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**THE LOCATION MODEL WITH TWO
PERIODS OF PRICE COMPETITION**

by Harry Webers

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The location model with two periods of price competition *

Harry Webers[†]

July 1994

Abstract

In this paper we characterize the subgame perfect Nash equilibria of a location-then-price game where firms first choose locations and after that compete for prices in two subsequent periods. Locations are thus seen as long term commitments. There are two types of consumers, each with possibly different valuations for the variants offered by the firms. Due to changes in the fractions of the consumer types, competition in both periods may differ. Firms anticipate that their location choice influences price competition in both periods and therefore maximize their two period profit. We prove the existence of a unique subgame perfect Nash equilibrium and give explicit expressions for the location choices for some specific cases.

JEL Classification: C72, D43, L13, R11.

Keywords: *location theory, overlapping generations, subgame perfect Nash equilibrium.*

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1 Introduction

In the literature the location decision of firms is assumed to be a strategic one, in other words, it is assumed that firms take into account the effect of their location decision on the price they can set.

In case of multiple periods, if price competition is the same in every period, it is sufficient to study the standard two stage location-then-price game due to Hotelling (1929). This is the procedure usually followed in the literature. If however price competition differs over periods, for example due to changes from the demand side over time, the outcome of the standard model, with only one period of price competition, is not appropriate.

In this paper we look at the situation where location is a two period commitment for each firm and so there are two periods of price competition. Usually firms are concerned about their (near) future and try to incorporate future changes into their decision (see for example Harrison (1987)). We adopt the view that firms take into account the impact of their decision on their per period profits. A two period location commitment would be the case if the costs of relocation are relatively high or a contract is signed by the firm to be located on the same spot for two periods. Friedman and Thisse (1993) look at n period location commitments, but they assume that firms will select their locations, knowing that a particular trigger price equilibrium will ensue.

In order to model the differences in price competition between the two periods, we consider an economy with two types of consumers, having possibly valuation differences for the products offered by the firms. The fractions of consumer types may differ over time. The model is motivated by the overlapping generations literature, that promotes the idea that consumers' valuations differ by age (see for example Samuelson (1958), Diamond (1965), or Weddepohl (1990)).

Each consumer buys one of the mutually exclusive variants. Consumers take a decision per period and buy from the cheapest source, i.e., the firm with the lowest overall price. As in Anderson, de Palma and Thisse (1992) we use an indirect utility function that involves both transportation costs and quality difference aspects. The valuations consumers have for the products of the firms can be seen as prices to compensate for (subjective) quality differences.

We will derive that there exists a unique equilibrium for the location-then-price game with two periods of price competition. Furthermore we give a complete characterization of this equilibrium. The equilibrium outcome of course depends on the consumers' valuations and the fractions of consumer types over time. We show in which direction firms' locations are forced compared to the situation with one period of price competition.

The contribution of our model to location theory is twofold. First the long term (strategic) effects of location choices are taken into account and second a framework is provided to study an economy where different consumer types coexist.

The paper is organized as follows. In Section 2 we present the standard location-then-price game in which consumers have valuation differences. We discuss this one period model quite extensively because it is needed to understand the rest of the paper. In Section 3 we formulate a location-then-price game with valuation differences, but now there are two periods of price competition. In Section 4 we calculate the unique subgame perfect Nash equilibrium for this game. In Section 5 we give an example and in Section 6 we draw some conclusions.

2 The basic model

There is a continuum of consumers distributed uniformly with density one along the line segment $[0, 1]$. A fraction α^Y of them is young and a fraction $\alpha^O = 1 - \alpha^Y$ of them is old, henceforth referred to as the young (Y) and the old (O). Both types of consumers are located uniformly along the line segment. The young and the old differ with respect to their valuations of the quality of a certain product offered by each of two firms.

Let a_i^θ denote the valuation consumers of type $\theta \in \Theta = \{Y, O\}$ have for firm i 's product, $i \in \{1, 2\}$. Each consumer of type θ has real income w^θ and buys one unit of a single variant.

Firm i locates at x_i and sells variant i at (real) price p_i . Firms are assumed to maximize profits in a two stage location-then-price game. We assume that firm 2 locates to the right of firm 1.

Assumption 1 *Firm 2 locates to the right of firm 1, i.e., $x_1 < x_2$.*

For a discussion of the impact of this assumption see Bester, de Palma, Leininger, von Thadden and Thomas (1991). Note that firms' locations may well be outside the interval $[0, 1]$. The degree of differentiation can be higher then as compared to the maximum differentiation result of d'Aspremont, Gabszewicz and Thisse (1979). Sometimes this is called excessive differentiation (see Tirole (1988, p.286) and Anderson, de Palma and Thisse (1992, p.299)). Excessive differentiation can be mitigated when demand is distributed more closely to the centre (see Webers (1994), for example).

Each type θ consumer buys one unit of the variant that offers the greatest conditional indirect utility, for firm i given by the additive form

$$V_i^\theta(x) = w^\theta - p_i + a_i^\theta - t(x, x_i), \text{ for } i \in \{1, 2\}, \quad (2.1)$$

where x is the consumer's location in the characteristics space and $t(x, x_i)$ is the transportation cost for shipping commodity i to the consumer's location. We assume this transportation cost to be quadratic, i.e., the square of the distance between the consumer's location and the firm's location. Note that $t(x, x_i)$ is individual specific and that w^θ and a_i^θ are group specific.

There are two different market spaces. The market space of variant i among consumers of type $\theta \in \Theta$ is defined as

$$M_i^\theta = \{x \in [0, 1] \mid V_i^\theta(x) \geq V_j^\theta(x), j \neq i\}, \quad (2.2)$$

that is the set of type θ consumers that prefer variant i over variant j .

The demand X_i for commodity i is the sum of the demands for variant i by both types of consumers, i.e.,

$$X_i = \sum_{\theta \in \Theta} \int_{M_i^\theta} \alpha^\theta dx, \text{ for } i \in \{1, 2\}. \quad (2.3)$$

By definition the sum of commodity demands, $X_1 + X_2$, equals 1.

We furthermore assume that the firms cannot influence the valuation consumers have for any of the commodities. The location of the type θ consumer indifferent between buying from firm 1 and buying from firm 2 is denoted by $\bar{x}^\theta$ and is given by

$$\bar{x}^\theta = \frac{x_i + x_j}{2} + \frac{p_j - p_i}{2(x_j - x_i)} + \frac{a_i^\theta - a_j^\theta}{2(x_j - x_i)}, \quad (2.4)$$

being the midpoint between the firms' locations corrected for price differences and consumers' valuation differences. We furthermore see that in general $\bar{x}^Y \neq \bar{x}^O$. Under the assumption that the consumers' valuation differences are not too large, both firms however will sell their products to both the young and the old.

From equation (2.3) it then follows

$$X_1 = \sum_{\theta \in \Theta} \alpha^\theta \bar{x}^\theta, \quad X_2 = 1 - X_1, \quad (2.5)$$

which means that firm 1's market share is a weighted sum of the location of the indifferent consumer of both types. The weights are the respective fractions in the population.

