Tilburg University

Existence of Optimal Auctions in General Environments

Page Jr., F.H.

Publication date:
1997

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Page Jr., F. H. (1997). *Existence of Optimal Auctions in General Environments*. (CentER Discussion Paper; Vol. 1997-28). CentER, Center for Economic Research.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

CBM
R

8414
1997
NR.28

entER

for
omic Research

Discussion paper

Tilburg University

Center
for
Economic Research

No. 9728

**EXISTENCE OF OPTIMAL AUCTIONS IN
GENERAL ENVIRONMENTS**

By Frank H. Page, Jr.

March 1997

ISSN 0924-7815

Existence of Optimal Auctions in General Environments

by

Frank H. Page, Jr.*
Department of Finance
University of Alabama
Tuscaloosa, Alabama 35487
U.S.A.

Abstract

We provide a unified approach to the problem of existence of optimal auctions for a wide variety of auction environments. We accomplish this by first establishing a general existence result for a particular Stackelberg revelation game. By systematically specializing our revelation game to cover various types of auctions, we are able to deduce the existence of optimal Bayesian auction mechanisms for single and multiple unit auctions, as well as for contract auctions with moral hazard and adverse selection. In all cases, we allow for externalities, risk aversion, and multidimensional, stochastically dependent types.

Current Version, February, 1997
Forthcoming in *Journal of Mathematical Economics*

Keywords: Optimal Auctions, Bayesian incentive compatibility, relative K-compactness.

*The author thanks two anonymous referees for helpful comments. The author also thanks Erik Balder, Andrzej Nowak, and Myrna Wooders for many helpful comments and discussions. An earlier version of this paper, entitled "Optimal Auction Design with Risk Aversion and Correlated Information," was completed while the author was visiting the CentER for Economic Research at Tilburg University and appears as Discussion Paper No. 94109, Center for Economic Research, Tilburg University. The author gratefully acknowledges CentER's support and hospitality.

1. Introduction

Overview

Much of the literature on auctions is devoted to the study of properties of various types of auctions (e.g., Milgrom and Weber (1982), and Maskin and Riley (1984)). Existence questions which arise naturally in general auction environments have only begun to be addressed.¹ In this paper, we provide a unified approach to the problem of existence of optimal auctions for a rich variety of auction environments. Our approach involves three steps. First, building on the work of Vickrey (1961), Harsanyi (1967/68), and Myerson (1979, 1982), we construct a general model of a Stackelberg revelation game.² Second, within the context of our model, we establish the existence of an optimal, feasible, interim individually rational and Bayesian incentive compatible revelation mechanism. Third, we deduce the existence of optimal Bayesian auction mechanisms for various types of auctions by showing that our Stackelberg revelation game can be systematically specialized to cover various types of auctions. Specifically, we show that our model covers (i) single unit auctions with risk neutral seller and buyers (e.g., Myerson (1981), Harris and Raviv (1981), Riley and Samuelson (1981), Milgrom and Weber (1982), and Cremer and McLean (1988)); (ii) multiple and single unit auctions with risk-averse seller and buyers (e.g., Matthews (1979, 1983), Maskin and Riley (1984), and Branco (1996)); and (iii) contract auctions with moral hazard and adverse selection (e.g., Laffont and Tirole (1987) and Page (1994)). In all cases we allow private information (in the form of buyer types) to be multidimensional and stochastically dependent. Thus, we show that the existence of optimal Bayesian auction mechanisms in a wide variety of auction environments follows from the existence of an optimal Bayesian revelation mechanism for a particular Stackelberg revelation game. This is the main contribution of the paper.

Stackelberg Revelation Games

In a Stackelberg revelation game, players' utilities depend on the private information of followers and on the choice made by the leader. Following Harsanyi (1967/68) (see also Vickrey (1961) and Myerson (1979, 1982)), we represent each follower's private information by a parameter called the follower's type. In the game, the leader selects a mechanism $f(\cdot): T \rightarrow K$ which specifies the leader's choice as a function

¹Recent papers on existence include Menezes and Monteiro (1995) which establishes existence in a discriminatory price auction and Lebrun (1996) which addresses the problem of existence in first price auctions assuming risk neutrality and independent types.

²Our model shares many of the characteristics of the principal-agent model with incomplete information (e.g., Myerson (1982), Page (1991a, 1992), and Balder (1996)).

of the h -tuple of types (i.e., the private information) reported by followers (here $f(\cdot)$ is the mechanism, T is the set of type profiles representing the private information of followers, and K is the set of choices available to the leader). If $t \in T$ is the type h -tuple (or type profile) reported by followers, then the leader is committed to making choice $f(t) \in K$.

In essence, the mechanism selected by the leader determines a revelation (or reporting) game to be played by the followers. The leader's problem, then, is to choose a feasible mechanism that maximizes his own expected utility subject to the constraints that the revelation game determined by the mechanism induces voluntary participation and truthful reporting by followers. Here we assume that each follower, in deciding whether or not to participate in the revelation game and in choosing a reporting strategy, seeks to maximize his own conditional expected utility, guided by his preferences and conditional probability beliefs concerning the types of others - assuming other followers will participate and report honestly. Thus, in order to ensure that the revelation game determined by the mechanism induces voluntary participation and truthful reporting, the leader must choose a mechanism that is interim individually rational (IRR) and Bayesian incentive compatible (BIC).

