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**REDUCING EXTERNAL DEBT IN A WORLD WITH
IMPERFECT ASSET AND IMPERFECT
COMMODITY SUBSTITUTION**

by Theo van de Klundert

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Reducing External Debt in a World with Imperfect Asset
and Imperfect Commodity Substitution

by

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Summary

Three alternatives for eliminating US external debt are analysed. Besides a reduction in government spending attention is paid to the possibility of eliminating debt by inflating the economy and to a financial crisis in case foreign investors loose confidence. The analysis is performed on the base of a two-country model with a portfolio choice between money, domestic and foreign assets which are imperfect substitutes on the one hand and imperfect commodity substitution on the other hand. The model deals with balance of payments dynamics, government debt dynamics, capital accumulation, monetary growth and exchange rate expectations. A simplified version of the model is solved analytically. The full version is applied by working through numerical exercises.

May, 1989

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1. Introduction

The continuing large US external deficit is one of the central themes of contemporary economics and for that matter of economic policy. There are a number of interesting questions to be asked. First, is it really necessary for a reduction in the US deficit that the dollar depreciates in nominal and in real terms? Second, what are the alternatives for realizing a decline of the deficit and subsequently of a stabilisation of the increase in foreign debt? Third, are the long-run consequences of a policy aimed at a reduction of the deficit different from the short or medium run impact?

Branson (1988) takes a strong view on the first question by criticizing the opinion held by the finance ministers of the Group of Seven countries, implying that exchange rates need not change to restore external balance in the world economy. What the Group of Seven argues for is that a fiscal reduction in the United States combined with a fiscal expansion elsewhere would make a dollar depreciation unnecessary. In this theoretical paper we share Branson's view that depreciation is the very mechanism through which a fiscal shift would restore international equilibrium.

A reduction in US government spending without any compensating measures in that country or elsewhere raises fears of too much deflation with unfavourable effects on output and employment. For that reason a policy mix is usually advocated which avoids the unpleasant side effects of the medicine. Therefore, Branson (1988) proposes to combine a tightening of fiscal policy in the United States with an easing of monetary policy in the Federal Republic of Germany and Japan. Such a scenario would avoid too sudden movements in exchange rates, while it would reduce real interest rates and developing country debt service. Here we disagree with Branson. It will be shown in this paper that the Branson-scenario may not work if its principal aim is to reduce US external debt.

An alternative to the fiscal-adjustment view is that inflationary finance can be used to wipe out the real value of accumulated external debt. Dornbusch (1989) considers this argument but quickly rejects it, because real interest rates would be negative for a long period and the public's willingness to accept the required high rates of inflation may be limited. The inflationary-finance argument seems to have more merit than Dornbusch is willing to admit. In the paper we therefore will carefully analyse the

consequences of a rise in the money growth rate in the United States and show that it is a valuable alternative for a fiscal reduction.

If policy makers do not succeed in taking appropriate measures the market may play its own decisive role. The unlimited ability of the US to finance current account imbalances by selling off assets may be put into question. If the share of US assets in foreign portfolios rises fast preferences might shift against dollar denominated securities. As observed by Dornbusch (1989) the discussion ought to be about two-way diversifications rather than about international one-way lending. A shift in asset preferences of wealth-owners outside the US will be the third alternative to reduction in external debt. To dramatize somewhat this alternative will be labelled as a financial crisis.

Although a reduction in the US foreign deficit could be accomplished within a few years it may be interesting and important to look at the long-run consequences of the proposed alternatives. Sometimes, punishments or rewards come with a long time lag. An analysis of long-run effects calls for a proper specification of the dynamics of the model. Recently much attention is given to the microfoundations of two-country models (e.g., Buiter, 1986; van der Ploeg, 1988; Giovannini, 1988; van de Klundert and van der Ploeg, 1989). In these models perfect capital mobility and uncovered interest parity are assumed. This does not seem the right scene for the problems we want to tackle. Recalling Dornbusch's statement about a two-way diversification on the international capital market we return to the time-honoured portfolio model, which despite its rudimentary micro-underpinnings still seems to be a good working horse. However, what we have in common with the recent micro-foundations literature is the emphasis put on the intrinsic dynamics of the economy. Capital accumulation, government debt dynamics and monetary growth will be considered along with the core aspects of international economics, i.e. balance of payments dynamics and exchange rate expectations.

Portfolio analysis has been given ample attention in open economy macroeconomics (e.g., Branson, 1979; Branson and Buiter, 1983; Buiter, 1983; Branson, 1985; Kawai, 1985). Discussions of these issues within the context of two-country models are, however, less numerous. Branson and Henderson (1985) provide an interesting and useful exception. Our model differs from theirs by a more extended specification of the dynamics of the economy. This

difference in specification also concerns the balance of payments equation by including gross interest payments. This allows a growing external debt to worsen the current account deficit.

A final point worth mentioning is that we do not intend to cope with cyclical fluctuations. Adjustment policies should succeed at full employment and capacity utilization. Business cycles as such have nothing to do with the problem of structural equilibrium in the world economy. It will therefore be assumed throughout the paper that prices are flexible so that all markets clear. In this respect we depart from Branson (1988), who applies a two-country version of the model in Dornbusch (1976). However, there seems not much to be gained for the purpose at hand by assuming prices to adjust with a time lag and at the same time allowing firms to produce at full capacity.

The paper is organized as follows. In Section 2 the model is presented in a general form. The linearized version of this model can be found in Appendix I. Because the model is rather complicated a full analytical solution is impracticable. Instead we will therefore solve a simplified version of the model, which following Branson (1988) will be called the "fundamentals" model. The analytical solution of the fundamentals model is discussed in Section 3. The technical details of it are given in Appendix II. In Section 4 we consider the three alternatives mentioned above for reducing the external debt of the US. To get these alternatives in proper perspective numerical examples based on the complete model will be presented. The paper closes with some conclusions.

2. A two-country model with imperfect assets and goods

Countries exchange goods and assets. In modelling their interdependence it may be useful to consider subsystems to cope with the complexity of the system. Here we distinguish three such subsystems. In the portfolio subsystem agents decide how to allocate their wealth between different securities. Following Branson and Henderson (1985) it will be assumed that agents in each country have a choice between holding cash balance in domestic currency units or to invest in domestic or foreign bonds, which are imperfect substitutes because of differences in risk. In the commodity

subsystem agents decide about consumption of home and foreign goods and the level of investment. In the dynamic subsystem the accumulation of investment goods and of assets is given proper treatment. The interactions are complex, but the main principles are familiar from general equilibrium theory. Short-run equilibrium is attained by flexible price and interest rate adjustments. In the medium and long run current account dynamics, government budget constraints and capital accumulation determine the movement of the backward-looking state variables. Exchange rate expectations are assumed to be formed rationally. The real exchange rate is therefore a forward-looking state variable. Through this hypothesis future developments are reflected fully in current equilibrium solutions. Although it is useful to distinguish nominal and real variables, the model can in certain respects be formulated in real terms ultimately. Problems of nominal price inertia are ignored to avoid the complications of cyclical fluctuations. So nominal prices and real money balances are free to jump.

The portfolio subsystem

Agents hold different assets in portfolio. It is assumed that the shares of assets held are independent of price levels. Asset demand equations can therefore be formulated in nominal or in real terms, both formulations being equivalent. It should be noted, however, that the demand for securities depends on nominal interest rates. The nominal interest rate ($\tilde{i}$) is the sum of the real interest rate ($\tilde{r}$) and expected price inflation ($\tilde{p}$). Thus for both countries the Fischer equations are:

$$\tilde{i} = \tilde{r} + \tilde{p}$$

$$\tilde{i}^* = \tilde{r}^* + \tilde{p}^*$$

Variables expressed as percentages are denoted by lower-case letters with a tilde. For instance, $\tilde{p}$, is the percentage increase of the domestic price level. Foreign variables are denoted by an asterisk in the sequel of the paper. Domestic residents split their non-human real wealth (W) between real cash balances (M), bonds issued by domestic firms and domestic government

(B_h) which are perfect substitutes and bonds issued by foreign firms and the foreign government (B_m) which are as such also perfect substitutes. Upper-case letters relate to real variables expressed in national commodity units. Nominal price levels are denoted by P (for domestic prices), P^* (for foreign prices) and P^e for the nominal exchange rate. The nominal exchange rate is measured in units of home currency per unit of foreign exchange. The real exchange rate is then defined by: $E = P^* P^e / P$.