Given the locations and prices firm i 's profit is equal to

$$\Pi_i(x_i, x_j, p_i, p_j) = p_i X_i, \quad (2.6)$$

where X_i is given by (2.5). Costs are normalized to zero.

We analyze a subgame perfect Nash equilibrium in which firms first choose locations and then choose prices. Given locations x_1 and x_2 in the first stage, the corresponding price subgame is solved by prices $p_1^*(x_1, x_2)$ and $p_2^*(x_1, x_2)$ such that

$$\Pi_i(x_i, x_j, p_i^*(x_1, x_2), p_j^*(x_1, x_2)) \geq \Pi_i(x_i, x_j, p_i, p_j^*(x_1, x_2))$$

for all $p_i \in [0, \infty)$ and $i \in \{1, 2\}$.

Profits, evaluated at the second-stage equilibrium $(p_1^*(x_1, x_2), p_2^*(x_1, x_2))$, are denoted by $\tilde{\Pi}_i(x_i, x_j) = \Pi_i(x_i, x_j, p_i^*(x_1, x_2), p_j^*(x_1, x_2))$.

The equilibrium of the location game is given then by the pair (x_1^*, x_2^*) , satisfying Assumption 1 and satisfying

$$\hat{\Pi}_i(x_i^*, x_j^*) \geq \hat{\Pi}_i(x_i, x_j^*) \text{ for all } x_i \in (-\infty, \infty) \text{ and } i \in \{1, 2\}.$$

A subgame perfect Nash equilibrium for the location-then-price game is defined by (x_1^*, x_2^*) and by $(p_1^*(x_1, x_2), p_2^*(x_1, x_2))$ for all location pairs (x_1, x_2) . The corresponding equilibrium outcome is (x_1^*, x_2^*) and (p_1^*, p_2^*) where $p_1^* = p_1^*(x_1^*, x_2^*)$ and $p_2^* = p_2^*(x_1^*, x_2^*)$.

Theorem 2.1 Define $\mathcal{A}_0 = \sum_{\theta \in \Theta} \alpha^\theta (a_1^\theta - a_2^\theta)$. The unique subgame perfect Nash equilibrium for the location-then-price game is the location pair $(x_1^*, x_2^*) = (-\frac{1}{4} + \frac{\mathcal{A}_0}{3}, \frac{5}{4} + \frac{\mathcal{A}_0}{3})$ and, for any (x_1, x_2) , the price pair $(p_1^*(x_1, x_2), p_2^*(x_1, x_2)) = (\frac{1}{3}((x_2 - x_1)(x_1 + x_2 + 2) + \mathcal{A}_0), \frac{1}{3}((x_2 - x_1)(4 - x_1 - x_2) - \mathcal{A}_0))$.

Proof First we look at the price stage. Let x_1 and x_2 be given. Maximizing equation (2.6) with respect to p_i gives $p_i(x_1, x_2, p_j) = 2(x_j - x_i)X_i$. For $i = 1$ this means $p_1(x_1, x_2, p_2) = \frac{1}{2}(p_2 + \mathcal{A}_0 + (x_2 - x_1)(x_2 + x_1))$ and for $i = 2$ this means $p_2(x_1, x_2, p_1) = \frac{1}{2}(p_1 - \mathcal{A}_0 + (x_2 - x_1)(2 - x_1 - x_2))$. But then $p_1^*(x_1, x_2) = \frac{1}{3}((x_2 - x_1)(2 + x_1 + x_2) + \mathcal{A}_0)$ and $p_2^*(x_1, x_2) = \frac{1}{3}((x_2 - x_1)(4 - x_1 - x_2) - \mathcal{A}_0)$. Next we look at the location stage. Given $p_1^*(x_1, x_2)$ and $p_2^*(x_1, x_2)$ firm i maximizes $\Pi_i(x_1, x_2)$ with respect to x_i . It is easy to verify that $\Pi_1(x_1, x_2) = \frac{1}{18(x_2 - x_1)}((x_2 - x_1)(2 + x_1 + x_2) + \mathcal{A}_0)^2$ and $\Pi_2(x_1, x_2) = \frac{1}{18(x_2 - x_1)}((x_2 - x_1)(4 - x_1 - x_2) - \mathcal{A}_0)^2$. From these expressions it follows that x_1 and x_2 must satisfy the equations $(x_2 - x_1)(2 + 3x_1 - x_2) = \mathcal{A}_0$ and $(x_2 - x_1)(-4 - x_1 + 3x_2) = \mathcal{A}_0$. This gives the solution $x_1^* = -\frac{1}{4} + \frac{\mathcal{A}_0}{3}$ and $x_2^* = \frac{5}{4} + \frac{\mathcal{A}_0}{3}$. One can verify that the second order conditions are satisfied.

It is easy to see that the equilibrium outcome is $(x_1^*, x_2^*) = (-\frac{1}{4} + \frac{\mathcal{A}_0}{3}, \frac{5}{4} + \frac{\mathcal{A}_0}{3})$ and $(p_1^*, p_2^*) = (\frac{3}{2} + \frac{2\mathcal{A}_0}{3}, \frac{3}{2} - \frac{2\mathcal{A}_0}{3})$ with profits $(\Pi_1^*, \Pi_2^*) = (\frac{1}{3}(\frac{3}{2} + \frac{2\mathcal{A}_0}{3})^2 \frac{1}{3}(\frac{3}{2} - \frac{2\mathcal{A}_0}{3})^2)$.

If $\mathcal{A}_0 = 0$ the consumers have 'on average' the same preference for both products. In this case the equilibrium outcome is $(x_1^*, x_2^*) = (-\frac{1}{4}, \frac{5}{4})$ and $(p_1^*, p_2^*) = (\frac{3}{2}, \frac{3}{2})$. Profits are $\frac{3}{4}$ per firm. If $\mathcal{A}_0 > 0$ consumers on average have a stronger preference for the product of firm 1, which enables firm 1 to set a higher price and to locate closer to the centre. Firm 2 has to set a lower price and moves from the centre in order to soften price competition. Consequently firm 1's profit increases and firm 2's profit decreases. The situation $\mathcal{A}_0 < 0$ is similar.

Note that equilibrium prices are non-negative for $\mathcal{A}_0 \in [-\frac{9}{4}, \frac{9}{4}]$. Furthermore firms' profits differ for $\mathcal{A}_0 \neq 0$. This does not necessarily mean that in order to guarantee firms equal profits consumers' valuations should be equal over firms, but that a higher valuation by one type of consumers is compensated through a lower valuation by the other type of consumers.

Recall that consumer valuations for the products are given exogenously. We thus abstract from the fact that firms have an incentive to influence $\mathcal{A}_0$ through the consumer valuations for their product.