Due to the unusual nature of the Bayesian incentive compatibility (BIC) constraints, the resolution of the existence question which arises in connection with the leader's maximization problem is difficult (see Page (1989, 1991a, 1991b, 1992, 1994) and Balder (1996) for additional discussion).³ Novel existence arguments are required. Here, we show that the leader's problem has a solution (Theorem 1). Key ingredients in our proof are a new notion of compactness for mechanisms, called K -compactness (Definition 4.2), an abstract Komlos Theorem due to Balder (1990, 1996) (also see Komlos (1967)), and a new result showing that the feasible set of interim individually rational and Bayesian incentive compatible revelation mechanisms is K -compact (Theorem 2).⁴ The latter result extends an earlier result on K -compactness due to the author (Page (1992), Theorem 4.1) by allowing for mechanisms that map into quite general choice sets. In particular, the K -compactness result given here considers mechanisms $f(\cdot): T \rightarrow K$ where the choice set K is given by a convex, compact, metrizable subset of a Hausdorff locally convex topological

³Page (1989, 1991a, 1991b, 1994) and Balder (1996) treat existence questions which arise in connection with the leader's maximization problem over mechanisms which are feasible, interim individually rational, and *dominant strategy incentive compatible*. Page (1992) treats the more difficult case of maximization over mechanism which are feasible, interim individually rational, and *Bayesian incentive compatible*. Here we extend the existence result in Page (1992) to cover more general environments and use this result to give a unified treatment of existence of optimal auctions.

⁴The notion of K -compactness was introduced in Page (1989) - see Definition 2.2.2. However, the term K -compactness was not used in Page (1989). The term K -compactness was introduced in Page (1992) - see Definition 2.3.2.

space. In Page (1992), the K-compactness result considers only those mechanisms where K is given by the set of probability measures $P(X)$ defined over a compact metric space X of choices. This latter set of mechanisms represents an important example of the set mechanisms covered by the K-compactness result given here (Theorem 2). The added generality afforded by Theorem 2 makes the corresponding existence result applicable in a wide variety of environments - and thus makes possible a unified treatment of existence. The K-compactness result given here also extends the earlier result in Page (1992) by taking as the feasible set of mechanisms the set of all measurable selections from a feasible choice correspondence. This refinement is useful. In some auction applications, the feasible choice correspondence specializes to an ex post budget constraint for the seller and buyers. This is the case, for example, in the multiple unit auction model developed below. In other applications, the feasible choice correspondence serves to define ex post individual rationality - as is the case in the contract auction model developed below. In Page (1992) the feasible set of mechanisms is simply taken to be the set of all measurable mappings from buyer type profiles into the set of probability measures over a compact metric space of choices. Together, Theorems 1 and 2 provide a unified approach to existence problems involving *Bayesian* incentive compatibility and extend earlier results by the author on existence in Stackelberg games with incomplete information (e.g., Page (1989, 1992)). This is an additional contribution of the paper.

Multiple and Single Unit Auctions

For our first application we specialize our revelation game to a single unit auction with risk neutral seller and buyers (see Section 3.1). The resulting model covers the classical auction models of Myerson (1981), Harris and Raviv (1981), Riley and Samuelson (1981), and Cremer and McLean (1988). Moreover, because we allow buyer types to be multidimensional and stochastically dependent our model also covers the auction environments analyzed in, for example, Milgrom and Weber (1982) where buyer types are allowed to be affiliated (also see, Cremer and McLean (1985), McAfee and Reny (1992), Funk (1993), Jehiel, Moldovanu, and Stacchetti (1994), Baye, Kovenock, and de Vries (1996)).

Single unit auctions in which the seller and buyers are risk neutral and buyers' types are stochastically independent have been intensely studied in the literature (see for example Myerson (1981), Riley and Samuelson (1981) and Harris and Raviv (1981)). Two main conclusions emerge from this work. First, the four most common forms of

auctions (Dutch, first-price, second-price, and English)⁵ generate the same expected revenue for the seller. Second, for many common distributions of buyers' types (including the normal, exponential, and uniform distributions) the four standard auction forms with suitably chosen reserve prices or entry fees are optimal from the perspective of the seller. These conclusions, however, are not robust with respect to changes in the assumptions of risk neutrality *or* stochastic independence. For example, Maskin and Riley (1984) show that if buyers' types are independently distributed but buyers are risk averse, then from the seller's viewpoint first-price and English auctions are not revenue equivalent - nor are they optimal. Alternatively, it follows from Milgrom and Weber (1982) that if buyers and the seller are risk neutral but types are dependent - and in particular affiliated - then English auctions generate the highest expected revenue to the seller, followed by the second-price auction, and finally the Dutch and first-price auctions. Even more striking are the results of Cremer and McLean (1985, 1988) and McAfee and Reny (1992). Assuming risk neutrality they show that if buyers' types are dependent then by using an optimal auction the seller can obtain the same expected payoff as that obtainable under conditions of complete information concerning buyers' types.

These studies indicate the importance of analyzing auction environments in which *both* risk aversion and stochastically dependent buyers' types are present. Thus for our second application, we specialize our revelation game to a multiple unit auction with risk averse players *and* stochastically dependent types (see section 3.2).⁶ The resulting model automatically covers the multiple unit auction model of Branco (1996) with risk neutral players and stochastically independent types. Moreover, by further specializing to a single unit auction, our model covers the auction models of Matthews (1979, 1983) and Maskin-Riley (1984) with risk-averse buyers and stochastically independent types.