Portfolio choices should take account of exchange rate expectations. The expected rate of return on foreign bonds bought by domestic residents then equals the nominal interest rate (i^*) corrected for expected nominal exchange rate depreciations: $\dot{i}^* + \dot{p}^e$ (where $\dot{p}^e = \dot{P}^e / P^e$). A dot over a variable indicates as usual a time derivative. From the definition of the real exchange rate we get: $\dot{E} / E = \dot{E} = \dot{p}^* + \dot{p}^e - \dot{p}$. Substituting for $\dot{i}^*$ as well as for $\dot{p}^e$ the expected rate of return on foreign securities held in the home country reads: $\dot{i}^* + \dot{p}^e = \dot{r}^* + \dot{p} + \dot{E}$. A similar reasoning applies to the expected rate of return on domestic bonds held by foreign residents. This rate is equal to $\dot{i} - \dot{p}^e = \dot{r} + \dot{p}^* - \dot{E}$. In the paper we assume that expectations are formed rationally which in the absence of uncertainty means that agents have perfect foresight.

Assuming that money is riskless and taking account of the transaction, demand for cash being related to output (Y), the asset demand equations for the domestic economy can be written as:

$$(2.1) \quad M = M(\tilde{r} + \tilde{p}, \tilde{r}^* + \tilde{p} + \tilde{E}, Y)W, \quad M_r < 0, \quad M_{r^*} < 0, \quad M_y > 0$$

$$(2.2) \quad B_h = B(\tilde{r} + \tilde{p}, \tilde{r}^* + \tilde{p} + \tilde{E}, Y)W, \quad B_r > 0, \quad B_{r^*} < 0, \quad B_y < 0$$

$$(2.3) \quad EB_m = F(\tilde{r} + \tilde{p}, \tilde{r}^* + \tilde{p} + \tilde{E})W, \quad F_r < 0, \quad F_{r^*} > 0.$$

To simplify somewhat changes in transaction balances are assumed to come exclusively out of domestic bond holdings, and therefore Y does not appear in equation (2.3). The balance-sheet identity $W = M + B_h + EB_m$ implies the following "adding-up" constraints:

$$M_r + B_r + F_r = 0$$

$$M_{r^*} + B_{r^*} + F_{r^*} = 0$$

$$M_y + B_y = 0$$

The corresponding asset demand equations for the foreign country are given by

$$(2.4) \quad M^* = M^* (\tilde{r} + \tilde{p}^* - \tilde{E}, \tilde{r}^* + \tilde{p}^*, Y^*) W^*, \quad M_r^* < 0, \quad M_{r^*}^* < 0, \quad M_y^* > 0$$

$$(2.5) \quad B_h^* = B^* (\tilde{r} + \tilde{p}^* - \tilde{E}, \tilde{r}^* + \tilde{p}^*, Y^*) W^*, \quad B_r^* < 0, \quad B_{r^*}^* > 0, \quad B_y^* < 0$$

$$(2.6) \quad \frac{B_m^*}{E} = F^* (\tilde{r} + \tilde{p}^* - \tilde{E}, \tilde{r}^* + \tilde{p}^*) W^*, \quad F_r^* > 0, \quad F_{r^*}^* < 0.$$

The "adding-up" constraints are similar to those presented above. The supply of bonds (B) consists of securities issued by firms and by the government. Firms finance net investment by issuing bonds. The stock of bonds supplied by firms is therefore equal to the capital stock (K). The amount of outstanding government bonds correspond to the government debt (D). Because bonds originating in the private and the public sector are perfect substitutes we can write for both countries:

$$(2.7) \quad B = K + D$$

$$(2.8) \quad B^* = K^* + D^*.$$

It is assumed that international securities markets are in equilibrium, which is expressed by

$$(2.9) \quad B = B_h + B_m^*$$

$$(2.10) \quad B^* = B_h^* + B_m.$$

There is no currency substitution.

The portfolio submodel can be completed by specifying real wealth. As observed above the balance-sheet identities are: $W = M + B_h + EB_m$ and $W^* = M^* + B_h^* + B_m^*/E$. Net foreign claims expressed in domestic commodities equals: $F = EB_m - B_m^*$. Substituting this definition in equations (2.9) and (2.10) and the result in the balance-sheet identities yields finally:

$$(2.11) \quad W = M + B + F$$

$$(2.12) \quad W^* = M^* + B^* - \frac{F}{E}$$

The commodity subsystem

To keep in line with the portfolio submodel the commodity expenditure equations are not based on explicit microfoundations in the form of intertemporal choices made by consumers and firms. A full microeconomic foundation of macroeconomics should integrate saving, investment and portfolio decisions. Such an approach is beyond the scope of the present paper. Moreover, for an analysis of policy shocks which are not pre-announced the present model may do reasonably well.

It is assumed that total consumption (C) depends on real income, net of lump-sum taxes (T), real non-human wealth and the real rate of interest. The consumption functions can be written as:

$$(2.13) \quad C = C((Y-T), W, r), \quad C_y > 0, \quad C_w > 0, \quad C_r < 0$$

$$(2.14) \quad C^* = C^*((Y^*-T^*), W^*, r^*), \quad C_{y^*}^* > 0, \quad C_{w^*}^* > 0, \quad C_{r^*}^* < 0.$$

Total consumption is split up between domestic and foreign goods, which are imperfect substitutes. Expenditures on both commodities therefore depend on total consumption and also on the terms of trade (which is the reciprocal of the real exchange rate). Denoting demand for home goods by domestic residents by C_h (resp. C_h^*) and demand for foreign goods (imports) by C_m (resp. C_m^*) the commodity demand equations read:

$$(2.15) \quad C_h = H(C, E), \quad H_C > 0, \quad H_E \geq 0$$

$$(2.16) \quad C_h^* = H^*(C^*, E), \quad H^* > 0, \quad H^* \leq 0$$

$\qquad \qquad \qquad c^* \qquad \qquad \qquad e$

$$(2.17) \quad EC_m = G(C, E), \quad G_c > 0, \quad G_e \leq 0$$

$$(2.18) \quad \frac{C_m^*}{E} = G^*(C^*, E), \quad G_{c^*}^* > 0, \quad G_e^* \geq 0.$$

The signs of the partial derivatives reflect the usual income and substitution effects from the static theory of consumer behaviour.

Investment (J) depends on the real interest rate and the existing stock of capital along the lines of standard neoclassical theory:

$$(2.19) \quad J = J(\tilde{r}, K), \quad J_r < 0, \quad J_k < 0$$

$$(2.20) \quad J^* = J^*(\tilde{r}^*, K^*), \quad J_{r^*}^* < 0, \quad J_{k^*}^* < 0$$

Output follows from a neoclassical production function in labour and capital, which obeys the Inada conditions. The labour market is always in equilibrium and the supply of labour is exogenous. Labour can therefore be suppressed and output equals:

$$(2.21) \quad Y = Y(K), \quad Y_k > 0, \quad Y_{kk} < 0$$

$$(2.22) \quad Y^* = Y^*(K^*), \quad Y_{k^*}^* > 0, \quad Y_{k^*k^*}^* < 0$$

The commodity markets are in equilibrium, which implies

$$(2.23) \quad Y = C_h + J + G + C_m^*$$

$$(2.24) \quad Y^* = C_h^* + J^* + G^* + C_m$$

where G denotes real government expenditure, which is exogenous and falls entirely on domestic goods in both countries ¹. A similar assumption is made with respect to investment outlays.

The dynamic subsystem

The rate of change of the capital stock can be obtained by subtracting the amount of depreciation from the volume of gross investment. Assuming exponential depreciation at a rate δ in both countries one gets:

$$(2.25) \quad \dot{K} = J - \delta K$$

$$(2.26) \quad \dot{K}^* = J^* - \delta K^*$$

The government buys goods on the domestic market, pays interest to domestic and foreign residents on outstanding debt. These outlays are financed by imposing lump-sum taxes, by selling bonds or by printing money. If the government sells bonds it increases its debt, which is shown by writing the budget constraint as:

$$(2.27) \quad \dot{D} = rD + G - T - \tilde{\theta}M$$

$$(2.28) \quad \dot{D}^* = r^*D^* + G^* - T^* - \tilde{\theta}^*M^*$$

where $\tilde{\theta}$ (resp. $\tilde{\theta}^*$) denotes the exogenous growth rate of nominal money supply at home (resp. abroad). These equations assume in effect that government debt is indexed and constitutes a sure claim on given amounts of future goods (e.g. Sargent, 1986). The same assumption applies to private debt.

¹ The case where government expenditure falls on domestic and foreign goods is discussed extensively in Frenkel and Razin (1987).