3 Changing fractions of consumer types over time

In the previous section we have looked at the situation where firms choose locations and prices in a two-stage game. Firms thereby take the fraction of consumer types and the consumer valuations into account. Now we want to extend the model to one where also changes over time in the fractions of consumer types are taken into account. Once firms have settled, they serve the market from this location in the subsequent periods. We assume that there are two periods, t and $t + 1$, in which firms set prices. Before the start of period t firms choose locations. Firm $i \in \{1, 2\}$ locates at x_i and sells commodity i at price $p_{i\tau}$ in period $\tau \in \mathcal{T} = \{t, t + 1\}$.

Again there is a continuum of consumers distributed uniformly with density one along the line segment $[0, 1]$. In period $\tau \in \mathcal{T}$ a fraction α_τ^Y of them is young and a fraction $\alpha_\tau^O = 1 - \alpha_\tau^Y$ of them is old. The valuation consumers of type $\theta \in \Theta$ have for firm i 's product is assumed to be constant over time and is given by a_i^θ as before. Real income for consumers of type $\theta \in \Theta$ is constant over time and is given by w^θ .

In period τ each type θ consumer at location x buys one unit of the variant that offers the greatest conditional indirect utility, for firm i given by

$$V_{i\tau}^\theta(x) = w^\theta - p_{i\tau} + a_i^\theta - t(x, x_i), \text{ for } i \in \{1, 2\}, \quad (3.7)$$

where the interpretation of the terms is the same as before. Note that only prices are time dependent. The market space of variant i among consumers of type $\theta \in \Theta$ in period $\tau \in \mathcal{T}$ is defined as

$$M_{i\tau}^\theta = \{x \in [0, 1] \mid V_{i\tau}^\theta(x) \geq V_{j\tau}^\theta(x), j \neq i\}, \quad (3.8)$$

i.e., the set of type θ consumers at time τ that prefer variant i over variant j .

The demand $X_{i\tau}$ for commodity i in period τ is the sum of the demands in period τ for variant i by both types of consumers, i.e.,

$$X_{i\tau} = \sum_{\theta \in \Theta} \int_{M_{i\tau}^\theta} \alpha_\tau^\theta dx, \text{ for } i \in \{1, 2\}. \quad (3.9)$$

At time τ the location of the type θ consumer indifferent between buying from firm 1 and buying from firm 2 is denoted by $\tilde{x}_\tau^\theta$ and is given by

$$\tilde{x}_\tau^\theta = \frac{x_i + x_j}{2} + \frac{p_{j\tau} - p_{i\tau}}{2(x_j - x_i)} + \frac{a_i^\theta - a_j^\theta}{2(x_j - x_i)}. \quad (3.10)$$

Similarly to (2.5) it then follows

$$X_{1\tau} = \sum_{\theta \in \Theta} \alpha_{\tau}^{\theta} x_{\tau}^{\theta}, \quad X_{2\tau} = 1 - X_{1\tau}. \quad (3.11)$$

Firm i 's 'lifetime' profit equals

$$\Pi_{iL}(x_i, x_j, p_{it}, p_{jt}, p_{it+1}, p_{jt+1}) = \sum_{\tau \in T} \rho^{\tau-t} p_{i\tau} X_{i\tau}, \quad (3.12)$$

where $X_{i\tau}$ is given by (3.11) and ρ is the time discount factor. For simplicity we take $\rho = 1$. Firm i 's lifetime profit is the (discounted) sum of the per period profits.

Firm i 's profit in period τ is equal to

$$\Pi_{i\tau}(x_i, x_j, p_{i\tau}, p_{j\tau}) = p_{i\tau} X_{i\tau}. \quad (3.13)$$

The price choice in period t does not affect the price choice in period $t + 1$. Therefore we can consider the price choices in both periods separately. The price game in period τ is solved by prices $p_{1\tau}^*(x_1, x_2)$ and $p_{2\tau}^*(x_1, x_2)$ such that

$$\Pi_{i\tau}(x_i, x_j, p_{i\tau}^*(x_1, x_2), p_{j\tau}^*(x_1, x_2)) \geq \Pi_{i\tau}(x_i, x_j, p_{i\tau}, p_{j\tau}^*(x_1, x_2))$$

for all $p_{i\tau} \in [0, \infty)$ and $i \in \{1, 2\}$. For ease of notation let $\hat{\Pi}_{iL}(x_i, x_j) = \Pi_{iL}(x_i, x_j, p_{it}^*(x_1, x_2), p_{jt}^*(x_1, x_2), p_{it+1}^*(x_1, x_2), p_{jt+1}^*(x_1, x_2))$.

The equilibrium of the location game is then given by (x_1^*, x_2^*) satisfying

$$\hat{\Pi}_{iL}(x_i^*, x_j^*) \geq \hat{\Pi}_{iL}(x_i, x_j^*) \text{ for all } x_i \in (-\infty, \infty) \text{ and } i \in \{1, 2\}.$$

A subgame perfect Nash equilibrium for the location-then-price game is defined by (x_1^*, x_2^*) and by $\{(p_{1t}^*(x_1, x_2), p_{2t}^*(x_1, x_2)), (p_{1t+1}^*(x_1, x_2), p_{2t+1}^*(x_1, x_2))\}$ for all location pairs (x_1, x_2) . The corresponding equilibrium path is (x_1^*, x_2^*) and $\{(p_{1t}^*(x_1^*, x_2^*), p_{2t}^*(x_1^*, x_2^*)), (p_{1t+1}^*(x_1^*, x_2^*), p_{2t+1}^*(x_1^*, x_2^*))\}$.

Lemma 3.1 For $\tau \in T$ define $\mathcal{A}_{\tau} = \sum_{\theta \in \Theta} \alpha_{\tau}^{\theta} (a_1^{\theta} - a_2^{\theta})$. At locations x_1 and x_2 the price game at time $\tau \in T$ is solved by prices $p_{1\tau}^*(x_1, x_2) = \frac{1}{3}((x_2 - x_1)(2 + x_1 + x_2) + \mathcal{A}_{\tau})$ and $p_{2\tau}^*(x_1, x_2) = \frac{1}{3}((x_2 - x_1)(4 - x_1 - x_2) - \mathcal{A}_{\tau})$.

Proof As noted before the price choice in period t does not affect the price choice in period $t + 1$. Therefore the solution of each price stage is found by Theorem 2.1.

As we saw already in Section 2 the firm selling the product for which the consumers on average have a stronger preference, is able to set a higher price while the other firm can only charge a lower price.

To determine a subgame perfect Nash equilibrium for the location-then-price game we also have to solve the location stage. This is done in the next section.

4 Subgame perfect Nash equilibria

In this section we show that there exists a unique subgame perfect Nash equilibrium for the model with changing fractions of consumer types. Furthermore we give a complete equilibrium characterization. This shows in which direction and to what extent firms adjust their locations in case price competition differs over the two periods.