Contract Auctions

For our final application, we consider the problem faced by a risk averse principal seeking to choose, via a contract auction, an agent or contractor from among several competing, risk averse agents to carry out a particular project (see Section 3.3). Laffont and

⁵The Dutch auction is conducted by an auctioneer who initially calls for a high price and then continuously lowers the price until some buyer stops the auction and claims the object at that price. In an English auction, the auctioneer begins by soliciting bids at a low price and then gradually raises the price until only one willing buyer remains. A first-price auction is a sealed-bid auction in which the buyer making the highest bid wins and pays the amount of his bid for the object. A second-price auction is also a sealed-bid auction in which the buyer making the highest bid wins and pays the amount of the second highest bid. All of these standard auction forms are representable via functions - and thus all are representable via Young measures.

⁶With further minor modifications, our model becomes a model of a simultaneous pooled auction (see Menezes and Monteiro (1996)).

Tirole (1987, 1993) characterize optimal Bayesian contract auctions assuming risk neutrality and independent agent types, while Page (1994) and Balder (1996) demonstrate the existence of an optimal, *dominant-strategy incentive compatible* contract auction mechanism assuming risk aversion and multidimensional, dependent agent types. Here, we establish the existence of an optimal, interim individually rational and Bayesian incentive compatible contract auction mechanism - thus, extending the existence results of Page (1994) and Balder (1996) to the more difficult Bayesian case.⁷

A distinguishing feature of contract auctions is the presence of *both* adverse selection and moral hazard. Thus in addition to being concerned about the adverse selection problem (as reflected in the incentive compatibility constraints) the principal must also be concerned about the actions taken by the agent after entering into the contract with the principal. These actions often affect contract performance and the economic well-being of the principal as well as the winning agent. Because these actions are only indirectly controllable via contract incentives, the principal must choose an auction mechanism that not only selects the best agent but also selects a contract that provides the winning agent with correct incentives.

In specializing our revelation game to a contract auction model, we show that the auction problem with adverse selection and moral hazard reduces to an equivalent problem with adverse selection only. While we continue to allow risk aversion and multidimensional, dependent types, in the contract auction model developed here (like the models in Laffont and Tirole (1987, 1993), Page (1994), and Balder (1996)), we do not allow informational externalities.⁸ This assumption is made for simplicity. In particular, in order to avoid the "informed principal problem" which would arise for the winning agent at the action selection stage if information externalities were allowed, we simply assume that each agent's utility depends only upon his own type. We save for future research the treatment of informational externalities and the informed principal problem in contract auctions.

*Outline **

We shall proceed as follows: In Section 2 we present a general model of a Stackelberg revelation and state our main existence result (Theorem 1). In Section 3 we consider applications. In particular, in Section 3.1, we specialize our revelation game to a

⁷The existence result given by Balder (1996) for dominant strategy incentive compatible auction mechanisms refines the earlier result due to Page (1994) by relaxing some of the topological and measure-theoretic assumptions made in Page (1994).

⁸Thus, here we assume that each agent's utility depends only upon his own type and not upon the types of other agents.

single unit auction model with risk neutral players and we deduce the existence of an optimal Bayesian auction mechanism (Corollary 1) from our main existence result. In Section 3.2, we specialize our revelation game to a multiple unit auction model with risk averse players and deduce the existence of an optimal Bayesian auction mechanism (Corollary 2). In Section 3.3, we specialize our revelation game to a contract auction model and show that the contract auction problem with adverse selection and moral hazard reduces to an equivalent auction problem with adverse selection only. We also establish the existence of an optimal Bayesian contract auction mechanism (Corollary 3). In Section 4 within the context of our general Stackelberg revelation game we show that the set of all feasible, interim individually rational and Bayesian incentive compatible revelation mechanisms is K-compact (Theorem 2). Finally in Section 4, we prove our main existence result (Theorem 1).

2. The Stackelberg Revelation Game

2.1 Basic Elements

We will assume that the basic elements of the game are known to all players.

Players and Types

To begin, let $I = \{0, 1, 2, \dots, h\}$ denote the set of players. Elements of I will be denoted by i or j , with 0 denoting the leader and $i = 1, 2, \dots, h$, denoting followers.

Now let,

- T_i = the set of i th follower types with elements denoted by t_i . Equip T_i with the σ -field Σ_i .
- T = $T_1 \times T_2 \times \dots \times T_h$ with elements denoted by $t = (t_1, t_2, \dots, t_h)$. Equip T with the product σ -field $\Sigma = \Sigma_1 \times \Sigma_2 \times \dots \times \Sigma_h$.
- T_{-i} = $T_1 \times \dots \times T_{i-1} \times T_{i+1} \times \dots \times T_h$ with elements denoted by $t_{-i} = (t_1, \dots, t_{i-1}, t_{i+1}, \dots, t_h)$. Equip T_{-i} with the product σ -field $\Sigma_{-i} = \Sigma_1 \times \dots \times \Sigma_{i-1} \times \Sigma_{i+1} \times \dots \times \Sigma_h$.
- $(t_i, t_{-i}) = (t_1, \dots, t_{i-1}, t_i, t_{i+1}, \dots, t_h) = t$.

Probability Beliefs

For each follower $i = 1, \dots, h$, let $q_i(\cdot | \cdot)$ denote the stochastic kernel representing the i th follower's conditional probability beliefs concerning other followers' types. Thus, for each $t_i \in T_i$, $q_i(\cdot | t_i)$ is a probability measure defined on Σ_{-i} specifying for each type t_{-i} , the i th follower's conditional probability beliefs concerning other followers' types, and for each $E \in \Sigma_{-i}$, $q_i(E | \cdot)$ is a real-valued, Σ_i -measurable function defined on T_i specifying for each of the i th follower's types, the probability weight the i th follower assigns to subset E .

