a regression of the vector of OLS residuals $\hat{u}$ on $(X, \hat{u}_{-1}, \dots, \hat{u}_{-p})$ (see Breusch and Pagan (1980)). Secondly, it is easy to redefine the null hypothesis A as meaning $R_j = \rho_j I_n$ for all j , rather than $R_j = O$ for all j . It should be pointed out, however, that this redefined null hypothesis is only slightly less restrictive than the total absence of serial correlation. All the results of Theorem 4 can be seen to apply to this case as well, upon replacing X by $X_* = X - \sum_{j=1}^p \hat{\rho}_j X_{-j}$ and Y by $Y_* = Y - \sum_{j=1}^p \hat{\rho}_j Y_{-j}$, as in Equations (18') and (19'). Note, in particular, that $\hat{U} = Y - \hat{B}_0 X$ must be replaced by $\hat{E} = Y_* - \hat{B}_0 X_*$; the two are no longer equal under the redefined null hypothesis.

Section 4. An empirical illustration

As announced in the introduction, we will illustrate the preceding theory with an estimation on quarterly British data of two well-known consumer demand systems. The first one is the AIDS model (Deaton and Muellbauer (1980)):

$$w_{it} = \alpha_i + \delta_i t + \sum_j \gamma_{ij} \log p_{jt} + \beta_i \log(x_t/P_t) + u_{it} \quad (37)$$

where p_{it} denotes the price of commodity i at time t ; $x_t = \sum_j p_{jt} q_{jt}$ is total expenditure; $w_{it} = p_{it} q_{it}/x_t$ is the budget share of commodity i ; and $\log P_t = \sum_j w_{jt} \log p_{jt}$. The second one is the Rotterdam system (Theil (1975); Barten (1969)), given by:

$$\bar{w}_{it} \Delta \log q_{it} = a_i + b_i \sum_j \bar{w}_{jt} \Delta \log q_{jt} + \sum_j c_{ij} \Delta \log p_{jt} + u_{it}^* \quad (38)$$

where $\bar{w}_{it} = (w_{it} + w_{i,t-4})/2$, $\Delta \log q_{it} = \log q_{it} - \log q_{i,t-4}$, and $\Delta \log p_{it} = \log p_{it} - \log p_{i,t-4}$.

The models differ in two substantial respects: firstly, they depend on two different parameterizations: for instance, the Rotterdam model assumes constant coefficients $b_i = p_{it}(\partial q_{it}/\partial x_t)$, whereas the AIDS model assumes constant $\beta_i = (\partial w_{it}/\partial \log x_t) = p_{it}(\partial q_{it}/\partial x_t) - w_{it}$. Secondly, and perhaps more importantly in the present context, the AIDS model is in levels whereas the Rotterdam model is in first differences; the properties of the disturbances u_{it} and u_{it}^* should therefore be quite different. Consequently, any comparative estimation of the two systems should model adequately the stochastic processes generating their respective disturbances. This will be amply illustrated by our numerical results.

The quarterly expenditures on nine classes of consumption goods (Food; Alcoholic Drink and Tobacco; Housing; Fuel and Light; Clothing and Footwear; Durable Household Goods; Cars and Motorcycles; Other Goods; Other Services) were taken from the *Economic Trends Annual*

Supplement, 1983, for the period going from 1955 (Quarter 1) to 1982 (Quarter 2). Price indices were taken from the *Monthly Digest of Statistics*.

In order to keep the number of estimated parameters within reasonable bounds, we estimated both models under the specification $R = (R_1 \ O \ O \ R_4)$. For this alternative hypothesis, Test number 10 of Table 1 yields Lagrange multiplier statistics of 391.33 for the AIDS model and 281.57 for the Rotterdam model. Both are highly significant, since the 1% critical value of the Chi-square distribution with 128 degrees of freedom is 168.13.

TABLE 2: LIKELIHOOD RATIO TESTS FOR REGULARITY

Model	Test no.	H_0	H_1	LR	LRC	CV1	CV5
AIDS	1	AHS	$AH\bar{S}$	112.40	98.35	48.28	41.34
AIDS	2	AHS	$A\bar{H}\bar{S}$	149.45	130.09	58.62	51.00
AIDS	3	$AH\bar{S}$	$A\bar{H}\bar{S}$	37.06	31.67	20.09	15.51
AIDS	11	$\bar{A}HS$	$\bar{A}\bar{H}\bar{S}$	138.36	98.88	58.62	51.00
Rotterdam	1	AHS	$AH\bar{S}$	86.50	76.10	48.28	41.34
Rotterdam	2	AHS	$A\bar{H}\bar{S}$	107.73	94.26	58.62	51.00
Rotterdam	3	$AH\bar{S}$	$A\bar{H}\bar{S}$	21.23	18.22	20.09	15.51
Rotterdam	7	$\bar{A}HS$	$\bar{A}\bar{H}\bar{S}$	90.59	65.06	48.28	41.34
Rotterdam	11	$\bar{A}HS$	$\bar{A}\bar{H}\bar{S}$	96.86	69.08	58.62	51.00
Rotterdam	12	$\bar{A}H\bar{S}$	$\bar{A}\bar{H}\bar{S}$	6.27	4.36	20.09	15.51

LR = likelihood ratio test statistic. LRC = likelihood ratio test statistic with small-sample correction. $CV1$ = critical value, 1% significance level. $CV5$ = critical value, 5% significance level.