Given the equilibrium price schemes $(p_{1t}^*(x_1, x_2), p_{2t}^*(x_1, x_2))$ and $(p_{1t+1}^*(x_1, x_2), p_{2t+1}^*(x_1, x_2))$ it is easy to check that $X_{it} = \frac{p_{it}^*(x_1, x_2)}{2(x_2 - x_1)}$. Firm i wants to maximize its profit $\frac{1}{18(x_2 - x_1)} \sum_{\tau \in T} (3p_{i\tau}^*(x_1, x_2))^2$ with respect to x_i .

Let $C = x_1 + x_2$ and define $\mathcal{A} = \mathcal{A}_t + \mathcal{A}_{t+1}$ and $\Delta = \mathcal{A}_t^2 + \mathcal{A}_{t+1}^2$. Given the equilibrium price schemes, the first order conditions for the location game are

$$\begin{aligned} 2(x_2 - x_1)^2 \{ (2 + 4x_1 - C)(C + 2) - \mathcal{A} \} &= \Delta \\ 2(x_2 - x_1)^2 \{ (4 - 4x_2 + C)(4 - C) + \mathcal{A} \} &= \Delta \end{aligned} \quad (4.14)$$

and the second order conditions for a maximum are

$$\begin{aligned} -2(4 + 3x_1 + x_2) + \frac{\Delta}{(x_2 - x_1)^3} &< 0 \\ -2(8 - x_1 - 3x_2) + \frac{\Delta}{(x_2 - x_1)^3} &< 0. \end{aligned} \quad (4.15)$$

The system of equations (4.14) is solved by $x_1(C)$ and $x_2(C)$ satisfying

$$\begin{aligned} 4(C - 1)x_1(C) &= 2C^2 - 8C + 6 + \mathcal{A} \\ 4(C - 1)x_2(C) &= 2C^2 + 4C - 6 - \mathcal{A}. \end{aligned} \quad (4.16)$$

For $(x_1(C), x_2(C))$ to be an equilibrium of the location game, C must solve

$$2(x_2(C) - x_1(C))^2 \{ (2 + 4x_1(C) - C)(C + 2) - \mathcal{A} \} - \Delta = 0. \quad (4.17)$$

For $C \neq 1$ this can be rewritten as

$$36(C - 1 - \frac{\mathcal{A}}{6})^2 \{ (C - 1)(C - 4)(C + 2) + 3\mathcal{A} \} - 2\Delta(C - 1)^3 = 0. \quad (4.18)$$

The lefthand side of equation (4.18) is a polynomial of degree five and is denoted by $F_{\mathcal{A}}(C)$. After some tedious calculations we get $F_{\mathcal{A}}(C) = 36C^5 - (180 + 12\mathcal{A})C^4 + (\mathcal{A}^2 + 48\mathcal{A} + 36 - 2\Delta)C^3 + (612 + 144\mathcal{A} - 3\mathcal{A}^2 + 6\Delta)C^2 - (42\mathcal{A}^2 + 384\mathcal{A} + 792 + 6\Delta)C + (3\mathcal{A}^3 + 44\mathcal{A}^2 + 204\mathcal{A} + 288 + 2\Delta) = 0$. Note that $0 \leq \mathcal{A}^2 \leq 2\Delta$ and moreover $\mathcal{A}^2 = 2\Delta$ for $\mathcal{A}_t = \mathcal{A}_{t+1}$.

In order to guarantee that both firms are in the market we assume that the degree of vertical differentiation is limited, i.e., the value of $\mathcal{A}_\tau$ for $\tau \in T$ is not too big.

Assumption 2 For $\tau \in T$, $\mathcal{A}_\tau^2 \leq 3$.

Without loss of generality we restrict ourselves to the situation $\mathcal{A} \geq 0$. This means that for the two-period situation consumers 'on average' prefer firm 1's product to firm 2's product. The meaning of 'on average' is slightly different then before. It is still possible that firm 2's product is preferred in one period, but this is offset by a stronger preference for firm 1's product in the other period.

Proposition 4.1 *Let $\mathcal{A}_t$ and $\mathcal{A}_{t+1}$ be given. All five roots of the equation $F_{\mathcal{A}}(C) = 0$ are real. These roots are in the interval $(-\infty, 1 + \frac{\mathcal{A}}{12}]$, $[1 + \frac{\mathcal{A}}{12}, 1 + \frac{2\mathcal{A}}{12}]$, $[1 + \frac{2\mathcal{A}}{12}, 1 + \frac{3\mathcal{A}}{12}]$, $[1 + \frac{3\mathcal{A}}{12}, 1 + \frac{4\mathcal{A}}{12}]$, and $[1 + \frac{4\mathcal{A}}{12}, \infty)$, respectively.*

Proof First note that $\lim_{C \rightarrow -\infty} F_{\mathcal{A}}(C) = -\infty$ and $\lim_{C \rightarrow \infty} F_{\mathcal{A}}(C) = \infty$. Substitution in equation (4.18) yields $F_{\mathcal{A}}(1 + \frac{\mathcal{A}}{12}) = (\mathcal{A}/12)^3(\mathcal{A}^2/4 + 972 - 2\Delta)$, $F_{\mathcal{A}}(1 + \frac{2\mathcal{A}}{12}) = -2\Delta(\mathcal{A}/6)^3$, $F_{\mathcal{A}}(1 + \frac{3\mathcal{A}}{12}) = (\mathcal{A}/12)^3(27\mathcal{A}^2/4 + 324 - 54\Delta)$, and $F_{\mathcal{A}}(1 + \frac{4\mathcal{A}}{12}) = (\mathcal{A}/3)^3(\mathcal{A}^2 - 2\Delta)$. Because both $\mathcal{A}_t^2$ and $\mathcal{A}_{t+1}^2$ are smaller than 3, we have $F_{\mathcal{A}}(1 + \frac{\mathcal{A}}{12}) \geq 0$, $F_{\mathcal{A}}(1 + \frac{2\mathcal{A}}{12}) \leq 0$, $F_{\mathcal{A}}(1 + \frac{3\mathcal{A}}{12}) \geq 0$, and $F_{\mathcal{A}}(1 + \frac{4\mathcal{A}}{12}) \leq 0$. Because $F_{\mathcal{A}} : (-\infty, \infty) \mapsto \mathbf{R}$ is continuous the intermediate value theorem says that for each of the intervals there exists a ψ in it such that $F_{\mathcal{A}}(\psi) = 0$.

The bounds for the first and the last interval can be narrowed easily. It is left to the reader to check that the first root is in the interval $[-3, 2]$ and the fifth root is in the interval $[3, 4]$.