Feedback rules for taxation, government spending or monetary growth are required, because in the absence of such rules the solvency of the government's finances is not guaranteed and therefore the government debt explodes. A sensible tax rule (cf. Buiter, 1986) is

$$\dot{T} = T_0 + \xi_1 \dot{D} - \xi_2 \ddot{D}$$

$$\dot{T}^* = T_0^* + \xi_1 \dot{D}^* - \xi_2 \ddot{D}^*$$

A similar rule could be given for government spending or monetary growth. There is no commercial banking system. Money market equilibrium in both regions is then given by

$$(2.29) \quad \dot{M} = (\tilde{\theta} - \tilde{p})M$$

$$(2.30) \quad \dot{M}^* = (\tilde{\theta}^* - \tilde{p}^*)M^*$$

The current account of the domestic economy is the sum of the balance of trade and the balance of interest payments on foreign assets held by domestic residents and domestic securities held by foreign residents. This sum equals the (net) increase in the wealth of the nation vis-à-vis the rest of the world.

$$\begin{aligned} (2.31) \quad \dot{F} &= \tilde{r}^* EB_m - \tilde{r} B_m^* + C_m^* - EC_m \\ &= \tilde{r} F + (\tilde{r}^* - \tilde{r}) EB_m + C_m^* - EC_m \end{aligned}$$

It should be noted that F can be negative in which case the country is a debtor nation. Consistency requires: $\dot{F}^* = -\frac{\dot{F}}{\bar{E}}$ as can be easily checked to hold.

There are 31 equations in 30 endogenous variables, viz. $M, B_h, B_m, B, W, D, K, Y, C, C_h, C_m, J, \tilde{r}, \tilde{p}, M^*, B_h^*, B_m^*, B^*, W^*, D^*, K^*, Y^*, C^*, C_h^*$.

C_m^* , J^* , $\tilde{r}^*$, $\tilde{p}^*$, E , F . Invoking Walras' law one of the equilibrium relations can be eliminated. Therefore, equation (2.10) which is the equilibrium condition for the market for foreign securities can be skipped. The model comprises eight state variables, of which the following five are backward-looking: K , K^* , D , D^* and F . The three remaining state variables, M , M^* and E are forward-looking. For given values of the backward-looking state variables short-run equilibrium requires jumps of the forward-looking state variables. These discrete jumps reflect price adjustments which together with interest rate changes generate equilibrium in the goods markets, the bonds markets and the money markets.

An analytical solution of the model is intractable even if symmetry is assumed and the model is linearized around a steady state solution. A linearized version of the model is presented in Appendix I. Solution of this linear model results in rather intricate expressions which are difficult to handle and to interpret. For this reason the models presented in Appendix I will be simplified. The analytical solution of the simplified model will be discussed in Section 3. Borrowing from Branson (1988) we call this the "fundamentals" model, because it fully reflects the main characteristics of the extended model. More specifically, our fundamentals model deals with current account and real exchange rate dynamics which are of utmost importance for the understanding of international economic relations and their development over time. The insights gained by solving this model can be used in the interpretation of simulation experiments with the complete model, which will be discussed in Section 4.

3. The fundamentals model

The fundamentals model is obtained by making the following simplifications. Investment and capital accumulation are ignored. If the stock of capital is fixed output will not change either. Therefore, the variable Y can be dropped from the consumption function and the asset demand equations. Further it is assumed that the nominal money stocks are constant ($\dot{g} = \dot{g}^* = 0$). As a consequence there will be no inflation and no difference between nominal and real interest rates. Moreover, we assume that in each country residents' demand for real cash balances is independent of the

return on the security issued in the other country. Branson and Henderson (1985) provide a microeconomic foundation for this assumption. Finally, it should be stated that the government budget constraint is not explicitly taken into consideration. In what manner real government expenditure is financed is now left unexplained.

The simplified model is just a special case of the linear, symmetrical two-country model presented in Appendix I. In the linearized models variables are measured as percentage deviations from their steady state values. Such relative deviations are denoted by lower-case letters. Variables which relate to percentages in the original model are measured as absolute deviations from steady state values. Such absolute deviations are denoted by variables without a tilde. Definitions of elasticities and other ratios can be found in Appendix I.

The fundamentals model can now be written in a compact form as follows ¹:

$$(3.1) \quad \lambda_m (m-w) = -M_r r$$

$$(3.2) \quad \lambda_m (m^*-w^*) = -M_r r^*$$

$$(3.3) \quad (1-\lambda_m)(b-w) = B_r r - B_{r^*}(r^* + \dot{e}) + F_r^* (r - \dot{e}) - F_{r^*}^* r^*$$

$$(3.4) \quad w = \lambda_m m + (1-\lambda_m)b + f$$

$$(3.5) \quad w^* = \lambda_m m^* + (1-\lambda_m)b^* - f$$

$$(3.6) \quad \mu[\nu w - \varphi r + (1-\mu)(\sigma-1)e] + (1-\mu)[\nu w^* - \varphi r^* + \{\mu(\sigma-1) + 1\} e]$$

$$+ g = 0$$

$$(3.7) \quad \mu[\nu w^* - \varphi r^* - (1-\mu)(\sigma-1)e] + (1-\mu)[\nu w - \varphi r - \{\mu(\sigma-1) + 1\} e]$$

¹ All coefficients are defined positively so that signs of derived expressions can be detected in a straightforward manner.

$$+ g^* = 0$$

$$(3.8) \quad \dot{f} = \tilde{r}f + x[\nu w^* - \rho r^* + \{2\mu(\sigma-1) + 1\}e - \nu w + \rho r]$$

Equations (3.1) and (3.2) imply equilibrium on the money market in both countries. Equilibrium on the market for domestic bonds is given in equation (3.3). The composition of real wealth follows from equations (3.4) and (3.5). It should be noted that f denotes deviations from the steady state value of F , which is assumed to be zero, expressed as a percentage of steady state wealth W . Equations (3.6) and (3.7) are the equilibrium conditions for the goods market after substitution of the behavioural relations for consumption of home goods and exports. The deviation of government expenditure (g), which is assumed to be zero in the steady state, is expressed as a percentage of steady state output (Y). Output itself does not deviate from its initial steady state value as explained above. Equation (3.8) gives the balance of payments condition after substituting the relevant expressions for c_m and c_m^* . To simplify further, it is assumed in deriving the expressions (3.3) and (3.8) that the share of foreign bonds in total wealth in the initial steady state is very small and may be neglected ($\lambda_f = 0$). Through this assumption a strong form of "local asset preference" is introduced. The supply of domestic bonds (b) and of foreign bonds (b^*) are exogenous in the fundamentals model.

The model can be reduced to a system of two simultaneous differential equations in e and f . Details of the solutions are given in Appendix II. The real exchange rate (e) is a forward-looking variable, whereas foreign debt (f) is a backward-looking variable. The interest rates r and r^* and the wealth variables w and w^* can be found from the equations (3.1), (3.2), (3.4), (3.5), (3.6) and (3.7). Substitution of the result in equations (3.3) and (3.8) then yields:

$$(3.9) \quad \dot{f} = \tilde{r}f + x \left[\frac{2\mu(\sigma-1) + 1}{(2\mu-1)} \right] e + \frac{x}{(2\mu-1)} (g - g^*)$$

$$(3.10) \quad \dot{e} = \frac{A_1}{A_2} \left\{ \left[\frac{2\nu}{(1-\lambda_m)} + \frac{\nu^M r + \rho(1-\lambda_m)}{(1-\lambda_m)A_1} \right] f + \frac{2(1-\mu)(2\mu(\sigma-1) + 1)}{(2\mu-1)} e \right\}$$

$$+ \nu(b-b^*) + \frac{1}{(2\mu-1)} (g-g^*) \Big]$$

where

$$A_1 = F_r + F_r^* > 0$$

$$A_2 = \left[\frac{\nu M_r}{1-\lambda_m} + \varphi \right] (F_{r^*} + F_r^*) > 0$$

The phase diagram for the system (3.9) and (3.10) is shown in Figure 1. It is assumed that there is "local goods preference" ($\mu > 0.5$), so that the slopes of the $\dot{e} = 0$ and $\dot{f} = 0$ loci are both negative¹.

Figure 1.

¹ In the full version model the slope of the $\dot{f} = 0$ locus is indeterminate. If the $\dot{f} = 0$ locus is upward sloping, a unique downward saddlepath exists (see for the one-country case: Branson and Buiter (1983), Branson (1985) and for the two-country case: Branson and Henderson (1985)).