Table 2 presents the likelihood ratio test statistics for regularity (Tests 1, 2, 3, 7, 11, and 12), both with and without small-sample corrections (Anderson (1958, pp. 207–210); Italianer (1985)). The regularity test statistics under the maintained hypothesis of no serial correlation (Tests 1, 2, and 3) are given for comparison purposes only. It is emphasized that the small-sample corrections have a rigorous theoretical basis only in the case of Test number 3. In the part of Table 2 that relates to the AIDS model, we could only perform Test number 11, due to the lack of convergence of all the optimization algorithms that were tried on the AIDS model with homogeneity only (the information matrix became singular, and the likelihood function discontinuous, during

the iterations). This problem can also occur in the single equation model if the autocorrelation coefficient becomes arbitrarily close to one, and has also been encountered in other contexts; see Quandt (1983, p. 744).

It is immediately apparent from Table 2 that taking autocorrelation into account systematically decreases the tests statistics for regularity. However, symmetry is still rejected by all models, even when heuristic small-sample corrections are made. Nevertheless, the change in the LR statistic for homogeneity in the Rotterdam system is quite dramatic: in the autocorrelated model, homogeneity is not rejected at the 38% significance level; in the model without correlation, it is rejected at the 5% level, though not at the 1% level if the Anderson small-sample correction is made.

Tables 3 and 4 present the results for the AIDS model under adding-up with and without serial correlation, respectively. Tables 5 and 6 present the Rotterdam model under homogeneity. In the case of the AIDS model, the differences are indeed quite substantial. Incidentally, the technique of introducing seasonal constants and seasonally variable income coefficients (see Blundell and Ray (1982)) does not fully deseasonalize the AIDS model: in this case, the LM test of $R = O$ against $R = (R_1 \ R_2 \ R_3 \ R_4)$ under the maintained hypothesis of adding-up yields a statistic of 435.11, larger than the 1% critical value of 311.56.

As mentioned at the beginning of this section, the different properties of the AIDS and Rotterdam disturbances provide an important check on the validity of our methodology. Indeed, it is of interest to ascertain whether the marginal propensities to spend and the diagonal elements of the Slutsky matrix implied by the two models are comparable (such a comparison would be meaningless for the off-diagonal elements, since symmetry is rejected). An approximate correspondence between the coefficients of the two models is given by:

$$b_i \approx \beta_i + \bar{w}_i \stackrel{\text{def}}{=} b_i^*$$

$$c_{ii} \approx \gamma_{ii} + \beta_i^2 \log(\bar{x}/\bar{P}) - \bar{w}_i + \bar{w}_i^2 \stackrel{\text{def}}{=} c_{ii}^*$$

where $\log \bar{P} = \sum_j \bar{w}_j \log \bar{p}_j$, and where $\bar{w}_i$, $\bar{x}$, and $\bar{p}_j$ denote sample means.

Table 7 presents the b_i , b_i^* , c_{ii} , and c_{ii}^* in the models with and without serial correlation, both estimated under adding-up. The corresponding elasticities (ϵ_i , ϵ_i^* , η_{ii} , η_{ii}^*), obtained by dividing b_i , b_i^* , c_{ii} , and c_{ii}^* by $\bar{w}_i$, are also given. There is a reasonable agreement when serial correlation is taken into account. Very substantial differences exist when it is not. For instance, the AIDS model without serial correlation classifies Fuel/Light and Vehicles as inferior goods, and predicts an incorrect sign for the own Slutsky coefficients in these two equations. It also improperly classifies Drink/Tobacco and Other Goods as luxuries.

Another observation in Table 7 is the surprisingly good performance of the Rotterdam model without serial correlation. With the exception of Food and Services, the marginal propensities b_i are quite close to the ones estimated when serial correlation is taken into account. Furthermore, the own price coefficients in the equations for Housing, Clothing and Durables are virtually the same. This can be taken as evidence that the definition of the first differences used in Equation (38) is appropriate. Indeed, the likelihood ratio test statistic for autocorrelation in the Rotterdam model with homogeneity is 167.07, slightly less than the 1% critical value of 168.13 (but still significant at the 5% level). By contrast, the same statistic for the AIDS model under adding-up is 854.46.