For the leader, let p_0 be a probability measure defined on (T, Σ) , the measure space of type profiles, representing the leader's probability beliefs concerning followers' type profiles.

Assumption 1: We will assume that there is a product measure $\mu = \mu_1 \times \dots \times \mu_h$ defined on (T, Σ) , with each μ_i σ -finite on Σ_i , such that

- (1) the leader's probability measure p_0 over T is absolutely continuous with respect to μ (denoted $p_0 \ll \mu$),

- (2) for each i and $t_i \in T$, the i th follower's conditional probability measure $q_i(\cdot | t_i)$ over T_{-i} is absolutely continuous with respect to μ_{-i} (denoted $q_i(\cdot | t_i) \ll \mu_{-i}$), where μ_{-i} is the product measure $\mu_1 \times \cdots \times \mu_{i-1} \times \mu_{i+1} \times \cdots \times \mu_h$.

We will refer to μ as the dominating measure.

Assumption 1 is satisfied automatically in any revelation game in which the leader's probability beliefs concerning type profiles are specified via a probability density function and followers' conditional probability beliefs are specified via a conditional probability density function (i.e., Lebesgue measure then serves as the dominating measure).

Example: Suppose there are two followers, $i = 1, 2$, with types $T_i = [0, \infty)$ for each i . Thus, the set of type profiles is given by $T = [0, \infty) \times [0, \infty)$. Let Σ be the Borel product σ -field, $B[0, \infty) \times B[0, \infty)$, in T and equip (T, Σ) with the Lebesgue product measure $\mu = \mu_1 \times \mu_2$. Suppose now that the leader's probability beliefs p_0 are given via a joint density function $h_0(\cdot)$, defined on T , so that for $E \in \Sigma$,

$$p_0(E) = \int_E h_0(t) \mu(dt).$$

Thus, $p_0 \ll \mu$. Suppose also that each follower's conditional probability beliefs $q_i(\cdot | t_i)$ are given via a conditional density $r_i(\cdot | t_i)$ corresponding to a joint probability density $h_i(\cdot)$, so that for $S \in \Sigma_{-i}$,

$$q_i(S | t_i) = \int_S r_i(t_{-i} | t_i) \mu_{-i}(dt_{-i}).$$

Thus, $q_i(\cdot | t_i) \ll \mu_{-i}$ for all $t_i \in T_i$.

Choices and Revelation Mechanisms

Let K be a subset of a Hausdorff locally convex topological space E . Elements of K , denoted by f , will be called choices and measurable functions from type profiles T into K will be called revelation mechanisms. If the leader selects revelation mechanism $f(\cdot): T \rightarrow K$, then the leader is committed to choice $f(t) \in K$ whenever followers report type profile $t \in T$. Here measurability is defined with respect to the product σ -field Σ on T and the Borel σ -field $B(K)$ on K . In particular, $f(\cdot): T \rightarrow K$ is said to be measurable if for

each $E \in \mathcal{B}(K)$, $f^{-1}(E) := \{t \in T : f(t) \in E\} \in \Sigma$. We will denote by $M(T, K)$ the set of all revelation mechanisms.

Assumption 2: We shall assume that the set of choices K is convex, compact, and metrizable for the relative topology inherited from E .

For each h -tuple $t \in T$ of reported types, let $K(t)$ be the set of *feasible* choices in K given reported types t .

Assumption 3: We will assume that the *feasible choice correspondence* $K(\cdot) : T \rightarrow 2^K$ is such that for each $t \in T$, $K(t)$ is a closed convex subset of K .

A revelation mechanism $f(\cdot) \in M(T, K)$ is said to be *feasible* if

$$f(t) \in K(t) \text{ a.e.}[\mu].$$

Note that the qualifier "a.e." (almost everywhere) is with respect to the dominating measure μ (see Assumption 1). Thus, for each feasible mechanism $f(\cdot)$ the leader and the followers will agree upon the set of type profiles where the mechanism may fail to produce a feasible choice and all players will assign this set probability measure zero.

We shall denote by Γ the feasible set of revelation mechanisms. Thus,

$$\Gamma = \{f(\cdot) \in M(T, K) : f(t) \in K(t) \text{ a.e.}[\mu]\}. \quad (1)$$

In some applications, the mapping $K(\cdot)$ specializes to an ex post budget constraint for the leader and followers. This is the case in the multiple unit auction model developed in section 3.2. In other applications, $K(\cdot)$ serves to define ex post individual rationality. This is the case in the contract auction model developed in section 3.3.

Followers' Utility Functions

For $i = 1, 2, \dots, h$, let $v_i(\cdot, \cdot) : T \times K \rightarrow \mathbb{R}$ be the i th follower's utility function. Given type profile $t \in T$ and choice $f \in K$, $v_i(t, f)$ is the corresponding utility level for the i th follower.