Furthermore, the slope of the $\dot{e}=0$ locus is assumed to be larger in absolute value than the slope of the $\dot{f}=0$ locus. As can easily be checked the system is saddlepoint stable in this case. The stable arm (separatrix) of the saddlepoint is indicated by SS'. For saddlepoint stability it is therefore required that

$$\frac{1}{(1-\lambda_m)(1-\mu)} \left[\nu + \frac{\nu M_r + (1-\lambda_m)\varphi}{2A_1} \right] > \frac{\tilde{r}}{\tilde{x}}$$

For the coefficient $\tilde{x}$ we may write: $\tilde{x} = \bar{x}(1-\mu)(1-\lambda_m)$ with $\bar{x} = \frac{C}{B} = 1$ by proper scaling of variables. Substitution of this expression and that for A_1 in the inequality results in:

$$\nu + \frac{\nu M_r + (1-\lambda_m)\varphi}{2(F_r + F_r^*)} > \tilde{r}$$

For reasonable values of the parameters this inequality will hold. For instance realistic values may be $\nu=0.1$ and $\tilde{r}=0.05$ which is sufficient to obtain the required condition. Therefore, it may be concluded that the model is saddlepoint stable. The inequality condition implies that the determinant of the state matrix is negative. Consequently, both eigenvalues (roots) have opposite signs, which is what one expects in the case where one of the state variables is backward-looking and the other one is a (forward-looking) jump variable. If the determinant of the state matrix is positive both eigenvalues are positive, because the trace of the state matrix is also positive. The model is then unstable, which may be checked by analysing the appropriate phase diagram.

The model can be used to analyse policy shocks. Identical shocks originating in both regions ($g=g^*$ and $b=b^*$) have no impact on the real exchange rate (e) and foreign debt ($-f$) as should be the case in a symmetric two-country model. The effects of a decrease in government spending in the foreign country, say the US, are shown in Figure 2. A fall in g^* leads to a downward shift of both the $\dot{f}=0$ and the $\dot{e}=0$ loci. As is reasonable to assume: $\chi < 1$, so that the shift of the $\dot{e}=0$ locus is relatively larger. A fall of government expenditure leads on impact to an appreciation of the domestic currency unit. This is indicated by the change from point A to point B in

Figure 2. The balance of payments of the US improves and f declines gradually along the stable arm of the new saddlepoint.

Figure 2.

The real exchange rate overshoots its long-run equilibrium and after the initial jump depreciates smoothly to maintain asset market equilibrium. Such a depreciation is required because the US interest rate falls under influence of a rise in national savings. In the new steady state (point A') external debt of the US is lower and the real exchange rate has depreciated compared with the initial situation as can be shown by considering the long-run solution of equations (3.9) and (3.10):

$$(3.11) \quad f = -\frac{1}{Q} \left[\frac{\gamma}{2} (b-b^*) + \frac{(1.5-\mu)}{(2\mu-1)} (g-g^*) \right]$$

$$(3.12) \quad e = \frac{1}{\Lambda} \left[\frac{\gamma}{2} (b-b^*) + \left[\frac{(1.5-\mu)}{(2\mu-1)} - \frac{\chi Q}{(2\mu-1)r} \right] (g-g^*) \right]$$

where

$$\Omega = \left[\frac{(1-\mu)\tilde{r}}{\chi} + \frac{\nu}{(1-\lambda_m)} + \frac{\nu M_r + p(1-\lambda_m)}{2A_1(1-\lambda_m)} \right]$$

$$\Lambda = \Omega \frac{2\mu(\sigma-1) + 1}{(2\mu-1)} \frac{\chi}{r}$$

From equation (3.11) it follows $\frac{\partial f}{\partial g^*} > 0$, so that a fall in government expenditure abroad induces a rise in foreign debt at home. As appears from equation (3.12)

$$\text{sign } \frac{\partial e}{\partial g^*} = -\text{sign} [\tilde{r}(1.5 - \mu) - \chi\Omega] = -\text{sign} \left[0.5\tilde{r} - \frac{\nu}{1-\lambda_m} \left(1 + \frac{M_r}{2A_1} \right) - \frac{p}{2A_1} \right]$$

As ν and $(1-\lambda_m)$ are roughly of the same magnitude it is reasonable to assume that the expression between brackets on the RHS is negative. Therefore, we may conclude: $\frac{\partial e}{\partial g^*} > 0$, so that a fall in foreign government expenditure leads to an appreciation of the real exchange rate in the long run from the European point of view. This result can be explained as follows. A decline in US government spending requires a depreciation of the dollar to crowd-in net exports. However, the "coupon" effect (cf. Stevenson, et al., 1988) works in the opposite direction because interest payments on foreign debt decrease, which puts less pressure on the dollar. Ultimately, the "crowding-in" effect still dominates the "coupon" effect and the real exchange rate (e) falls.

The effects of an autonomous increase in domestic government debt (i.e., an increase in b) are presented in Figure 3. A change in b has here no effect on the $\dot{f}=0$ locus. The $\dot{e}=0$ locus shifts downwards and the new steady state equilibrium is at point A'. Compared with the initial steady state foreign debt is higher and the real exchange rate has depreciated. In

Figure 3.

contrast the real exchange rate appreciates on impact of the asset supply shock (movement from $A \rightarrow B$ in Figure 3). The real exchange rate depreciation following the initial negative jump is required to maintain asset market equilibrium. A rise in the supply of domestic securities raises the interest rate at home, so that agents shift to domestic assets. To restore portfolio equilibrium the rate of return on foreign securities must rise, which requires a real exchange rate depreciation. The initial appreciation induces a balance of payments deficit in the home country (Europe), which increases foreign debt ($-f$).

In analysing fiscal policy account should be taken of the government budget constraint. For instance, if a decline in government spending ($g^* \downarrow$) in the US is financed by redeeming public debt ($b^* \downarrow$) the net result will be a substantial reduction in foreign debt as can be seen by combining the results in Figures 2 and 3. (Because of the symmetry an increase in b has the same effect as a reduction in b^* .) The real exchange rate movements in the short run (appreciation on impact) and in the long run (depreciation) are also reinforced.

The dynamic implications of the government budget constraint along with other complications of the fundamentals model will be taken into account in

the next section. An analytical solution of the full-size model is somewhat intractable. Therefore, the effect policy shocks will be analysed by studying numerical examples. As will become clear the conclusions of the present section are very useful in explaining the numerical results.

4. Reducing US external debt

There are several alternatives which lead to reduction of the current account deficit of the US economy. As shown in Section 3 a cut in government expenditure will do the job. Another way to get rid of foreign debt is to inflate the economy by increasing the growth of money. If policy measures are not effectuated, a financial crisis may force the economy on the right track. These different options will be studied in this Section by presenting numerical exercises ¹.

A cut in government spending

The consequences of a fall in government expenditure abroad in the full-scale model are illustrated in Table 1. It is assumed that the government cuts its expenses with 5% in relation to output in the initial steady state. The fall in public expenditure is used to reduce the government budget deficit. Therefore, the supply of foreign bonds declines in accordance with the dynamics of the government budget constraint.

¹ Computations are based on the algorithm developed in Markink and van der Ploeg (1989).

Table 1: A cut in government spending abroad ($g^* = -5$)

variable ^{a)}	period	0	3	∞
<u>Europe</u>				
real cash balances (m)		-1.58	-1.68	-5.14
real interest rate (r)		-0.01	-0.00	0.01
real wealth (w)		-0.16	-0.44	-4.11
capital stock (k)		0	0.03	-0.24
inflation (p)		0.04	0.03	0
consumption domestic goods (c_h)		-0.23	-0.46	-1.16
consumption foreign goods (c_m)		3.16	2.96	3.06
stock domestic bonds (b_h)		0.95	1.74	-1.45
stock foreign bonds (b_m)		-0.20	-2.67	-6.98
<u>US</u>				
real cash balances (m^*)		12.06	12.46	18.89
real interest rate (r^*)		-0.12	-0.10	-0.14
real wealth (w^*)		1.21	-1.92	1.99
capital stock (k^*)		0	0.74	3.86
inflation (p^*)		-0.10	-0.15	0
consumption domestic goods (c_h^*)		2.96	4.86	7.63
consumption foreign goods (c_m^*)		-0.42	1.44	3.41
stock domestic bonds (b_h^*)		0.10	-4.42	-2.19
stock foreign bonds (b_m^*)		-1.90	-3.44	2.58
<u>World</u>				
real exchange rate (e)		-1.69	-1.71	-2.11
foreign debt US (f)		0	-0.28	-3.50

^{a)} All variables are expressed as percentage deviation from their initial steady values (except f for which it is the deviation as a percentage of W in the initial steady state and r , r^* , for which it is 100 times the absolute deviation).