Because the equation $F_{\mathcal{A}}(C) = 0$ has five real roots it is possible to construct two pairs of symmetric roots. We denote these roots as $C_1 = L - \epsilon$, $C_2 = L + \epsilon$, $C_3 = M - \delta$, $C_4 = M + \delta$. The remaining root equals $C_5 = 5 + \frac{\mathcal{A}}{3} - 2M - 2L$ then. Next we define $G_{\mathcal{A}}(C) = 36(C - C_1)(C - C_2)(C - C_3)(C - C_4)(C - C_5)$. Rewriting yields $G_{\mathcal{A}}(C) = 36\{C^5 - (5 + \mathcal{A}/3)C^4 + \gamma_3 C^3 + \gamma_2 C^2 + \gamma_1 C + \gamma_0\}$, with $\gamma_0 = (L^2 M^2 - L^2 \delta^2 - M^2 \epsilon^2 + \epsilon^2 \delta^2)(-5 - \mathcal{A}/3 + 2M + 2L)$, $\gamma_1 = (2L\delta^2 + 2M\epsilon^2 - 2LM^2 - 2ML^2)(-5 - \mathcal{A}/3 + 2M + 2L) + (L^2 M^2 - L^2 \delta^2 - M^2 \epsilon^2 + \epsilon^2 \delta^2)$, $\gamma_2 = (M^2 + 4ML + L^2 - \epsilon^2 - \delta^2)(-5 - \mathcal{A}/3 + 2M + 2L) + (2L\delta^2 + 2M\epsilon^2 - 2LM^2 - 2ML^2)$, and $\gamma_3 = (-2M - 2L)(-5 - \mathcal{A}/3 + 2M + 2L) + (M^2 + 4ML + L^2 - \epsilon^2 - \delta^2)$.

Comparing $F_{\mathcal{A}}(C)$ and $G_{\mathcal{A}}(C)$ gives the following.

Lemma 4.2 *For any C , $G_{\mathcal{A}}(C) = F_{\mathcal{A}}(C)$ for L, M, ϵ and δ satisfying*

- (i) $\gamma_3 = \mathcal{A}^2 + 48\mathcal{A} + 36 - 2\Delta$
- (ii) $\gamma_2 = 612 + 144\mathcal{A} - 3\mathcal{A}^2 + 6\Delta$
- (iii) $\gamma_1 = -42\mathcal{A}^2 - 384\mathcal{A} - 792 - 6\Delta$
- (iv) $\gamma_0 = 3\mathcal{A}^3 + 44\mathcal{A}^2 + 204\mathcal{A} + 288 + 2\Delta$.

Note that the existence of an L, M, ϵ and δ satisfying conditions (i) to (iv) is guaranteed for any $\mathcal{A}$ and Δ because the equation $F_{\mathcal{A}}(C) = 0$ has five real roots.

It is not possible however to give explicit analytical expressions for M, L, ϵ and δ in the general case, and therefore we have to compute these numbers numerically.

For the special situations $\mathcal{A}_t = \mathcal{A}_{t+1}$ and $\mathcal{A}_t = -\mathcal{A}_{t+1}$ we give analytical expressions in the following two corollaries. We omit the proof of these corollaries because they can be seen as a direct consequence of Lemma 4.2.

Corollary 4.3 Suppose $\mathcal{A}_t = \mathcal{A}_{t+1}$. Then $L = -\frac{1}{2}$, $M = \frac{5}{2}$, $\epsilon^2 = \frac{9}{4} + \frac{A}{2}$, and $\delta^2 = \frac{9}{4} - \frac{A}{2}$ solve (i) to (iv). Furthermore $C_1 < C_2 \leq C_3 \leq C_5 < C_4$.

For the special situation $\mathcal{A}_t = \mathcal{A}_{t+1} = 0$ we only have three different roots. This result also holds for the more general case $\mathcal{A}_t = -\mathcal{A}_{t+1}$.

Corollary 4.4 Suppose $\mathcal{A}_t = -\mathcal{A}_{t+1}$. Then $L = 1 - \frac{1}{2}(9 + \Delta/18)^{\frac{1}{2}}$, $M = 1 + \frac{1}{2}(9 + \Delta/18)^{\frac{1}{2}}$, and $\epsilon^2 = \delta^2 = \frac{9}{4} + \frac{\Delta}{72}$ solve (i) to (iv). Furthermore $C_1 < C_2 \leq C_3 \leq C_5 < C_4$.

Given the values C_k , $k \in \{1, \dots, 5\}$, of C that solve (4.17), $x_1(C_k)$ and $x_2(C_k)$ can be determined from equation (4.16) for all k . We distinguish for $k \in \{1, \dots, 5\}$ between the situations $C_k = 1$ and $C_k \neq 1$ for some k .

For $C_k = 1$ equation (4.16) requires that $\mathcal{A} = 0$, so $\mathcal{A}_t = -\mathcal{A}_{t+1}$. Denote $\mu = 1 - \frac{1}{3}\mathcal{A}_t^2$. Then equation (4.17) can be rewritten as

$$x_1(C_k)^3 - \frac{3}{4}x_1(C_k)^2 + \frac{\mu}{16} = 0. \quad (4.19)$$

After substituting $x_1(C_k) = y + \frac{1}{4}$ we have

$$y^3 - \frac{3}{16}y + \frac{2\mu - 1}{32} = 0. \quad (4.20)$$

Assumption 2 implies $\mu(\mu - 1) \leq 0$, so the roots for equation (4.20) are

$$y_1 = \frac{1}{2}\cos(\frac{\phi}{3}), \quad y_2 = \frac{1}{2}\cos(\frac{\phi}{3} + 120), \quad y_3 = \frac{1}{2}\cos(\frac{\phi}{3} + 240), \quad (4.21)$$

where $\phi = \arctan(2\frac{(\mu - \mu^2)^{\frac{1}{2}}}{1 - 2\mu})$ for $\mu > \frac{1}{2}$ on the condition that $\phi \in [90, 180]$ and $\phi = \arccos(1 - 2\mu)$ for $\mu \leq \frac{1}{2}$ on the condition that $\phi \in [0, 90]$ ¹. For $\mathcal{A}_t = \mathcal{A}_{t+1} = 0$ we have $y_1 = y_3 = \frac{1}{4}$ and $y_2 = -\frac{1}{2}$. For more details about the derivations we refer the reader to Uspensky (1948).

There is only an equilibrium when $\mu(\mu - 1) \leq 0$, i.e., $0 \leq \mathcal{A}_t^2 \leq 3$. Intuitively this is what we expected. If the consumers' preference for one of the firms is very large, there cannot be an equilibrium where both firms are in the market.

¹Without Assumption 2 the situation $\mu(\mu - 1) > 0$ can occur. Then the real root is

$$y_1 = \left\{ \frac{1 - 2\mu}{64} + \frac{1}{32}(\mu(\mu - 1))^{\frac{1}{2}} \right\}^{\frac{1}{3}} + \left\{ \frac{1 - 2\mu}{64} - \frac{1}{32}(\mu(\mu - 1))^{\frac{1}{2}} \right\}^{\frac{1}{3}}.$$

But then $x_1(C) > 3/4$ and $x_2(C) < 1/4$, which contradicts Assumption 1. For the situation $C = 1$ we thus essentially do not need Assumption 2.

Note that this is equivalent to the condition in Assumption 2. With the above results we can prove the following proposition.