Assumption 4: For each $i = 1, 2, \dots, h$, we will assume that

- (1) $v_i(t, \cdot)$ is affine and continuous on $K(t)$ for each $t \in T$,
- (2) $v_i(\cdot, f(\cdot))$ is Σ -measurable for each $f(\cdot) \in M(T, K)$, and
- (3) for each $t_i \in T_i$ there exists a $q_i(\cdot | t_i)$ -integrable function $\xi_{t_i}(\cdot) : T_{-i} \rightarrow \mathbb{R}$ such that $|v_i(t_i, t_{-i}, f(t_i, t_{-i}))| \leq \xi_{t_i}(t_{-i})$ a.e. $[q_i(\cdot | t_i)]$ on T_{-i} for $f(\cdot) \in M(T, K)$.

Bayesian Incentive Compatibility and Interim Individual Rationality

Let B denote the subset of mechanisms in $M(T, K)$ such that corresponding to each $f(\cdot) \in B$ there are h sets $C_1, \dots, C_i, \dots, C_h$, with $C_i \in \Sigma_i$ and $\mu_i(C_i) = 0$, such that for each follower $i = 1, 2, \dots, h$,

$$\int_{T_{-i}} v_i(t_i, t_{-i}, f(t_i, t_{-i})) q_i(dt_{-i} | t_i) \geq \int_{T_{-i}} v_i(t_i, t_{-i}, f(t'_i, t_{-i})) q_i(dt_{-i} | t_i) \quad (2)$$

for all $t_i \in T_i \setminus C_i$ and all $t'_i \in T_i$. Note that the measure μ_i is the i th component of the dominating product measure $\mu = \mu_1 \times \dots \times \mu_h$ (see Assumption 1). Under any revelation mechanism in B , given each follower's conditional probability beliefs concerning other followers' types, and given almost any follower type, each follower will do at least as well by reporting his true type t_i to the mechanism as by reporting t'_i given that all other followers report truthfully. We will refer to B as the set of *Bayesian incentive compatible* (BIC) mechanisms. Thus, for any mechanism in B truthful reporting is a Nash equilibrium of the revelation game induced by the mechanism.

Let Λ denote the subset of mechanisms in $M(T, K)$ such that corresponding to each $f(\cdot) \in \Lambda$ there are h sets $Q_1, \dots, Q_i, \dots, Q_h$, with $Q_i \in \Sigma_i$ and $\mu_i(Q_i) = 0$, such that for each follower $i = 1, 2, \dots, h$,

$$\int_{T_{-i}} v_i(t_i, t_{-i}, f(t_i, t_{-i})) q_i(dt_{-i} | t_i) \geq r_i(t_i) \quad (3)$$

for all $t_i \in T_i \setminus Q_i$. Here $r_i(\cdot) : T_i \rightarrow \mathbb{R}$ is the i th follower's interim reservation utility function. Under any revelation mechanism in Λ , given each follower's conditional probability beliefs concerning other followers' types, and given almost any follower type, each follower will do at least as well by participating in the mechanism as by abstaining from participation, given that all other followers participate and report truthfully. We will refer to Λ as the set of *interim individually rational* (IIR) mechanisms.

Assumption 5: We shall assume the following:

- (1) $\Gamma \cap B \cap \Lambda$ is nonempty.
- (2) The set of choices K contains an $\bar{f}$ such that for each follower $i = 1, 2, \dots, h$,

$$\int_{T_{-i}} v_i(t_i, t_{-i}, \bar{f}) q_i(dt_{-i} | t_i) \leq r_i(t_i) \text{ for all } t_i \in T_i.$$

We shall refer to any choice $\bar{f} \in K$ satisfying Assumption 5(2) as a *penalty* choice.

Assumption 5(1) is nontriviality assumption: without it the revelation game is uninteresting. Assumption 5(1) will be satisfied if, for example, the set K of choices contains an f'' such that for each $t \in T$, $f'' \in K(t)$ and for each follower $i = 1, 2, \dots, h$,

$$\int_{T_{-i}} v_i(t_i, t_{-i}, f'') q_i(dt_{-i} | t_i) \geq r_i(t_i) \text{ for all } t_i \in T_i.$$

The Leader's Utility

Now let $u(\cdot, \cdot) : T \times K \rightarrow \mathbb{R}$ be the leader's utility function. Given type profile $t \in T$ and choice $f \in K$, $u(t, f)$ is the corresponding level of utility for the leader.

Assumption 6: We will assume that

- (1) $u(t, \cdot)$ is concave and upper-semicontinuous on $K(t)$ for each $t \in T$,
- (2) $u(\cdot, f(\cdot))$ is Σ -measurable for each $f(\cdot) \in \Gamma \cap B \cap \Lambda$, and
- (3) there exists a p_0 -integrable function $\zeta(\cdot)$ such that $u(t, f(t)) \leq \zeta(t)$ a.e. $[p_0]$ on T for $f(\cdot) \in \Gamma \cap B \cap \Lambda$.

2.2 The Leader's Problem and The Existence Result

The leader's problem is to choose a revelation mechanism $f(\cdot)$ from the set of feasible, interim, individually rational and Bayesian incentive compatible mechanisms $\Gamma \cap B \cap \Lambda$ that maximizes the leader's expected utility

$$\int_T u(t, f(t)) p_0(dt).$$

Stated compactly, the leader's problem is given by

$$\max_{f(\cdot) \in \Gamma \cap B \cap \Lambda} \int_T u(t, f(t)) p_0(dt). \quad (4)$$

Theorem 1 (*Existence of an Optimal Revelation Mechanism*):

Under Assumptions 1-6 there exists an optimal revelation mechanism solving the leader's problem (4).