Space considerations do not allow us to report the full sets of autocorrelation coefficients (there are 128 such coefficients for each model). The matrices R_1 and R_4 do not appear to be diagonal: for the Rotterdam system, R_1 includes $n_{1,D} = 5$ significant diagonal elements, and $n_{1,O} = 8$ significant off-diagonal elements; and R_4 includes $n_{4,D} = 2$ significant diagonal elements, and $n_{4,O} = 14$ significant off-diagonal elements. The corresponding figures for the AIDS model are $n_{1,D} = 4$, $n_{1,O} = 20$, $n_{4,D} = 7$, and $n_{4,O} = 25$. All the coefficients were less than one in absolute value, some of them being quite close to unity: in the AIDS model, for instance, the fourth and fifth diagonal elements of R_4 were equal to 0.83 and 0.82, with "t-ratios" of 14.14 and 12.00, respectively. This confirms our previous observation that autocorrelation is quite severe in the case of AIDS.

Section 5. Conclusions

This paper has attempted to provide a methodology for jointly testing autocorrelation and regularity in allocation systems. It has been argued that both issues cannot be treated separately. Most of the tests that we propose require only the estimation of a linear allocation system, and hence do not require an inordinate amount of computational effort. By contrast, the estimation of a large autocorrelated allocation system is quite difficult, and convergence proved impossible to obtain in one case.

It is unusual to include commodities such as Durables and Vehicles in classical allocations systems such as (37) or (38), since the corresponding expenditures have strong dynamic components. Nevertheless, we have shown that the parameters of those equations can be plausibly estimated, provided that autocorrelation is taken into account.

We have also shown that an allocation system can be reliably estimated from quarterly data; the estimates presented in Tables 3 and 5 are both quite significant and plausible. We have argued that the estimation technique underlying these estimates is superior to the mere introduction of seasonal dummies. Indeed, these do not appear to fully deseasonalize the data.

Autocorrelation was found to be less severe in the Rotterdam model than in the AIDS model. In fact, the estimated regression coefficients in the correlated and uncorrelated versions of the Rotterdam system are quite close. This is perhaps not too surprising since neglecting autocorrelation produces consistent, albeit asymptotically inefficient, estimated coefficients. Nevertheless, Table 7 amply illustrates that severe inefficiency can produce results that are quite implausible. Furthermore, as shown in Table 2, the inconsistency of the estimated variances has serious consequences on the various test statistics for regularity.

Given the computational cost of obtaining our estimates, however, we do not advocate the systematic estimation of autocorrelated allocation systems. It is much more appealing to reduce autocorrelation by augmenting the list of regressors, i.e. introducing conditioning variables or price expectations in Equations (37) and (38). The various tests of Table 1 should enable the investigator to ascertain whether the added explanatory variables fully account for the observed dynamic behavior of the explained variables, and should therefore be quite useful in this context.

TABLE 3: AUTOCORRELATED AIDS MODEL UNDER ADDING-UP

	α_i	β_i	δ_i	γ_{i1}	γ_{i2}	γ_{i3}
Food	.1814 3.6995	-.0862 -6.7164	-.0008 -3.2237	.0826 6.9759	-.0187 -2.1502	-.0663 -6.2310
Drink/Tob.	.2911 6.7488	-.0699 -5.3771	.0011 4.5439	-.0168 -1.3335	.0316 3.6246	-.0744 -7.6442
Housing	-.0177 -5.071	-.0456 -3.9849	.0004 2.3007	.0217 1.7640	.0246 2.9943	.0751 8.3207
Fuel/Light	-.0028 -.0504	-.0278 -1.9824	.0000 .0320	.0159 1.2387	.0030 .3085	.0084 .7300
Clothing	.2500 6.3472	.0211 2.0036	.0005 2.1408	-.0261 -2.7975	.0018 .2702	-.0120 -1.4670
Durables	.0401 1.3868	.0963 10.7363	-.0010 -6.4140	.0326 3.6542	-.0188 -3.0753	.0564 8.6591
Vehicles	-.0287 -4.015	.2274 11.4648	-.0018 -4.5921	-.0299 -1.6058	.0186 1.3187	.0153 .9587
Oth. Goods	.0669 1.7746	-.0041 -.4354	.0004 2.0147	-.0292 -3.2570	.0097 1.4364	.0101 1.2816
Oth. Serv.	.2198 3.1837	-.1112 -7.7494	.0012 3.2674	-.0509 -3.8048	-.0517 -5.2016	-.0126 -1.0417
	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{i9}
Food	-.0330 -3.6193	-.0257 -1.3455	.0119 .6800	.0079 .5142	.0203 1.2958	.0309 2.5020
Drink/Tob.	-.0028 -.3028	-.0007 -.0350	-.0300 -1.6523	.0403 2.4717	-.0158 -.9460	.0234 1.7120
Housing	-.0209 -2.2711	.0495 2.6257	-.0631 -3.4522	-.0466 -3.0201	-.0296 -1.7596	.0049 .3509
Fuel/Light	.0295 2.9882	.0056 .2686	-.0054 -.2875	-.0309 -1.8114	-.0001 -.0085	-.0199 -1.4452
Clothing	-.0129 -1.8289	.0070 .4726	.0242 1.8050	.0203 1.6736	-.0219 -1.7697	-.0106 -1.0636
Durables	.0087 1.3099	-.0097 -.7098	-.0217 -1.6606	-.0195 -1.7251	.0271 2.2845	-.0421 -4.1593
Vehicles	.0599 4.1495	-.0133 -.4581	.0375 1.3211	-.0290 -1.1968	-.0059 -.2374	-.0088 -.4405
Oth. Goods	-.0046 -.6538	.0215 1.4755	-.0180 -1.3079	.0240 2.1075	.0214 1.7864	-.0232 -2.4194
Oth. Serv.	-.0238 -2.3291	-.0341 -1.5542	.0647 3.2598	.0336 1.9561	.0045 .2564	.0455 3.2409