Proposition 4.5 *When $C_k = 1$ there is a unique solution to the location stage. Firm 1 chooses location $\frac{1}{4} + \frac{1}{2}\cos(\frac{\phi}{3} + 120)$ and firm 2 chooses location $\frac{3}{4} - \frac{1}{2}\cos(\frac{\phi}{3} + 120)$ where ϕ as specified before.*

Proof For all $0 \leq \mu \leq 1$ it holds according to (4.21) that $y_2 \leq y_3 \leq y_1$. For $0 \leq \mu < \frac{1}{2}$ we have $y_2 \in [-\frac{1}{2}\sqrt{3}, -\frac{1}{2}]$, $y_3 \in [-\frac{1}{2}, 0]$, and $y_1 \in [0, \frac{1}{2}]$. For $\frac{1}{2} \leq \mu \leq 1$ we have $y_2 \in [-\frac{1}{2}\sqrt{3}, 0]$, $y_3 \in [0, \frac{1}{2}]$, and $y_1 \in [\frac{1}{2}, \frac{1}{2}\sqrt{3}]$. Then $x_2(C) - x_1(C) = \frac{1}{2} - 2y$ is minimal for y_2 and prices are maximal. Because $X_1 = X_2 = \frac{1}{2}$ in all three situations, profits for both firms are maximized for $y = y_2$. One can check that the second order conditions for a maximum for both firms are indeed satisfied for $y = y_2$.

Next we consider the situation $C_k \neq 1$. Equation (4.16) can be rewritten then as

$$\begin{aligned} x_1(C) &= \frac{2(C-1)(C-3)+A}{4(C-1)} \\ x_2(C) &= \frac{2(C-1)(C+3)-A}{4(C-1)}. \end{aligned} \quad (4.22)$$

From the second order conditions in the location stage we will derive that the equilibrium locations are found for C in the interval $[1 + \frac{3A}{12}, 1 + \frac{4A}{12}]$.

Lemma 4.6 *When $C_k < 1 + \frac{A}{12}$ and $C_k > 1 + \frac{4A}{12}$ there does not exist a location equilibrium.*

Proof It is clear that there only exists a location equilibrium if the second order conditions for both firms are satisfied. We will prove that for $C_k < 1 + \frac{A}{12}$ and $C_k > 1 + \frac{4A}{12}$ the second order conditions are not satisfied for both firms. From (4.15) we see that the second order conditions for a maximum are $2(x_1(C) - x_2(C))^3(4 + 3x_1(C) + x_2(C)) + \Delta < 0$ for firm 1 and $2(x_1(C) - x_2(C))^3(8 - x_1(C) - 3x_2(C)) + \Delta < 0$ for firm 2. With the help of equation (4.22) we can rewrite these conditions as $S_1(C) < 0$ and $S_2(C) < 0$ where $S_1(C) = 2(\frac{A}{2(C-1)} - 3)^3(1 + 2C + \frac{A}{2(C-1)}) + \Delta$ and $S_2(C) = 2(\frac{A}{2(C-1)} - 3)^3(5 - 2C + \frac{A}{2(C-1)}) + \Delta$. First note that $S_1(C) > 0$ for $C < -\frac{1}{2}$ because both $\frac{A}{2(C-1)} - 3$ and $1 + 2C + \frac{A}{2(C-1)}$ are negative then. Furthermore for $C > 3$, $S_2(C) > 0$, because both $\frac{A}{2(C-1)} - 3 < 0$ and $5 - 2C + \frac{A}{2(C-1)} < 0$. It is easy to see that $C = \frac{7}{4} + \frac{\sqrt{(9+4A)}}{4}$ solves $5 - 2C + \frac{A}{2(C-1)} = 0$. For $C > \frac{7}{4} + \frac{\sqrt{(9+4A)}}{4}$ it holds that $5 - 2C + \frac{A}{2(C-1)} < 0$. From Assumption 1 we furthermore know that $\frac{7}{4} + \frac{\sqrt{(9+4A)}}{4} < 3$. But this is what we wanted to prove.

Now we can formulate the following theorem.

Theorem 4.7 Let $\mathcal{A} = 0$ and define $\Phi = \frac{3}{2} + \frac{3}{2}\cos(\frac{\Phi}{3} + 120)$. The (unique) subgame perfect Nash equilibrium for the location-then-price game is the location pair $(x_1^*, x_2^*) = (-\frac{1}{4} + \frac{\Phi}{3}, \frac{5}{4} - \frac{\Phi}{3})$ and for any location pair (x_1, x_2) the price pairs $\{(\frac{1}{3}((x_2 - x_1)(2 + x_1 + x_2) + \mathcal{A}_t), \frac{1}{3}((x_2 - x_1)(4 - x_1 - x_2) - \mathcal{A}_t)), (\frac{1}{3}((x_2 - x_1)(2 + x_1 + x_2) + \mathcal{A}_{t+1}), \frac{1}{3}((x_2 - x_1)(4 - x_1 - x_2) - \mathcal{A}_{t+1}))\}$.

Proof This is an immediate consequence of Lemma 3.1, Proposition 4.5 and Lemma 4.6.

When $\mathcal{A} = 0$, meaning that consumers' preference for firm i in period t equals consumers' preference for firm j in period $t + 1$, firm i can charge a higher price in period t and firm j can charge a higher price in period $t + 1$. In case $\mathcal{A}_t \neq 0$, both firms will locate more close to the centre and price competition is enlarged. Profits for both firms are $(\frac{3}{2} - \frac{2}{3}\Phi) + \frac{\Delta}{18(\frac{3}{2} - \frac{2}{3}\Phi)}$. One can check that profits are maximal for $\mathcal{A}_t = \mathcal{A}_{t+1} = 0$. Oscillating preferences of consumers thus decrease both firms' profits.

Lemma 4.8 Let $\mathcal{A} > 0$ be given. For $C_k \in [1 + \frac{\mathcal{A}}{12}, 1 + \frac{2\mathcal{A}}{12}]$ there does not exist an equilibrium for the location stage.

Proof For $C_k \in [1 + \frac{\mathcal{A}}{12}, 1 + \frac{2\mathcal{A}}{12}]$ it holds that $x_2(C) < x_1(C)$, which contradicts Assumption 1. Furthermore profits for both firms are negative.

The proof of the following lemma is omitted. It follows from combining equation (4.14) and equation (4.15).

Lemma 4.9 Let $\mathcal{A} > 0$ be given. For $C_k \in [1 + \frac{2\mathcal{A}}{12}, 1 + \frac{3\mathcal{A}}{12}]$ the second order conditions for a maximum are not satisfied, whereas these conditions are satisfied for $C_k \in [1 + \frac{3\mathcal{A}}{12}, 1 + \frac{4\mathcal{A}}{12}]$.

Proposition 4.10 For $C_k \neq 1$ there is a solution to the location stage if and only if $C_k \in [1 + \frac{3\mathcal{A}}{12}, 1 + \frac{4\mathcal{A}}{12}]$.

Proof See Lemma 4.6, Lemma 4.8, and Lemma 4.9.