3. Auctions

By carefully specifying the choice set K , the feasible choice correspondence, $K(\cdot): T \rightarrow 2^K$, and the leader's and followers' utility functions many auction problems can be formally treated as special cases of the leader's problem (4). Thus in many auction environments the existence of an optimal auction mechanism for the seller (acting as the leader) can be deduced from Theorem 1. In order to illustrate this we begin by considering single unit auctions of an indivisible good.

3.1 Bayesian Single Unit Auctions with Risk Neutral Players

For our first application we specialize our revelation game to a single unit auction with risk neutral seller and buyers. In particular, we consider a seller (leader) who faces h risk neutral potential buyers (followers) and is interested in adopting an auction procedure (a revelation mechanism) that maximizes the expected revenue from the sale of a single object. The resulting model covers the classical auction models of Myerson (1981), Harris and Raviv (1981), Riley and Samuelson (1981), and Cremer and McLean (1988). Moreover, because we allow buyer types to be multidimensional and stochastically dependent our model also covers the auction environments analyzed in, for example, Milgrom and Weber (1982) where buyer types are allowed to be affiliated (also see, Cremer and McLean (1985), McAfee and Reny (1992), Funk (1993), Jehiel, Moldovanu, and Stacchetti (1994), and Baye, Kovenock, and de Vries (1996)).

We begin by assuming that each players' probability beliefs concerning type profiles satisfy Assumption 1. A standard assumption in the auction literature is that buyers' types are identically and independently distributed (e.g., see Myerson (1981) and Riley and Samuelson (1981)). Here we depart from this assumption, allowing for stochastically dependent types.

Following Myerson (1981) (see also Page (1991b)), a single unit auction mechanism is a pair of Σ -measurable functions, $p(\cdot): T \rightarrow \Delta$, $x(\cdot): T \rightarrow C$, such that if $t = (t_1, \dots, t_h)$ is the h -tuple of reported buyer types then $p_i(t)$ is the probability that the i th

buyer wins the object and $x_i(t)$ is the amount of money the i th buyer must pay to the seller. Here C is a closed bounded convex subset of $\mathbb{R}^h$ containing the origin and

$$\Delta = \{(p_1, \dots, p_h) \in \mathbb{R}^h : p_i \geq 0 \text{ and } \sum_{i=1}^h p_i \leq 1\}. \quad (5)$$

Thus, the choice set K is given by $K = \Delta \times C$ ($E = \mathbb{R}^h \times \mathbb{R}^h$), and the set of revelation mechanisms $M(T, \Delta \times C)$ consists of functions of the form

$$f(\cdot) = (p(\cdot), x(\cdot)) = (p_1(\cdot), \dots, p_h(\cdot), x_1(\cdot), \dots, x_h(\cdot)). \quad (6)$$

We assume that the feasible choice correspondence $K(\cdot): T \rightarrow 2^{\Delta \times C}$ is given by $K(t) = \Delta \times C$ for all t . Thus, Assumptions 2 and 3 are satisfied.

As in Myerson (1981), each buyer is risk neutral with utility function $v_i(\cdot, \cdot): T \times (\Delta \times C) \rightarrow \mathbb{R}$ given by

$$v_i(t, (p, x)) = p_i \cdot v_i(t) - x_i. \quad (7)$$

We also assume that the h -tuple of buyer valuation functions $v(\cdot) = (v_1(\cdot), \dots, v_h(\cdot))$ is such that the range of valuations

$$V := \{(v_1(t), \dots, v_h(t)) \in \mathbb{R}^h : t \in T\} \quad (8)$$

is contained in C and that each buyer's valuation function is Σ -measurable. The seller - also risk neutral - has valuation function $v_0(\cdot): T \rightarrow \mathbb{R}$, bounded and Σ -measurable, and utility function $u(\cdot, \cdot): T \times (\Delta \times C) \rightarrow \mathbb{R}$ given by

$$u(t, (p, x)) = \left(1 - \sum_{i=1}^h p_i\right) \cdot v_0(t) + \left(\sum_{i=1}^h x_i\right). \quad (9)$$

Thus, Assumptions 4 and 6 are satisfied.

To complete our description of the single unit auction model, we assume that each buyer's interim reservation utility function $r_i(\cdot): T_i \rightarrow \mathbb{R}$ is identically zero.

A single unit auction mechanism $(p(\cdot), x(\cdot)) \in M(T, \Delta \times C)$ is Bayesian incentive compatible (i.e., is contained in the set B) if there are h sets $C_1, \dots, C_i, \dots, C_h$, with $C_i \in \Sigma_i$ and $\mu_i(C_i) = 0$, such that for each buyer $i = 1, 2, \dots, h$,

$$\begin{aligned}
& \int_{T_{-i}} [p_i(t_i, t_{-i}) \cdot v_i(t_i, t_{-i}) - x_i(t_i, t_{-i})] q_i(dt_{-i} | t_i) \\
& \geq \int_{T_{-i}} [p_i(t'_i, t_{-i}) \cdot v_i(t_i, t_{-i}) - x_i(t'_i, t_{-i})] q_i(dt_{-i} | t_i)
\end{aligned} \tag{10}$$

for all $t_i \in T_i \setminus C_i$ and all $t'_i \in T_i$.