Loglikelihood = 4196.423. The figures show the estimated coefficients and the coefficients divided by their estimated asymptotic standard errors.

TABLE 4: UNCORRELATED AIDS MODEL UNDER ADDING-UP

	α_i	β_i	δ_i	γ_{i1}	γ_{i2}	γ_{i3}
Food	.2110 6.2479	-.0966 -13.5048	-.0004 -2.7668	.0267 1.7750	-.0259 -2.6528	-.0222 -2.1533
Drink/Tob.	.2417 4.9989	.1482 14.4704	-.0009 -4.0625	.0066 .3054	.0901 6.4418	-.0638 -4.3236
Housing	.0321 1.2933	-.0755 -14.3469	.0009 7.8075	.0204 1.8435	.0128 1.7742	.0415 5.4700
Fuel/Light	-.1085 -1.1894	-.1036 -5.3609	.0004 .9627	-.0908 -2.2345	-.0876 -3.3192	-.0005 -.0181
Clothing	.2102 4.2510	.1828 17.4506	-.0012 -5.3868	-.0072 -.3250	-.0081 -.5690	-.0252 -1.6666
Durables	.0706 2.2850	.0552 8.4243	-.0005 -3.3112	.0105 .7659	-.0636 -7.1071	.0135 1.4354
Vehicles	.1639 2.1796	-.0936 -5.8745	.0010 2.8443	.0992 2.9623	-.0326 -1.4964	.0401 1.7487
Oth. Goods	.1479 3.8118	.0659 8.0157	.0005 3.1517	-.0476 -2.7560	.0012 .1059	-.0247 -2.0861
Oth. Serv.	.0310 .2667	-.0828 -3.3588	.0002 .3480	-.0179 -.3447	.1137 3.3789	.0412 1.1606
	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{i9}
Food	.0394 3.2435	.0157 .6659	.0532 2.0553	.0227 .9520	-.0437 -1.7981	-.0625 -3.4251
Drink/Tob.	-.0208 -1.1939	-.0352 -1.0415	-.0708 -1.9120	.0248 .7272	-.0091 -.2607	.0716 2.7406
Housing	-.0311 -3.4834	.0172 .9893	-.0368 -1.9312	-.0225 -1.2827	-.0205 -1.1445	.0195 1.4553
Fuel/Light	.1805 5.5018	.0986 1.5476	.0855 1.2240	-.1538 -2.3920	-.0055 -.0843	-.0042 -.0854
Clothing	.0492 2.7656	-.0483 -1.3983	.0324 .8544	-.0308 -.8841	.0029 .0818	.0328 1.2278
Durables	.0525 4.7284	-.0209 -.9675	.0371 1.5659	-.0396 -1.8173	.0058 .2608	.0043 .2580
Vehicles	-.0545 -2.0141	-.0679 -1.2927	-.0322 -.5589	.0603 1.1374	.0902 1.6658	-.1471 -3.6183
Oth. Goods	.0196 1.4073	-.0143 -.5261	.0788 2.6491	-.0155 -.5666	-.0475 -1.7010	.0518 2.4721
Oth. Serv.	-.2348 -5.6160	.0550 .6774	-.1471 -1.6510	.1545 1.8841	.0274 .3275	.0336 .5352

Loglikelihood = 3769.192. The figures show the estimated coefficients and the coefficients divided by their estimated asymptotic standard errors.