A fall in government expenditure in the US (g^*) leads to a real exchange rate appreciation on impact in accordance with the short-run result obtained in Section 3. Excess supply in the US goods market is choked off by a fall in the real interest rate which leads to crowding-in of domestic expenditure. The rise of US net exports reinforces this result. The real exchange rate appreciation induces excess supply in the European goods market, which is also choked off by a fall in the real interest rate. The nominal interest rate declines in the US which induces a shift in the portfolio from bonds towards real cash balances. Asset equilibrium in the world market requires a rise in the European nominal interest rate, which leads to an opposite shift, i.e. away from real cash balances towards bonds.

The increase in net exports of the US-economy caused by a real exchange rate depreciation of the dollar leads to a decline of external debt in the course of time as can be seen from the figures for the medium-run ($t=3$) presented in Table 1. The stock of capital increases in the US because investment is fostered by lower real interest rates. The real exchange rate depreciates somewhat in the medium run as should be expected from the analysis in Section 3, but the effect is rather weak and even not visible in the Table. This can be explained by the rise in output which relates to the increase in the capital stock. An increase in the supply of goods in the US (which is higher than that in Europe) puts pressure on the real dollar exchange rate, which was not the case in our earlier analysis.

These developments are even more pronounced in the long run. The US capital stock rises gradually to a new steady state value which is substantially higher than the initial value. As a consequence, the change in the real exchange rate is reversed and for $t \rightarrow \infty$ there is even a further appreciation above the initial jump at $t=0$. External debt of the US-economy is significantly reduced, which induces a small positive wealth effect despite the decline in bonds issued by the domestic government. The cut in government spending is favourable for the US in terms of the long-run consumption of both domestic and foreign goods. Europe incurs a loss in real wealth, but gains from an increase in its terms of trade. The long-run consumption of domestic goods falls, but long-run imports rise as foreign goods become relatively cheaper. Bond holdings in Europe decline, whereas the stock of European bonds held by US residents increases, which manifests the changed international composition of wealth.

Inflating the problem away

There are, of course, alternatives for US policy makers to reduce foreign debt. Another way out is to inflate the economy¹. The effects of an increase in the growth rate of money in the US are presented in Table 2. A rise in g^* induces a substantial upward price jump on impact in the US. The price increase in Europe is far less and the real exchange rate appreciates, which leads to an improvement of the US current account. Although the initial effect determined by the term $(c_m^* - e - c_m)$ is not much different from that in Table 1, the decline of the US foreign debt over time is now more pronounced. This can be explained by a negative supply effect. The real interest rates rises in the foreign economy to choke off excess demand in the goods market, caused by the depreciation of the real dollar exchange rate. Therefore, the US capital stock falls in the medium term so that there is a negative income effect leading to lower imports from Europe.

Monetary policy, as conceived here, reduces consumption in the US, but stimulates consumption in Europe in the short and medium term. It is therefore a kind of "beggar-thy-self" policy. This remarkable result follows from a fall in real interest rates in Europe. In this region the real interest rate must decline to eliminate excess supply in the goods market.

Inflation in Europe is a spill-over effect from the US economy originating in the portfolio subsystem. A rise in nominal interest rates in the US induces a rise in nominal interest rates in Europe to restore asset market equilibrium. An increase in domestic nominal interest rates corresponds to a fall in domestic real cash balances. With the money stock in Europe fixed this means that prices must rise there. Another point to note is that in both regions the allocation of wealth shifts towards foreign bonds in the short and medium run. This is more difficult to explain. First, because of the inflation generated in the US economy we have $i^* > i$ and $\frac{\dot{E}}{E} < 0$. The dollar is expected to depreciate as inflation accelerates in the US. Second, for the choice of securities within each region what matters is the

¹ Dornbusch (1989) qualifies this possibility as an alternative no-need-to-adjust view, but refrains from a careful analysis of this issue.

Table 2: A rise in money growth abroad ($\theta^* = 1$)

variable ^{a)}	period	0	3	∞
<u>Europe</u>				
real cash balances (m)		-6.64	-7.15	-19.00
real interest rate (r)		-0.02	-0.02	0.02
real wealth (w)		-0.67	-1.46	-16.92
capital stock (k)		0	0.13	-0.48
inflation (p)		0.14	0.14	0
consumption domestic goods (c_h)		0.07	0.08	-1.72
consumption foreign goods (c_m)		1.88	1.44	-5.01
stock domestic bonds (b_h)		-2.47	-3.07	-15.84
stock foreign bonds (b_m)		5.83	4.35	-20.02
<u>US</u>				
real cash balances (m^*)		-47.12	-46.65	-34.40
real interest rate (r^*)		0.01	0.01	-0.04
real wealth (w^*)		-4.71	-4.06	11.54
capital stock (k^*)		0	-0.05	0.93
inflation (p^*)		0.84	0.84	1.00
consumption domestic goods (c_h^*)		-0.38	-0.29	1.75
consumption foreign goods (c_m^*)		-2.19	-1.64	5.04
stock domestic bonds (b_h^*)		-2.92	-2.48	10.28
stock foreign bonds (b_m^*)		4.93	6.29	31.03
<u>World</u>				
real exchange rate (e)		-0.90	-0.68	1.65
foreign debt US (f)		0	-0.79	-14.82

a) See Table 1.

influence of rates of return on asset demand. In our exercise the increase in demand for foreign bonds in Europe depends on the equality $i < 2 (i^* + \frac{\dot{E}}{E})$. In the US the increase in demand for foreign bonds is explained by the

inequality $i^* < 2 \left(i - \frac{\dot{E}}{E} \right)$. Both inequalities may hold simultaneously as can be easily checked.

The long-run outcomes ($t \rightarrow \infty$) are dominated by the "coupon" effect. The terms of trade move against Europe, because the US becomes more of a rentier economy. Consumption of both goods rises in the US and declines in Europe. Inflation, therefore, pays in the long run. US prices increase proportionally to the money stock, while the European price level is stable. The real interest rate in the US falls and the capital stock increases. Both movements are reversed in Europe. The change in the long-run real (and nominal) interest rates is caused by the reallocation of domestic and foreign bonds from Europe to the US. Because regions have a preference for their own securities ("local asset preference": $\lambda_b > \lambda_f$) the interest rate in Europe rises and the US interest rate falls.

A financial crisis

Ongoing deficits on the US current account may lead to a loss of confidence in the rest of the world. This may be manifested by a sudden change of preferences away from foreign bonds towards domestic bonds in Europe¹. The effects of such a financial crisis are presented in Table 3. (To get figures which are broadly of the same magnitude as the outcomes in other Tables we set $b_{au} = 5$).

The results in both countries are mirror images. This holds also for the mutations in bond holdings if the figures for Europe are corrected for the direct impact of the change in preferences $\left(\frac{b_{au}}{\lambda_b} \right)$ for the stock of domestic bonds, b_h , and $\frac{b_{au}}{\lambda_f}$ for the stock of foreign bonds b_m). This

¹ Our exogenous variable b_{au} corresponds to the "safe haven" parameter in Branson (1985).

characteristic of the outcome is understandable, because a change in preferences takes the form of a zero-sum shock in an otherwise symmetrical

Table 3: A change in European preferences towards domestic bonds ($b_{au} = 5$)

variable ^{a)}	period	0	3	∞
<u>Europe</u>				
real cash balances (m)		-9.22	-9.80	-25.79
real interest rate (r)		-0.03	-0.03	0.02
real wealth (w)		-0.92	-2.02	-23.04
capital stock (k)		0	0.18	-0.63
inflation (p)		0.19	0.19	0
consumption domestic goods (c_h)		0.06	0.09	-2.32
consumption foreign goods (c_m)		2.93	2.27	-6.81
stock domestic bonds (b_h)		4.53	3.63	-13.66
stock foreign bonds (b_m)		-7.62	-9.62	-43.13
<u>US</u>				
real cash balances (m*)		9.22	9.80	25.79
real interest rate (r*)		0.03	0.03	-0.02
real wealth (w*)		0.92	2.02	23.04
capital stock (k*)		0	-0.18	0.63
inflation (p*)		-0.19	-0.19	0
consumption domestic goods (c_h^*)		-0.06	-0.09	2.32
consumption foreign goods (c_m^*)		-2.93	-2.27	6.81
stock domestic bonds (b_h^*)		3.81	4.71	22.00
stock foreign bonds (b_m^*)		-9.05	-7.04	26.46
<u>World</u>				
real exchange rate (e)		-1.43	-1.10	2.25
foreign debt US (f)		0	-1.10	-20.21

a) See Table 1.

two-country model. In Europe nominal interest rates rise in the short and medium run to induce agents to substitute domestic for foreign securities. The real exchange rate appreciates as foreign bonds become less attractive. As a consequence the European goods market is in excess supply and the real interest rate must decline to restore equilibrium. Exact the opposite holds in the US where a rise in the real interest rate must choke off excess demand in the goods market. The real dollar depreciation reduces the US foreign deficit and therefore external debt.