A single unit auction mechanism $(p(\cdot), x(\cdot)) \in M(T, \Delta \times C)$ is interim individually rational (i.e., is contained in the set Λ) if there are h sets $Q_1, \dots, Q_i, \dots, Q_h$, with $Q_i \in \Sigma_i$ and $\mu_i(Q_i) = 0$, such that for each buyer $i = 1, 2, \dots, h$,

$$\int_{T_{-i}} [p_i(t_i, t_{-i}) \cdot v_i(t_i, t_{-i}) - x_i(t_i, t_{-i})] q_i(dt_{-i} | t_i) \geq 0, \tag{11}$$

for all $t_i \in T_i \setminus Q_i$, where $Q_i \in \Sigma_i$ and $\mu_i(Q_i) = 0$.

Note that the auction mechanism $(\bar{p}(\cdot), \bar{x}(\cdot))$ given by $(\bar{p}(t), \bar{x}(t)) = (0, 0)$ for all t is feasible, interim individually rational and Bayesian incentive compatible (i.e., is contained in $\Gamma \cap B \cap \Lambda$). Note also that under the mechanism $(\bar{p}(\cdot), \bar{x}(\cdot))$, we have for each buyer

$$\int_{T_{-i}} [\bar{p}_i(t_i, t_{-i}) \cdot v_i(t_i, t_{-i}) - \bar{x}_i(t_i, t_{-i})] q_i(dt_{-i} | t_i) = 0 \text{ for all } t_i \in T_i. \tag{12}$$

Thus, Assumption 5 is satisfied.

The seller's single unit auction problem can be stated compactly as

$$\max_{(p(\cdot), x(\cdot)) \in \Gamma \cap B \cap \Lambda} \int_T \left[\left(1 - \sum_{i=1}^h p_i(t) \right) \cdot v_0(t) + \left(\sum_{i=1}^h x_i(t) \right) \right] p_0(dt). \tag{13}$$

It is easy to see that the seller's single unit auction problem (13) is a version of the leader's problem (4). Moreover, because our single unit auction model satisfies Assumptions 1-6, it follows directly from Theorem 1 that the seller's problem has a solution.

Corollary 1 (*Existence of an Optimal Single Unit Auction Mechanism*)

Given the single unit auction model above, there exists an auction mechanism $(p^*(\cdot), x^*(\cdot)) \in \Gamma \cap B \cap \Lambda$ solving the seller's problem (13).

The result above extends to the Bayesian case the single unit existence result for *dominant strategy incentive compatible* auction mechanisms given in Page (1991b).

3.1 Bayesian Multiple Unit Auctions with Risk Averse Players

Now consider a risk averse seller who faces h risk averse buyers and is interested in adopting an auction procedure that maximizes his expected utility from the sale of *several* indivisible units of a homogeneous good.

Again we assume that players' probability beliefs concerning type profiles satisfy Assumption 1. As indicated by our example in Section 2 this assumption is easily satisfied.

Let m denote the total number of units the seller wishes to put up for auction, and let

$$Q = \{0, 1, 2, \dots, m\}.$$

Each player, $i = 0, 1, \dots, h$ (seller as well as buyers), in the auction can win from 0 to m units of the good. Now let

$$Z = \underbrace{Q \times \dots \times Q}_{h+1 \text{ times}},$$

with typical element $z = (z_0, z_1, \dots, z_h)$. Each $h+1$ -tuple $z \in Z$ gives a profile of winnings by the players in the auction. Note that if $z_0 > 0$ then the seller keeps some of the good (i.e., some units are not sold during the auction).

Now let $C = \{c \in \mathbb{R} : 0 \leq c \leq w\}$ for some large positive number w , and for each buyer i and buyer type $t_i \in T_i$ let $w_i(t_i) \in C$ and

$$C_i(t_i) = \{c \in \mathbb{R} : 0 \leq c \leq w_i(t_i)\}.$$

Here $w_i(t_i)$ represents the maximum amount a type t_i i th buyer can pay at auction. The mapping $C_i(\cdot) : T_i \rightarrow 2^C$ is the i th buyer's budget correspondence. Now let

$$D = \underbrace{C \times \dots \times C}_{h \text{ times}}.$$

The set D equipped with the usual Euclidean metric (here denoted by d_D) is a compact metric space. Finally, for each type profile t let

$$D(t) = C_1(t_1) \times \cdots \times C_h(t_h).$$

Each h -tuple $c = (c_1, \dots, c_h) \in D(t)$ represents a profile of payments from buyers to the seller given reported types t and $D(\cdot): T \rightarrow 2^D$ is the ex post budget correspondence for buyers.

A tuple $(z, c) = (z_0, z_1, \dots, z_h, c_1, \dots, c_h) \in Z \times D(t)$ is a feasible outcome of the auction if the sum of the units allocated via the auction $\sum_{i=0}^h z_i$ equals m and if the sum of the payments from buyers to seller $\sum_{i=1}^h c_i$ equals or exceeds some minimum revenue requirement $r_0(t)$ dependent on the types reported by buyers. Formally the feasible set of auction outcomes corresponding to each reported type profile t is given by

$$F(t) = \{(z, c) = (z_0, z_1, \dots, z_h, c_1, \dots, c_h) \in Z \times D(t) : \sum_{i=0}^h z_i = m \text{ and } \sum_{i=1}^h c_i \geq r_0(t)\}. \quad (14)$$

Note that for each t , $F(t)$ is a closed subset of the compact metric space $Z \times D$.⁹