Let C be represented as a linear combination of the two endpoints of the interval $[1 + \frac{3\mathcal{A}}{12}, 1 + \frac{4\mathcal{A}}{12}]$, i.e., $C = 1 + \frac{(4 - \alpha^*)\mathcal{A}}{12}$ where $\alpha^* \in [0, 1]$ is the unique value for which $F_{\mathcal{A}}(C) = 0$. Substitution in equation (4.22) yields

$$\begin{aligned} x_1(C) &= \frac{12 + (4 - \alpha^*)\mathcal{A}}{24} - \frac{6 - 3\alpha^*}{2(4 - \alpha^*)} \\ x_2(C) &= \frac{12 + (4 - \alpha^*)\mathcal{A}}{24} + \frac{6 - 3\alpha^*}{2(4 - \alpha^*)}. \end{aligned} \quad (4.23)$$

It is easy to see that firms' locations are symmetric around $\frac{12 + (4 - \alpha^*)\mathcal{A}}{24}$.

For $\mathcal{A} = 0$ this means that firms' locations are symmetric around $\frac{1}{2}$. After some standard calculations we get $x_1^* = -\frac{1}{4} + \frac{(4-\alpha^*)\mathcal{A}}{24} + \frac{3\alpha^*}{4(4-\alpha^*)}$ and $x_2^* = \frac{5}{4} + \frac{(4-\alpha^*)\mathcal{A}}{24} - \frac{3\alpha^*}{4(4-\alpha^*)}$.

Note that $C_k \neq 1$ only holds for $\mathcal{A} > 0$. This enables us to formulate the following theorem.

Theorem 4.11 *Suppose $\mathcal{A} > 0$. Define $\Phi_{\mathcal{A}} = \frac{(4-\alpha^*)\mathcal{A}}{8}$ and $\Psi_{\mathcal{A}} = \frac{3\alpha^*}{4(4-\alpha^*)}$. The (unique) subgame perfect equilibrium for the location-then-price game is the location pair $(x_1^*, x_2^*) = (-\frac{1}{4} + \frac{\Phi_{\mathcal{A}}}{3} + \Psi_{\mathcal{A}}, \frac{5}{4} + \frac{\Phi_{\mathcal{A}}}{3} - \Psi_{\mathcal{A}})$ and for any location pair (x_1, x_2) the price pairs $\{(\frac{1}{3}((x_2 - x_1)(2 + x_1 + x_2) + \mathcal{A}_i), \frac{1}{3}((x_2 - x_1)(4 - x_1 - x_2) - \mathcal{A}_i)), (\frac{1}{3}((x_2 - x_1)(2 + x_1 + x_2) + \mathcal{A}_{i+1}), \frac{1}{3}((x_2 - x_1)(4 - x_1 - x_2) - \mathcal{A}_{i+1}))\}$.*

Proof This follows from Proposition 4.10 and equation (4.23).

When $\mathcal{A} > 0$ the consumers' preference for firm 1's product gives this firm some monopoly power, which enables it to gain higher profits. The other firm gains lower profits. Firm 1's profits are equal to $(\frac{3}{2} - 2\Psi_{\mathcal{A}})(1 + \frac{2}{9}\Phi_{\mathcal{A}})^2 + \frac{\mathcal{A}}{3}(1 + \frac{2}{9}\Phi_{\mathcal{A}}) + \frac{\Delta}{18(\frac{3}{2} - 2\Psi_{\mathcal{A}})}$ and firm 2's profits are equal to $(\frac{3}{2} - 2\Psi_{\mathcal{A}})(1 - \frac{2}{9}\Phi_{\mathcal{A}})^2 - \frac{\mathcal{A}}{3}(1 - \frac{2}{9}\Phi_{\mathcal{A}}) + \frac{\Delta}{18(\frac{3}{2} - 2\Psi_{\mathcal{A}})}$.

Now we have analyzed all the basic ingredients of the problem, we are able to prove the following theorem.

Theorem 4.12 *For all $\mathcal{A}$ satisfying Assumption 2 there exists a unique subgame perfect Nash equilibrium for the location-then-price game.*

Proof For $\mathcal{A} = 0$ we have the special situation that $C = 1$. The unique solution for the location stage is given then by Proposition 4.5. For $\mathcal{A} > 0$ we have the situation $C \in [1 + \frac{3\mathcal{A}}{12}, 1 + \frac{4\mathcal{A}}{12}]$. The solution for the location stage is given then by equation (4.22). The situation $\mathcal{A} < 0$ holds by symmetry. In all situations the price stage is solved by prices according to Lemma 3.1.

For $\mathcal{A} = 0$ both firms adjust their location choices 'symmetrically', i.e., one firm adds some fixed amount to its competitive location choice and the other firm subtracts the same amount from its competitive location choice. For $\mathcal{A} > 0$ the location choices are adjusted asymmetrically. Firms do also take into account their relative monopoly power. Consequently firms' profits in equilibrium are the same for $\mathcal{A} = 0$, whereas they differ for $\mathcal{A} \neq 0$.

5 Comparative statics: an example

In this section we look at an example where the fraction of young and old consumers oscillates over two periods. The fraction of young consumers in period t equals the fraction of old consumers in period $t + 1$, i.e., $\alpha_t^Y = \alpha_{t+1}^O$. Automatically this means that the fraction of old consumers in period t equals the fraction of young consumers in period $t + 1$. Recall that t and $t + 1$ are the two periods of price competition. Furthermore we assume that $a_1^Y - a_2^Y = \frac{1}{2}$ and that $a_1^O - a_2^O = -\frac{1}{2}$. This means that young consumers have a higher valuation for the product of firm 1 and that old consumers have a higher valuation for the product of firm 2. If one thinks for example of two firms that sell bicycles, then firm 1 would be the one that sells mountainbikes and firm 2 is the one that sells traditional bikes. By construction $\mathcal{A}_t = -\mathcal{A}_{t+1}$ where $\mathcal{A}_t = (\alpha_t^Y - \alpha_t^O)(a_1^Y - a_2^Y) = \frac{\alpha_t^Y - \alpha_t^O}{2}$. We can apply Theorem 4.7 to calculate the subgame perfect Nash equilibrium outcome.

In Table 1 we have listed the values of $\mathcal{A}_t$, $\mathcal{A}_{t+1}$, μ , and Φ for different values of α_t^Y and α_t^O and in Table 2 we have given the corresponding equilibrium outcomes of x_1 , x_2 , p_{1t} , p_{1t+1} , p_{2t} , and p_{2t+1} .