Capital is accumulated in Europe and decumulated in the US in the medium run. The corresponding supply effects lead to a gradual depreciation of the real exchange rate. The "coupon" effect reinforces these developments. The long-run situation is again dominated by the "coupon" effect. The real exchange rate has depreciated compared with the initial steady state. This results in a reversal of signs with respect to the interests rates, consumption levels and capital stocks. The reason is, of course, that the excess demand in the European goods market must be choked off by a rise of the real interest rate, whereas the opposite situation of excess supply holds in the US.

A financial crisis leads to a substantial improvement of the US external debt position. However, it should be observed that if the shock of confidence is sudden and large a significant balance of payment and foreign debt improvement may already be realised in the medium run. A severe financial crisis will lead to a substantial depreciation of the dollar real exchange rate. After things are settled preferences may go back to their original values. Therefore, the short and medium run effects in Table 3 may be more important than the long-run outcomes, which may not be realised after all.

The Branson scenario

It is frequently advocated by policy makers that a fiscal expansion in Europe and Japan should match a fiscal contraction in the US to prevent a deflationary development in the world economy as a whole. Moreover, it is argued that this policy mix could be a substitute for a real dollar depreciation. This view is strongly attacked by Branson (1988), who makes

perfectly clear that a change in the real exchange rate is a complement rather than a substitute to a fiscal shift in the world economy, except under most unusual circumstances. Our analysis strongly supports this view. The effects of an increase in government expenditure at home ($g > 0$) are exactly the same as the outcomes of a reduction in government spending abroad ($g^* < 0$). A fiscal shift as advocated by the Group of Seven therefore simply doubles the results presented in Table 1.

To avoid sudden movements of the exchange on impact of policy shocks Branson 1988) suggests a different scenario. Short-run exchange stability could be attained by combining a fiscal contraction in the US with a monetary expansion in Europe and Japan. This scenario has the additional benefit of lower world real interest rates and a stimulus to demand in Europe and Japan to offset the potential contraction in the US. Let us see what this means in terms of our model. Because the model is linear and symmetric the effects of a policy mix can be found by adding the outcomes of separate shocks. The consequences of a higher growth rate of money in Europe can be determined by interchanging country results and altering the signs of e and f in Table 2. The results of the Branson scenario can therefore be obtained by combining this with the figures presented in Table 1.

As can be seen a short-run stabilisation of the real exchange rate can be obtained by an appropriate mix of both policies. However, such a policy would not work! The real dollar exchange rate would indeed gradually depreciate, but the US foreign deficit and debt would increase. In Europe the decline in real cash balances depresses consumption. In contrast, US consumption is strongly pushed by the substantial fall in the real interest rate. Combining these absorption effects leads to a rise in the US deficit on current account. These effects are reinforced in the medium run, whereas in the long run the coupon effect is not strong enough to offset these developments. The Branson scenario scores on all points (lower real interest rates, less deflation, gradual dollar depreciation) in our exercise, but fails at the main point that is a reduction of US external debt. This raises the question in what respect Branson's model differs from ours. The decisive point is that Branson ignores the role of absorption effects in the import equations of both countries. The developments of the trade balances are therefore exclusively determined by real exchange rate movements in his model.

5. Conclusions

It is shown that a reduction in US external debt can be attained by a fiscal contraction or by a rise in the monetary growth rate. Both options serve the same goal but there are important differences as may be expected. A fiscal contraction in the US leads to a decline in prices in the short and medium run. In contrast, an increase in the monetary growth rate raises inflation in the US in the short and long run. Such an inflation erodes non-human wealth and depresses consumption. From a short-run perspective the inflationary solution seems therefore less attractive. But the long-run consequences may not be so bad. A substantial reduction in external debt induces positive wealth and income effects, which lead to a real dollar appreciation and an increase in consumption of domestic and foreign goods. In the case of a fiscal contraction there is also a rise in long-run consumption in the US, but this time it is more strongly supported by capital accumulation. As a result of this the European economy suffers less from a reduction in US foreign debt in the long run than in the case of a monetary expansion. For this reason a fiscal contraction may be preferable after all.

As is often argued by policy makers fiscal contraction in the US should be matched by fiscal expansion in Europe and Japan to avoid a deflationary development in the world as a whole. In our symmetric model with flexible prices a fiscal expansion in Europe generates exactly the same effects as a fiscal reduction in the US. A fiscal shift is therefore no substitute for a real dollar depreciation as argued forcefully by Branson (1988). The symmetry argument carries over to other cases. A reduction in monetary growth in Europe will have the same effects as a rise in monetary growth in the US. This is the very reason why the Branson scenario may not work. Branson (1988) proposes a combination of fiscal contraction in the US and easing of monetary policy in Europe and Japan. Such a policy-mix may prevent sudden exchange rate changes, but then the final goal of a reduction in US external debt may not be attained as we have shown in this paper.

The present analysis can be extended in several directions. From a policy point of view it could be attractive to add more realism to the model. One way to do this is to pay proper attention to labour markets in different economies and the institutional differences that may exist between

the US and Europe (e.g. van de Klundert and van der Ploeg, 1989). Another point is hysteresis in export markets which may complicate the problem of international adjustment after a disturbance of the equilibrium as observed by Dornbusch (1989). From a theoretical point of view it may be desirable to search for microfoundations which cover the case of imperfect capital mobility and portfolio diversification.

References

- Branson, W.H. (1979), "Exchange Rate Dynamics and Monetary Policy" in A. Lindbeck (ed.), Inflation and Employment in Open Economies, Amsterdam: North-Holland, pp. 190-224.
- Branson, W.H. (1985), "The Dynamic Interaction of Exchange Rates and Trade Flows", in Th. Peeters, P. Praet and P. Reding (eds.), International Trade and Exchange Rates in the Late Eighties, Amsterdam: North-Holland.
- Branson, W.H. (1988), "International Adjustment and the Dollar: Policy Illusions and Economic Constraints" in W. Guth (mod.), Economic Policy Coordination, IMF, Washington, D.C., pp. 44-84.
- Branson, W.H. and W.H. Buiter (1983), "Monetary and Fiscal Policy with Flexible Exchange Rates" in J.S. Bhandari and B.H. Putnam (eds.), Economic Interdependence and Flexible Exchange Rates, Cambridge, MA, the MIT Press, pp. 251-285.
- Branson, W.H. and D.W. Henderson (1985), "The Specification and Influence of Asset Markets" in R.W. Jones and P.B. Kenen (eds.), Handbook of International Economics, Vol. II, Elsevier Science Publishers B.V., pp. 749-805.
- Buiter, W.H. (1986), "Fiscal Policy in Open, Interdependent Economies" in A. Razin and E. Sadka (eds.), Economic Policy in Theory and Practice, London: MacMillan.
- Dornbusch, R.P. (1976), "Exchange Rate Dynamics", Journal of Political Economy, 84, pp. 1161-1176.
- Dornbusch, R.P. (1989), "The Need for Further Dollar Depreciation" in J.J. Sijben (ed.), Financing the World Economy in the Nineties, Dordrecht: Kluwer Academic Publishers, pp. 19-34.

- Frenkel, J.A. and A. Razin (1987), Fiscal Policies and the World Economy, Cambridge, MA, the MIT Press.
- Giovannini, A. (1988), "The Real Exchange Rate, the Capital Stock, and Fiscal Policy", European Economic Review, 32, pp. 1747-1767.
- Kawai, M. (1985), "Exchange Rates, the Current Account and Monetary-Fiscal Policies in the Short Run and in the Long Run", Oxford Economic Papers, 37, pp. 391-425.
- Klundert, Th. van de (1986), "Economic Resilience: A Two-Country Analysis", De Economist, 134, pp. 25-41.
- Klundert, Th. van de and F. van der Ploeg (1989), "Fiscal Policy and Finite Lives in Interdependent Economies with Real and Nominal Wage Rigidity", Oxford Economic Papers (forthcoming).
- Markink, A.J. and F. van der Ploeg (1989), "Dynamic Policy Simulation of Linear Models with Rational Expectations of Future Events: A Computer Package", Center for Economic Research, Tilburg University, Discussion Paper No. 8906.
- Ploeg, F. van der (1988), "Monetary and Fiscal Policy in Interdependent Economies with Capital Accumulation, Death and Population Growth", Centre for Economic Policy Research, London, Discussion Paper No. 270.
- Sargent, Th.J. (1986), Rational Expectations and Inflation, New York, Harper and Row.
- Stevenson, A., V. Muscatelli and M. Gregory (1988), Macroeconomic Theory and Stabilisation Policy, Oxford, Philip Allan Publishers Ltd.