TABLE 5: AUTOCORRELATED ROTTERDAM MODEL UNDER HOMOGENEITY

	a_i	b_i	c_{i1}	c_{i2}	c_{i3}	c_{i4}
Food	-.0016 -2.3449	.1150 6.9746	-.1233 -11.0301	.0362 3.9227	.0126 1.1458	-.0010 -.0930
Drink/Tob.	.0002 .3445	.0919 6.7574	.0149 1.5205	-.0547 -6.7501	-.0144 -1.4854	-.0116 -1.2569
Housing	.0032 5.3123	.0642 4.7438	.0376 3.8015	.0162 2.0004	-.0777 -7.9706	.0041 .4408
Fuel/Light	.0003 .4828	.0112 .8310	.0103 1.1151	.0058 .7805	.0216 2.4975	-.0055 -.6588
Clothing	.0005 1.1710	.0957 8.4605	-.0207 -2.7450	.0194 3.1021	-.0106 -1.4055	-.0091 -1.2644
Durables	-.0035 -8.0144	.1465 13.2415	.0512 7.5416	-.0149 -2.6543	.0367 5.2238	-.0023 -.3524
Vehicles	-.0067 -6.3992	.3161 15.8247	.0297 1.9622	-.0102 -.7849	.0538 3.5796	.0518 3.5974
Oth. Goods	.0036 6.7486	.1071 12.1959	.0012 .1601	.0001 .0097	.0168 2.2170	-.0006 -.0874
Oth. Serv.	.0040 3.4862	.0523 3.6572	-.0008 -.0603	.0022 .1964	-.0389 -3.1057	-.0259 -2.2648
	c_{i5}	c_{i6}	c_{i7}	c_{i8}	c_{i9}	
Food	.0527 3.1979	-.0024 -.1128	.0379 2.5648	-.0253 -1.3518	.0127 .8045	
Drink/Tob.	.0167 1.1967	-.0085 -.4440	.0099 .7924	-.0144 -.9055	.0621 4.8760	
Housing	.0586 4.1154	-.0714 -3.6810	-.0151 -1.2265	.0374 2.2841	.0103 .7363	
Fuel/Light	.0225 1.7759	-.0309 -1.8306	-.0136 -1.1569	.0283 1.9839	-.0385 -3.3101	
Clothing	-.0204 -1.8562	.0090 .6046	.0207 2.0568	.0072 .5736	.0045 .4268	
Durables	-.0278 -2.8083	-.0736 -5.2995	-.0071 -.7805	.0354 3.0647	.0024 .2352	
Vehicles	-.0969 -4.3711	.0636 2.1388	-.1003 -5.3663	.0121 .4880	-.0036 -1.793	
Oth. Goods	.0027 .2447	.0470 3.3623	.0172 1.8187	-.0875 -7.5325	.0031 .3486	
Oth. Serv.	-.0081 -.4576	.0671 2.9742	.0505 3.2564	.0068 .3624	-.0530 -3.7452	

Loglikelihood = 3991.885. The figures show the estimated coefficients and the coefficients divided by their estimated asymptotic standard errors.

TABLE 6: UNCORRELATED ROTTERDAM MODEL UNDER HOMOGENEITY

	a_i	b_i	c_{i1}	c_{i2}	c_{i3}	c_{i4}
Food	-.0016 -2.6933	.0872 5.9778	-.0978 -8.9539	.0085 .9679	-.0051 -.5964	-.0172 -1.6785
Drink/Tob.	.0008 1.4094	.0686 5.2377	.0221 2.2520	-.0697 -8.8077	-.0053 -.6852	-.0168 -1.8289
Housing	.0037 6.1440	.0611 4.1626	.0230 2.0913	.0221 2.4931	-.0810 -9.4063	.0010 .0990
Fuel/Light	.0007 1.4619	.0105 .8704	.0073 .8127	.0108 1.4853	.0069 .9723	.0044 .5187
Clothing	.0006 1.4894	.0967 9.7633	-.0169 -2.2758	.0107 1.7857	-.0019 -.3209	-.0002 -.0274
Durables	-.0033 -7.4824	.1487 13.7137	.0456 5.6183	-.0093 -1.4195	.0214 3.3675	.0053 .6951
Vehicles	-.0063 -6.8018	.3044 13.3576	.0427 2.5024	.0066 .4822	-.0087 -.6515	.0349 2.1796
Oth. Goods	.0045 8.6759	.0879 6.8636	-.0176 -1.8342	.0168 2.1745	.0319 4.2393	.0048 .5389
Oth. Serv.	.0010 1.3970	.1349 8.0527	-.0084 -.6705	.0034 .3400	.0418 4.2557	-.0162 -1.3775
	c_{i5}	c_{i6}	c_{i7}	c_{i8}	c_{i9}	
Food	.0240 1.5581	-.0005 -.0279	.0138 .9323	.0286 1.7047	.0458 3.7480	
Drink/Tob.	.0259 1.8772	-.0124 -.7289	.0130 .9782	-.0006 -.0387	.0438 3.9991	
Housing	.0312 2.0204	-.0173 -.9048	-.0118 -.7962	.0129 .7623	.0200 1.6295	
Fuel/Light	.0141 1.1086	-.0034 -.2146	-.0219 -1.7970	.0045 .3274	-.0227 -2.2535	
Clothing	-.0210 -2.0085	.0214 1.6587	.0105 1.0478	-.0050 -.4355	.0022 .2711	
Durables	-.0261 -2.2809	-.0731 -5.1751	-.0026 -.2352	.0349 2.7980	.0039 .4289	
Vehicles	-.1029 -4.2842	.0146 .4909	-.0554 -2.4008	.0586 2.2385	.0096 .5033	
Oth. Goods	.0523 3.8779	.0327 1.9588	.0290 2.2363	-.1327 -9.0226	-.0172 -1.6074	
Oth. Serv.	.0024 .1362	.0381 1.7458	.0255 1.5024	-.0012 -.0629	-.0854 -6.0968	

Loglikelihood = 3908.350. The figures show the estimated coefficients and the coefficients divided by their estimated asymptotic standard errors.