α_t^Y	α_t^O	$\mathcal{A}_t$	$\mathcal{A}_{t+1}$	μ	Φ
0.00	1.00	-0.50	0.50	0.9167	0.0285
0.25	0.75	-0.25	0.25	0.9792	0.0070
0.50	0.50	0.00	0.00	1.0000	0.0000
0.75	0.25	0.25	-0.25	0.9792	0.0070
1.00	0.00	0.50	-0.50	0.9167	0.0285

Table 1: Parameter values

α_t^Y	α_t^O	x_1	x_2	$p_{1t} = p_{2t+1}$	$p_{1t+1} = p_{2t}$
0.00	1.00	-0.2405	1.2405	1.3143	1.6477
0.25	0.75	-0.2477	1.2477	1.4117	1.5788
0.50	0.50	-0.2500	1.2500	1.5000	1.5000
0.75	0.25	-0.2477	1.2477	1.5788	1.4117
1.00	0.00	-0.2405	1.2405	1.6477	1.3143

Table 2: Equilibrium locations and prices

From these figures we see that firms' locations are closer the less equal the distribution of consumer types is. In case consumer fractions are constant, the

competitive outcome results². In all other cases firms' prices differ. The firm having the more (less) attractive variant for the majority of the consumers, can set a higher (lower) price than in the competitive situation.

In Table 3 we have given the equilibrium values of $\bar{x}_t^Y, \bar{x}_{t+1}^Y, \bar{x}_t^O, \bar{x}_{t+1}^O, X_{1t}, X_{1t+1}, X_{2t}$ and X_{2t+1} . We see that the demand a firm has in a certain period is higher the greater the fraction of consumers for which its product is more attractive than the other firm's product is. Because the fraction of young and old consumers oscillates, also firms' demand oscillates.

α_t^Y	$\bar{x}_t^Y$	$\bar{x}_{t+1}^Y$	$\bar{x}_t^O$	$\bar{x}_{t+1}^O$	$X_{1t} = X_{2t+1}$	$X_{1t+1} = X_{2t}$
0.00	0.7813	0.5563	0.4437	0.2187	0.4437	0.5563
0.25	0.7231	0.6113	0.3887	0.2767	0.4723	0.5277
0.50	0.6667	0.6667	0.3333	0.3333	0.5000	0.5000
0.75	0.6113	0.7231	0.2767	0.3887	0.5277	0.4723
1.00	0.5563	0.7813	0.2187	0.4437	0.5563	0.4437

Table 3: Indifferent consumers and firms' demand in equilibrium

In Table 4 we have given the equilibrium values of $\Pi_{1t}, \Pi_{1t+1}, \Pi_{2t}, \Pi_{2t+1}, \Pi_{1L}$ and Π_{2L} . As noted before profits are maximal in case consumer fractions are constant. The equilibrium locations are $-\frac{1}{4}$ and $\frac{5}{4}$ then. If consumer fractions are not constant, per period profits differ over firms. The reason is that firm 1 has a product that is relatively more attractive to young consumers than the product of firm 2. The higher the fraction of young consumers, the greater firm 1's profit, and the higher the fraction of old consumers, the greater firm 2's profit. Because the consumer fractions oscillate, one firm earns higher profits in the one period and the other firm earns higher profits in the other period.

α_t^Y	α_t^O	$\Pi_{1t} = \Pi_{2t+1}$	$\Pi_{1t+1} = \Pi_{2t}$	$\Pi_{1L} = \Pi_{2L}$
0.00	1.00	0.5832	0.9166	1.4998
0.25	0.75	0.6667	0.8332	1.4999
0.50	0.50	0.7500	0.7500	1.5000
0.75	0.25	0.8332	0.6667	1.4999
1.00	0.00	0.9166	0.5832	1.4998

Table 4: Equilibrium profits

The situation becomes more interesting if we change $a_1^O - a_2^O$ from $-\frac{1}{2}$ to $-\frac{1}{4}$. Then $\mathcal{A} = \frac{1}{4}$ and we can apply Theorem 4.11 to calculate the subgame perfect

²With the competitive outcome, we mean the outcome in the situation without valuation differences.

Nash equilibrium outcome. This is summarized in Tables 5, 6 and 7. We see that firms' profits differ and furthermore firm 1's profits are maximal in case consumer fractions are constant, whereas firm 2's profits are higher the greater the change in consumer fractions. The total surplus that firms attract from the consumers is higher than in the competitive case.

α_t^Y	$\mathcal{A}_t$	$\mathcal{A}_{t+1}$	Δ	α^*	$\Phi_{\mathcal{A}}$	$\Psi_{\mathcal{A}}$
0.00	-0.2500	0.5000	0.3125	0.026446	0.124174	0.004992
0.25	-0.0625	0.3125	0.1016	0.006841	0.124786	0.001285
0.50	0.1250	0.1250	0.0313	0.000000	0.125000	0.000000
0.75	0.3125	-0.0625	0.1016	0.006841	0.124786	0.001285
1.00	0.5000	-0.2500	0.3125	0.026446	0.124174	0.004992

Table 5: Parameter values

α_t^Y	x_1	x_2	p_{1t}	p_{2t}	p_{1t+1}	p_{2t+1}
0.00	-0.2036	1.2864	1.4478	1.5322	1.6978	1.2822
0.25	-0.2071	1.2903	1.5181	1.4767	1.6431	1.3517
0.50	-0.2083	1.2917	1.5833	1.4167	1.5833	1.4167
0.75	-0.2071	1.2903	1.6431	1.3517	1.5181	1.4767
1.00	-0.2036	1.2864	1.6978	1.5322	1.4478	1.5322

Table 6: Equilibrium locations and prices

α_t^Y	X_{1t}	X_{2t}	X_{1t+1}	X_{2t+1}	Π_{1L}	Π_{2L}
0.00	0.4858	0.5142	0.5697	0.4303	1.6707	1.3395
0.25	0.5069	0.4931	0.5486	0.4513	1.6710	1.3383
0.50	0.5277	0.4722	0.5277	0.4722	1.6713	1.3380
0.75	0.5486	0.4513	0.5069	0.4931	1.6710	1.3383
1.00	0.5697	0.4303	0.4858	0.5142	1.6707	1.3395

Table 7: Equilibrium demands and profits

6 Conclusions

In this paper we examined the situation where two firms compete in prices for two periods and differences in consumer valuations may occur. Firms take both multiperiod price competition and differences in consumer valuations into account in determining their optimal location. It is clear that there will be a trade-off between price and quantity.

Whenever the effect of the differences in valuations is the same in both periods, i.e., $\mathcal{A}_t = \mathcal{A}_{t+1}$, both firms move in the same direction. Whenever the effect of the differences in valuations is the opposite in both periods, i.e., $\mathcal{A}_t = -\mathcal{A}_{t+1}$, both firms move in the opposite direction.

Although we have not been able to give explicit analytical expressions for the firms' location choices, it was possible to prove the existence of a unique solution for the location-then-price game.

In case $\mathcal{A}_t = -\mathcal{A}_{t+1}$ both firms earn the same profits and changing fractions of consumer types over time decrease both firms' profits. In case $\mathcal{A}_t \neq -\mathcal{A}_{t+1}$, one firm earns strictly higher profits.

An interesting topic for future research is to extend the model by allowing for the possibility that a firm does not necessarily attract both types of consumers in both periods. This basically means that we should leave the (strong) assumption that consumers buy at any price.

Furthermore one can weaken the assumption that firms cannot influence the valuations consumers have for their product.

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