Appendix I: The linear model

The model outlined in Section 2 is linearized around a steady state solution which is symmetrical¹. Variables are written as percentage deviations (lower-case letters), for instance $m = \frac{dM}{M}$. For variables which are expressed as percentages in the original model, absolute deviations are taken (the tilde is now dropped). The numbering of equations corresponds to the numbers given in Section 2. All coefficients are defined positive.

Portfolio submodel

Asset demand equations:

$$(A.1) \quad \lambda_m(m-w) = -M_r(r+p) - M_{r*}(r^*+p+\dot{e}) + \epsilon_m y, \quad \epsilon_m = \frac{M_y Y}{W}$$

$$(A.2) \quad \lambda_b(b_h-w) = B_r(r+p) - B_{r*}(r^*+p+\dot{e}) - \epsilon_b y + b_{au}, \quad \epsilon_b = \frac{B_y Y}{W}$$

$$(A.3) \quad \lambda_f(b_m^*+e-w) = -F_r(r+p) + F_{r*}(r^*+p+\dot{e}) - b_{au}$$

$$(A.4) \quad \lambda_m(m^*-w^*) = -M_r^*(r+p^*-\dot{e}) - M_{r*}^*(r^*+p^*) + \epsilon_m^* y^*, \quad \epsilon_m^* = \frac{M^* y^* Y^*}{W^*}$$

$$(A.5) \quad \lambda_b(b_h^* - w^*) = -B_r^*(r+p^*-\dot{e}) + B_{r*}^*(r^*+p^*) - \epsilon_b^* y^*, \quad \epsilon_b^* = \frac{B^* y^* Y^*}{W^*}$$

$$(A.6) \quad \lambda_f(b_m^*+e-w^*) = F_r^*(r+p^*-\dot{e}) - F_{r*}^*(r^*+p^*)$$

From the adding-up condition we have: $\lambda_m + \lambda_b + \lambda_f = 1$, where the λ 's relate to the shares of assets in total wealth in the initial steady state. The exogenous variable b_{au} indicates a shift in preferences from foreign assets

¹ The symmetry assumption is made for analytical convenience. The problem of reducing external debt suggests an asymmetrical world to begin with. For theoretical purposes, however, a symmetrical initial solution is not harmful.

towards domestic assets in the home country. Exogenous variables are underlined>. The partial derivatives on the RHS of the equations are taken from the equations in Section 2. Symmetry requires:

$$M_r = M_r^*, M_{r^*} = M_r^*, B_r = B_r^*, F_r = F_r^*, B_{r^*} = B_r^* \text{ and } F_{r^*} = F_r^*.$$

In the neighbourhood of a stationary equilibrium: $d\tilde{E} = \tilde{E} = \dot{e}$.

Composition of stock of bonds

$$(A.7) \quad b = \eta k + (1-\eta)d$$

$$\eta = \frac{K}{B} = \frac{K^*}{B^*}$$

$$(A.8) \quad b^* = \eta k^* + (1-\eta)d^*$$

Equilibrium in the bonds markets is given by

$$(A.9) \quad b = \zeta b_h + (1-\zeta)b_m^*$$

$$\zeta = \frac{B_h}{B} = \frac{B_h^*}{B^*}$$

$$(A.10) \quad b^* = \zeta b_h^* + (1-\zeta)b_m$$

Definition of real wealth

$$(A.11) \quad w = \lambda_m m + (1-\lambda_m)b + f$$

$$(A.12) \quad w^* = \lambda_m m^* + (1-\lambda_m)b^* - f$$

In the initial steady state it is assumed $F=0$, so that the relative deviation is defined as $f = \frac{dF}{W}$.

Commodity subsystem

Consumption function:

$$(A.13) \quad c = \gamma (y-t) + \nu w - \rho r$$

$$(A.14) \quad c^* = \gamma (y^*-t^*) + \nu w^* - \rho r^*$$

The parameters γ , ν and ρ are elasticities of consumption with respect to disposable income, real wealth and the real interest rate. Changes in taxes are expressed as a percentage of output ($t = \frac{DT}{Y}$).

$$(A.15) \quad c_h = c + (1-\mu)(\sigma-1)e$$

$$(A.16) \quad c_h^* = c^* - (1-\mu)(\sigma-1)e$$

$$(A.17) \quad c_m + e = c - \mu(\sigma-1)e$$

$$(A.18) \quad c_m^* - e = c^* + \mu(\sigma-1)e$$

The choice between domestic and foreign consumption goods is based on a CES utility function with σ denoting the elasticity of substitution. The parameter μ is the initial share of consumption of home goods in total consumption: $\mu = \frac{c_h}{c}$

Investment function:

$$(A.19) \quad j = k - \frac{\kappa}{\delta} \left[\frac{1}{\alpha} \left(\frac{r}{r} - \underline{a} \right) + (k-1) \right]$$

$$(A.20) \quad j^* = k^* - \frac{\kappa}{\delta} \left[\frac{1}{\alpha} \left(\frac{r^*}{r} - \underline{a}^* \right) + (k^*-1) \right]$$

Investment demand follows from neoclassical theory (e.g. van de Klundert, 1986). The desired stock of capital ($\hat{k}$) can be found by equating the marginal product of capital and the user cost of capital (real interest rate plus rate of depreciation). The marginal product of capital is derived from

a Cobb-Douglas production function with α denoting the production elasticity of labour. Therefore, we may write

$$\hat{k} = \frac{1}{\alpha} \left(\underline{a} - \frac{r}{r} \right) + \underline{l}$$

Firms adjust the actual stock of capital with a time lag:

$$j - k = \frac{\kappa}{\delta} (\hat{k} - k)$$

where κ is an acceleration coefficient. Substitution of the expression for $\hat{k}$ and taking account of the equality between actual and desired capital stock in the initial steady state results in the formula given in equations (A.19) and (A.20). Exogenous productivity shocks are denoted by $\underline{a}$, whereas autonomous change in labour supply is given by $\underline{l}$ as appears from the production function:

$$(A.21) \quad y = \alpha \underline{l} + (1-\alpha)k + \underline{a}$$

$$(A.22) \quad y^* = \alpha \underline{l}^* + (1-\alpha)k^* + \underline{a}^*$$

Equilibrium in the goods markets is given by

$$(A.23) \quad y = \mu_c c_h + \mu_j j + g + (1-\mu_c - \mu_j) c_m^*$$

$$(A.24) \quad y^* = \mu_c c_h^* + \mu_j j^* + g^* + (1-\mu_c - \mu_j) c_m$$

where $\mu_c = \frac{C_h}{Y}$ is the share of domestic consumption of domestic goods in output and $\mu_j = \frac{I}{Y}$ is the share of investment goods in output. The change in government spending is expressed as a percentage of initial output: $g = \frac{dG}{Y}$.

Dynamic subsystem

Capital accumulation:

$$(A.25) \quad \dot{k} = \delta(i-k)$$

$$(A.26) \quad \dot{k}^* = \delta(i^* - k^*)$$

Government budget constraint:

$$(A.27) \quad \dot{d} = \tilde{r}d + r + \psi(g - t) - \rho\theta$$

$$\psi = \frac{Y}{D}, \quad \rho = \frac{M}{D}$$

$$(A.28) \quad \dot{d}^* = \tilde{r}d^* + r^* + \psi(g^* - t^*) - \rho\theta^*$$

It is assumed that government debt is positive in the initial steady state. The initial real rate of interest $\tilde{r}$ is assumed equal across countries. It should be recalled that changes in G and T are expressed as percentages of output. Solvency of the government's finances requires:

$$t = t_0 + \frac{\xi_1}{\psi} \dot{d} - \frac{\xi_2}{\psi} \dot{d}$$

$$t^* = t_0^* + \frac{\xi_1}{\psi} \dot{d}^* - \frac{\xi_2}{\psi} \dot{d}^*$$

Money market equilibrium is given by

$$(A.29) \quad \dot{m} = (\theta - p)m$$

$$(A.30) \quad \dot{m}^* = (\theta^* - p^*)m^*$$

In the initial steady state it is assumed: $\tilde{\theta} = \tilde{\theta}^* = 0$.