**TABLE 7: IMPLIED AND ACTUAL ROTTERDAM MODEL COEFFICIENTS
(Adding-up)**

No serial correlation								
	b_i	b_i^*	c_{ii}	c_{ii}^*	ϵ_i	ϵ_i^*	η_{ii}	η_{ii}^*
Food	0.086	0.118	-0.097	-0.144	0.402	0.549	-0.453	-0.672
Drink/Tob.	0.065	0.270	-0.071	-0.023	0.533	2.216	-0.582	-0.185
Housing	0.063	0.046	-0.079	-0.067	0.518	0.379	-0.650	-0.549
Fuel/Light	0.009	-0.057	0.005	0.133	0.193	-1.225	0.107	2.863
Clothing	0.094	0.269	-0.029	-0.135	1.092	3.124	-0.337	-1.574
Durables	0.146	0.103	-0.072	-0.009	3.079	2.163	-1.519	-0.187
Vehicles	0.299	-0.060	-0.044	0.026	8.998	-1.817	-1.324	0.781
Oth. Goods	0.093	0.211	-0.126	-0.172	0.643	1.455	-0.871	-1.191
Oth. Serv.	0.145	0.102	-0.076	-0.118	0.787	0.551	-0.412	-0.643
Serial correlation								
	b_i	b_i^*	c_{ii}	c_{ii}^*	ϵ_i	ϵ_i^*	η_{ii}	η_{ii}^*
Food	0.128	0.128	-0.140	-0.088	0.598	0.598	-0.654	-0.409
Drink/Tob.	0.089	0.052	-0.057	-0.077	0.730	0.427	-0.467	-0.629
Housing	0.063	0.076	-0.075	-0.032	0.518	0.625	-0.617	-0.265
Fuel/Light	0.012	0.019	-0.008	-0.015	0.258	0.403	-0.172	-0.324
Clothing	0.092	0.107	-0.030	-0.072	1.069	1.245	-0.349	-0.834
Durables	0.144	0.144	-0.070	-0.069	3.037	3.031	-1.476	-1.460
Vehicles	0.315	0.261	-0.102	-0.074	9.480	7.845	-3.070	-2.234
Oth. Goods	0.108	0.141	-0.086	-0.102	0.746	0.971	-0.594	-0.708
Oth. Serv.	0.048	0.073	-0.056	-0.108	0.260	0.397	-0.304	-0.586

FOOTNOTES

1. I wish to thank A.P. Barten and the participants of the Econometrics Seminar at CentER for helpful comments. Any error is my own.
2. If R_j^* denotes the j -th matrix of autocorrelation coefficients in the *full* system, we have:

$$R_j = (I_n \quad O_{n \times 1}) R_j^* (I_n \quad -\iota_n)'$$

In the sequel, we denote by I_r an identity matrix of order r , by $O_{r \times s}$ a null matrix with r rows and s columns, and by ι_r a column vector of r ones. "vec" denotes the stack operator and "tr" denotes trace. The identities $\text{vec}(ABC) = (C' \otimes A) \text{vec } B$ and $\text{tr}(AB) = (\text{vec } A')' \text{vec } B$ will be used repeatedly.

3. Following Breusch and Pagan (1980), we interpret the expectations in $\mathcal{I}_{bc}$ and $\mathcal{I}_{cc}$ as being conditional on U_1 .

REFERENCES

- J. AITCHISON and S.D. SILVEY (1960), "Maximum Likelihood Estimation Procedures and Associated Tests of Significance", *Journal of the Royal Statistical Society, Series B*, 22, 154-171
- T.W. ANDERSON (1958), *An Introduction to Multivariate Statistical Analysis*, Wiley, New York
- P. BALESTRA (1976), *La Dérivation matricielle*, Sirey, Paris
- A.P. BARTEN (1969), "Maximum Likelihood Estimation of a Complete System of Demand Equations", *European Economic Review* 1, 7-73
- E.R. BERNDT and N.E. SAVIN (1975), "Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive disturbances", *Econometrica* 43, 937-957
- E.R. BERNDT and N.E. SAVIN (1977), "Conflict Among Criteria for Testing Hypotheses in the Multivariate Linear Regression Model", *Econometrica* 45, 1263-1277
- R. BLUNDELL and R. RAY (1982), "A Non-separable Generalization of the Linear Expenditure System Allowing Non-linear Engel Curves", *Economics Letters* 9, 349-354
- T.R. BREUSCH and A.R. PAGAN (1980), "The Lagrange Multiplier Test and its Applications to Model Specification in Econometrics", *Review of Economic Studies* 47, 239-253
- A.S. DEATON and J. MUELLBAUER (1980), "An Almost Ideal Demand System", *American Economic Review* 70, 312-326
- P.J. DESCHAMPS (1988), "A Note on the Maximum Likelihood Estimation of Allocation Systems", *Computational Statistics and Data Analysis* 6, 109-112
- L.G. GODFREY (1978), "Testing against general autoregressive and moving average error models when the regressors include lagged dependent variables", *Econometrica* 46, 1293-1301
- D.F. HENDRY (1971), "Maximum likelihood estimation of systems of simultaneous regression equations with errors generated by a vector autoregressive process", *International Economic Review* 12, 257-272
- A. ITALIANER (1985), "A Small Sample Correction for the Likelihood Ratio Test", *Economics Letters* 19, 315-317