Balance of payments constraint:

$$(A.31) \quad \dot{f} = \tilde{r}f + \lambda_f(r^* - r) + \chi(c_m^* - e - c_m), \quad \chi = \frac{C_m^*}{W} = \frac{EC_m}{W}$$

$$[\chi = (1 - \mu_c - \mu_j)(1 - \eta)(1 - \lambda_m)\psi]$$

It should be noted that changes in F are expressed as percentages of W , because in the initial situation we have $F = 0$.

This completes the model. There are now 31 equations in 30 unknown variables. Applying Walras's law equation (A.10) can be eliminated. Equilibrium condition (A.9) will be rewritten for convenience. Subtracting w from both sides of the equation gives:

$$b-w = \zeta(b_h - w) + (1-\zeta)(b_m^* - e - w^*) + (1-\zeta)(w^*+e-w)$$

Substitution of equations (A.2) and (A.6) in the above expression yields:

$$(b-w) = \frac{\zeta}{\lambda_b} [B_r(r+p) - B_{r^*}(r^*+p+\dot{e}) - \epsilon_b y + b_{au}] + \frac{1-\zeta}{\lambda_f} [F_r^*(r+p^*-\dot{e}) - F_{r^*}^*(r^*+p^*)] + (1-\zeta)(w^*+e-w)$$

From the definitions of the coefficients it can be deduced that $\frac{\zeta}{\lambda_b} = \frac{1-\zeta}{\lambda_f} = \frac{1}{1-\lambda_m}$, assuming $W = W^*$ and $F=0$ in the initial steady state. Further by definition $(1-\lambda_m)(1-\zeta) = \lambda_f$. Substituting these relations in the above expression results in:

$$(1-\lambda_m)(b-w) = B_r(r+p) - B_{r^*}(r^*+p+\dot{e}) - \epsilon_b y + b_{au} + F_r^*(r+p^*-\dot{e}) - F_{r^*}^*(r^*+p^*) + \lambda_f(w^*+e-w)$$

Rearranging the equation gives:

$$(A.32) \quad \dot{e} = \frac{1}{B_{r^*} + F_{r^*}} \left[B_r(r+p) - B_{r^*}(r^*+p) + F_r^*(r+p^*) - F_{r^*}^*(r^*+p^*) - \epsilon_b y + b_{au} - (1-\lambda_m)b + \lambda_b w + \lambda_f(w^*+e) \right]$$

The model now consists of the output equations (A.1)-(A.8), (A.11)-(A.24) and the state equations (A.25)-(A.32). These (30) equations can be used to solve for:

$m, b_h, b_m, b, w, d, k, y, c, c_h, c_m, j, r, p$

$m^*, b_h^*, b_m^*, b^*, w^*, d^*, k^*, y^*, c^*, c_h^*, c_m^*, j^*, r^*, p^*, e, f.$

There are 5 backward-looking state variables (viz. k, k^*, d, d^*, f) and three forward-looking state variables (viz. m, m^*, e). For saddlepoint stability to hold one should therefore have 5 stable (negative) roots and 3 unstable (positive) roots.

In the numerical solutions of Section 4 the following parameter values have been applied:

Portfolio subsystem

$$\begin{array}{llll}
 -M_r = -M_r^* = -5 & -M_{r^*} = -M_r^* = 0 & \lambda_m = 0.1 & \epsilon_m = 1 \\
 B_r = B_{r^*} = 10 & -B_{r^*} = -B_r^* = -10 & \lambda_b = 0.6 & -\epsilon_b = -1 \\
 -F_r = -F_{r^*} = -5 & F_{r^*} = F_r^* = 10 & \lambda_f = 0.3 & \begin{array}{c} \text{---} \rightarrow \\ 0 \end{array} \\
 \begin{array}{c} \text{---} \rightarrow \\ 0 \end{array} & \begin{array}{c} \text{---} \rightarrow \\ 0 \end{array} & \begin{array}{c} \text{---} \rightarrow \\ 1 \end{array} &
 \end{array}$$

$$\eta = 0.5$$

Commodity subsystem

Consumption: $\gamma = 0.6, \nu = 0.1, \varphi = 20, \mu = 0.75, \sigma = 2$

Investment: $\kappa = 0.1, \delta = 0.025, \alpha = 0.75$

Output shares: $\mu_c = 0.6, \mu_j = 0.2$

Dynamic subsystem

Government budget constraint: $\psi = 1, \rho = \frac{2}{9}, \xi_1 = 0.5, \xi_2 = 0$

Balance of payments constraint: $\chi = 0.09$

Real interest rate: $\tilde{r} = 0.05$

Exogenous variables are equal to zero unless otherwise specified (see Tables).

The roots of the system satisfy the required saddlepath stability condition:

-0.4548, -0.4523, -0.0678, -0.0517, -0.0139
0.0188, 0.0271, 0.1269

Appendix II: Solution of the fundamentals model.

The simplified model is obtained from the system of equations presented in Appendix I by assuming:

$$M_{r^*} = M_r^* = 0 \quad \epsilon_m = \epsilon_b = \epsilon_m^* = \epsilon_b^* = 0 \quad b_{au} = 0$$

$$\eta = 0, \quad \gamma = 0, \quad \lambda_f = 0, \quad \theta = \theta^* = 0$$

In addition, the following assumptions are made. There is no capital (and investment), the government budget constraint is eliminated (bonds are exogenous) and money does not grow. The fundamentals model is given in Section 3. Here we present the method of solution.

Combining equations (3.1), (3.2), (3.4) and (3.6) result in:

$$(B.1) \quad (w-w^*) = -\frac{M_r}{1-\lambda_m} (r-r^*) + (b-b^*) + \frac{2f}{1-\lambda_m}$$

$$(B.2) \quad (w+w^*) = -\frac{M_r}{1-\lambda_m} (r+r^*) + (b+b^*)$$

From equations (3.6) and (3.7) one gets by adding and subtracting:

$$(B.3) \quad \nu(w-w^*) - \varphi(r-r^*) = -\frac{4\mu(1-\mu)(\sigma-1) + 2(1-\mu)}{(2\mu-1)} e - \frac{(g-g^*)}{2\mu-1}$$

$$(B.4) \quad \nu(w+w^*) - \varphi(r+r^*) = -(g+g^*)$$

Equations (B.1)-(B.4) yield

$$(B.5) \quad (r+r^*) = \frac{(1-\lambda_m)}{\nu M_r + \varphi(1-\lambda_m)} [\nu(b+b^*) + (g+g^*)]$$

$$(b.6) \quad (r-r^*) = \frac{(1-\lambda_m)}{\nu M_r + \varphi(1-\lambda_m)} [\nu(b-b^*) + \frac{2\nu}{(1-\lambda_m)} f + \frac{2(1-\mu)(2\mu(\sigma-1)+1)}{(2\mu-1)} e + \frac{(g-g^*)}{(2\mu-1)}]$$

The balance of payments equation (3.8) is:

$$(B.7) \quad \dot{f} = \tilde{r}f + \chi [\varphi(r-r^*) - \nu(w-w^*) + (2\mu(\sigma-1) + 1)e]$$

Substitution of equations (B.3) and (B.6) in equation (B.7) leads to:

$$(B.8) \quad \dot{f} = \tilde{r}f + \chi \left[\frac{2\mu(\sigma-1)+1}{(2\mu-1)} e + \frac{\chi}{(2\mu-1)} (g-g^*) \right]$$

which is equation (3.9).

Equation (3.3) can be rewritten taking account of equations (3.1) and (3.4) as

$$(B.9) \quad \dot{e} = \frac{1}{B_r^* + F_r^*} [(B_r + F_r^* - M_r)r + (B_r^* + F_r^*)r^* + f]$$

Applying short-hand notation and using the "adding-up" constraints (see Appendix I) equation (B.9) changes into

$$(B.10) \quad \dot{e} = \frac{1}{A_0} [A_1(r-r^*) + f]$$

where

$$A_0 = F_r^* + F_r^*$$

$$A_1 = F_r + F_r^*$$

Substitution of equation (B.6) in (B.10) then gives:

$$(B.11) \quad \dot{e} = \frac{A_1}{A_2} \left[\left\{ \frac{2\nu}{(1-\lambda_m)} + \frac{\nu M_r + \varphi(1-\lambda_m)}{(1-\lambda_m) A_1} \right\} f + \frac{2(1-\mu)(2\mu(\sigma-1)+1)}{(2\mu-1)} e \right]$$

$$+ \nu(b-b^*) + \frac{1}{(2\mu-1)} (g-g^*)]$$

$$\text{where } A_2 = \frac{\nu M_r + \varphi(1-\lambda_m)}{(1-\lambda_m)} A_0$$

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