- L.J. LAU (1978), "A Note on the Compatibility of a System of Difference Equations and a Time-Independent Linear Equation", *Economics Letters* 1, 243-247
- R.E. QUANDT (1983), "Computational Problems and Methods", in: *Handbook of Econometrics*, vol. I, ed. by Z. GRILICHES and M.D. INTRILIGATOR, North Holland, Amsterdam, 699-764
- H. THEIL (1975), *Theory and Measurement of Consumer Demand*, vols. I and II, North Holland, Amsterdam
- H. WHITE (1982), "Maximum Likelihood Estimation of Misspecified Models", *Econometrica* 50, 1-26

Discussion Paper Series, CentER, Tilburg University, The Netherlands:

(For previous papers please consult previous discussion papers.)

No.	Author(s)	Title
8932	E. van Damme, R. Selten and E. Winter	Alternating Bid Bargaining with a Smallest Money Unit
8933	H. Carlsson and E. van Damme	Global Payoff Uncertainty and Risk Dominance
8934	H. Huizinga	National Tax Policies towards Product- Innovating Multinational Enterprises
8935	C. Dang and D. Talman	A New Triangulation of the Unit Simplex for Computing Economic Equilibria
8936	Th. Nijman and M. Verbeek	The Nonresponse Bias in the Analysis of the Determinants of Total Annual Expenditures of Households Based on Panel Data
8937	A.P. Barten	The Estimation of Mixed Demand Systems
8938	G. Marini	Monetary Shocks and the Nominal Interest Rate
8939	W. Güth and E. van Damme	Equilibrium Selection in the Spence Signaling Game
8940	G. Marini and P. Scaramozzino	Monopolistic Competition, Expected Inflation and Contract Length
8941	J.K. Dagsvik	The Generalized Extreme Value Random Utility Model for Continuous Choice
8942	M.F.J. Steel	Weak Exogeneity in Misspecified Sequential Models
8943	A. Roell	Dual Capacity Trading and the Quality of the Market
8944	C. Hsiao	Identification and Estimation of Dichotomous Latent Variables Models Using Panel Data
8945	R.P. Gilles	Equilibrium in a Pure Exchange Economy with an Arbitrary Communication Structure
8946	W.B. MacLeod and J.M. Malcomson	Efficient Specific Investments, Incomplete Contracts, and the Role of Market Alternatives
8947	A. van Soest and A. Kapteyn	The Impact of Minimum Wage Regulations on Employment and the Wage Rate Distribution
8948	P. Kooreman and B. Melenberg	Maximum Score Estimation in the Ordered Response Model

No.	Author(s)	Title
8949	C. Dang	The D ₃ -Triangulation for Simplicial Deformation Algorithms for Computing Solutions of Nonlinear Equations
8950	M. Cripps	Dealer Behaviour and Price Volatility in Asset Markets
8951	T. Wansbeek and A. Kapteyn	Simple Estimators for Dynamic Panel Data Models with Errors in Variables
8952	Y. Dai, G. van der Laan, D. Talman and Y. Yamamoto	A Simplicial Algorithm for the Nonlinear Stationary Point Problem on an Unbounded Polyhedron
8953	F. van der Ploeg	Risk Aversion, Intertemporal Substitution and Consumption: The CARA-LQ Problem
8954	A. Kapteyn, S. van de Geer, H. van de Stadt and T. Wansbeek	Interdependent Preferences: An Econometric Analysis
8955	L. Zou	Ownership Structure and Efficiency: An Incentive Mechanism Approach
8956	P. Kooreman and A. Kapteyn	On the Empirical Implementation of Some Game Theoretic Models of Household Labor Supply
8957	E. van Damme	Signaling and Forward Induction in a Market Entry Context
9001	A. van Soest, P. Kooreman and A. Kapteyn	Coherency and Regularity of Demand Systems with Equality and Inequality Constraints
9002	J.R. Magnus and B. Pesaran	Forecasting, Misspecification and Unit Roots: The Case of AR(1) Versus ARMA(1,1)
9003	J. Driffill and C. Schultz	Wage Setting and Stabilization Policy in a Game with Renegotiation
9004	M. McAleer, M.H. Pesaran and A. Bera	Alternative Approaches to Testing Non-Nested Models with Autocorrelated Disturbances: An Application to Models of U.S. Unemployment
9005	Th. ten Raa and M.F.J. Steel	A Stochastic Analysis of an Input-Output Model: Comment
9006	M. McAleer and C.R. McKenzie	Keynesian and New Classical Models of Unemployment Revisited
9007	J. Osiewalski and M.F.J. Steel	Semi-Conjugate Prior Densities in Multivariate t Regression Models

No.	Author(s)	Title
9007	J. Osiewalski and M.F.J. Steel	Semi-Conjugate Prior Densities in Multi-variate t Regression Models
9008	G.W. Imbens	Duration Models with Time-Varying Coefficients
9009	G.W. Imbens	An Efficient Method of Moments Estimator for Discrete Choice Models with Choice-Based Sampling
9010	P. Deschamps	Expectations and Intertemporal Separability in an Empirical Model of Consumption and Investment under Uncertainty
9011	W. Güth and E. van Damme	Gorby Games - A Game Theoretic Analysis of Disarmament Campaigns and the Defense Efficiency-Hypothesis
9012	A. Horsley and A. Wrobel	The Existence of an Equilibrium Density for Marginal Cost Prices, and the Solution to the Shifting-Peak Problem
9013	A. Horsley and A. Wrobel	The Closedness of the Free-Disposal Hull of a Production Set
9014	A. Horsley and A. Wrobel	The Continuity of the Equilibrium Price Density: The Case of Symmetric Joint Costs, and a Solution to the Shifting-Pattern Problem
9015	A. van den Elzen, G. van der Laan and D. Talman	An Adjustment Process for an Exchange Economy with Linear Production Technologies
9016	P. Deschamps	On Fractional Demand Systems and Budget Share Positivity
9017	B.J. Christensen and N.M. Kiefer	The Exact Likelihood Function for an Empirical Job Search Model
9018	M. Verbeek and Th. Nijman	Testing for Selectivity Bias in Panel Data Models
9019	J.R. Magnus and B. Pesaran	Evaluation of Moments of Ratios of Quadratic Forms in Normal Variables and Related Statistics
9020	A. Robson	Status, the Distribution of Wealth, Social and Private Attitudes to Risk
9021	J.R. Magnus and B. Pesaran	Evaluation of Moments of Quadratic Forms in Normal Variables

No.	Author(s)	Title
9022	K. Kamiya and D. Talman	Linear Stationary Point Problems
9023	W. Emons	Good Times, Bad Times, and Vertical Upstream Integration
9024	C. Dang	The D_2 -Triangulation for Simplicial Homotopy Algorithms for Computing Solutions of Nonlinear Equations
9025	K. Kamiya and D. Talman	Variable Dimension Simplicial Algorithm for Balanced Games
9026	P. Skott	Efficiency Wages, Mark-Up Pricing and Effective Demand
9027	C. Dang and D. Talman	The D_1 -Triangulation in Simplicial Variable Dimension Algorithms for Computing Solutions of Nonlinear Equations
9028	J. Bai, A.J. Jakeman and M. McAleer	Discrimination Between Nested Two- and Three- Parameter Distributions: An Application to Models of Air Pollution
9029	Th. van de Klundert	Crowding out and the Wealth of Nations
9030	Th. van de Klundert and R. Gradus	Optimal Government Debt under Distortionary Taxation
9031	A. Weber	The Credibility of Monetary Target Announce- ments: An Empirical Evaluation
9032	J. Osiewalski and M. Steel	Robust Bayesian Inference in Elliptical Regression Models
9033	C. R. Wichers	The Linear-Algebraic Structure of Least Squares
9034	C. de Vries	On the Relation between GARCH and Stable Processes
9035	M.R. Baye, D.W. Jansen and Q. Li	Aggregation and the "Random Objective" Justification for Disturbances in Complete Demand Systems
9036	J. Driffill	The Term Structure of Interest Rates: Structural Stability and Macroeconomic Policy Changes in the UK
9037	F. van der Ploeg	Budgetary Aspects of Economic and Monetary Integration in Europe

No.	Author(s)	Title
9038	A. Robson	Existence of Nash Equilibrium in Mixed Strategies for Games where Payoffs Need not Be Continuous in Pure Strategies
9039	A. Robson	An "Informationally Robust Equilibrium" for Two-Person Nonzero-Sum Games
9040	M.R. Baye, G. Tian and J. Zhou	The Existence of Pure-Strategy Nash Equilibrium in Games with Payoffs that are not Quasiconcave
9041	M. Burnovsky and I. Zang	"Costless" Indirect Regulation of Monopolies with Substantial Entry Cost
9042	P.J. Deschamps	Joint Tests for Regularity and Autocorrelation in Allocation Systems

PO BOX 90153 5000 LE TILBURG THE NETHERLANDS

Bibliotheek K. U. Brabant

17 000 01117584 2