We are now in a position to define the choice set K and the feasible choice correspondence, $K(\cdot): T \rightarrow 2^K$ corresponding to the multiple unit auction. Let $P(Z \times D)$ denote the set of all probability measures defined on the product σ -field $\mathfrak{I}(Z) \times B(D)$ of the compact metric space $Z \times D$ of auction outcomes. Here, $B(D)$ denotes the Borel σ -field generated by the Euclidean metric d_D and $\mathfrak{I}(Z)$ denotes the σ -field consisting of all subsets of the finite set Z . Also let the set of choices K be given by $P(Z \times D)$. Here the space E is taken to be the set of all bounded signed measures on $Z \times D$ equipped with the narrow topology (i.e., the topology of weak convergence measures - see Dellacherie and Meyer (1978), III.54). The choice set $K = P(Z \times D)$ is clearly convex. By III.60 in Dellacherie and Meyer (1978), $K = P(Z \times D)$ satisfies Assumption 2. Denote by ϕ a typical element of the choice set $P(Z \times D)$ and denote by $\phi(\cdot)$ or $\phi(d(z, c) | \cdot)$ a typical element of the set of auction (revelation) mechanisms $M(T, P(Z \times D))$.

⁹The results of this section remain unchanged if in the definition of the set of feasible outcomes we require $\sum_{i=1}^h c_i = r_0(t)$ (i.e., we require the sum of buyer payments to the seller to equal $r_0(t)$). With this modification, the feasible set of auction outcomes $F(t)$ remains a closed subset of the compact metric space $Z \times D$.

Let the feasible choice correspondence, $K(\cdot) : T \rightarrow 2^K$ be given by

$$K(t) := \{\varphi \in P(Z \times D) : \varphi(F(t)) = 1\}. \quad (15)$$

Given type profile t , any probability measure $\varphi \in K(t)$ assigns probability 1 to the set of auction outcomes $F(t)$. For each t , $K(t)$ is convex. Thus since each $F(t)$ is compact it follows from III.58 and III.60 in Dellacherie and Meyer (1978) that the feasible choice correspondence $K(\cdot) : T \rightarrow 2^K$ defined via (6) satisfies Assumption 3. The feasible set of auction mechanisms is thus given by

$$\Gamma = \{\varphi(\cdot) \in M(T, P(Z \times D)) : \varphi(t) \in K(t) \text{ a.e. } [\mu]\}. \quad (16)$$

For each potential buyer $i = 1, \dots, h$ let $v_i(\cdot, \cdot, \cdot) : T \times Z \times D \rightarrow \mathbb{R}$ be the i th buyer's utility function over type profiles and auction outcomes. We will assume that

- (a) $v_i(t, \cdot, \cdot)$ is continuous on $Z \times D$ for each $t \in T$,
- (b) $v_i(\cdot, (z, c))$ is Σ -measurable on T for each feasible auction outcome $(z, c) \in Z \times D$, and
- (c) for each $t_i \in T_i$ there exists a $q_i(\cdot | t_i)$ -integrable function $\psi_{t_i}(\cdot) : T_{-i} \rightarrow \mathbb{R}$ such that $|v_i(t_i, t_{-i}, (z, c))| \leq \psi_{t_i}(t_{-i})$ for all $(t_{-i}, (z, c)) \in T_{-i} \times Z \times D$.

Given choice $\varphi \in P(Z \times D)$, the i th buyer's (expected) utility is given by

$$v_i(t, \varphi) = \int_{Z \times D} v_i(t, (z, c)) \varphi(d(z, c)). \quad (17)$$

For each $t \in T$, $\varphi \rightarrow v_i(t, \varphi)$ is continuous on $P(Z \times D)$ with respect to the narrow topology (Dellacherie and Meyer (1978), III.54), and for each $\varphi \in P(Z \times D)$, $t \rightarrow v_i(t, \varphi)$ is Σ -measurable on T . Thus, each buyer's expected utility function $v_i(\cdot, \cdot) : T \times P(Z \times D) \rightarrow \mathbb{R}$ is $\Sigma \times B(P(Z \times D))$ -measurable (Castaing and Valadier (1977), III.14). Here, $B(P(Z \times D))$ denotes the Borel σ -field in $P(Z \times D)$ generated by the narrow topology. Given these observations and the fact that each buyer's utility function, $v_i(\cdot, \cdot, \cdot)$, satisfies assumptions (a)-(c) above, it is easy to verify that each buyer's expected utility function $v_i(\cdot, \cdot) : T \times P(Z \times D) \rightarrow \mathbb{R}$ given in (17) satisfies Assumption 4.

Under auction mechanism $\varphi(\cdot) \in M(T, P(Z \times D))$ the i th buyer's interim expected utility given true type $t_i \in T_i$ and reported type $t'_i \in T_i$ is

$$\int_{T_{-i}} v_i(t_i, t_{-i}, \varphi(t'_i, t_{-i})) q_i(dt_{-i} | t_i). \quad (18)$$

A multiple unit auction mechanism $\varphi(\cdot) \in M(T, P(Z \times D))$ is Bayesian incentive compatible (i.e., is contained in the set B) if there are h sets $C_1, \dots, C_i, \dots, C_h$, with $C_i \in \Sigma_i$ and $\mu_i(C_i) = 0$, such that for each buyer $i = 1, 2, \dots, h$,

$$\begin{aligned} \int_{T_{-i}} \int_{Z \times D} v_i(t_i, t_{-i}, (z, c)) \varphi(d(z, c) | t_i, t_{-i}) q_i(dt_{-i} | t_i) \\ \geq \int_{T_{-i}} \int_{Z \times D} v_i(t_i, t_{-i}, (z, c)) \varphi(d(z, c) | t'_i, t_{-i}) q_i(dt_{-i} | t_i), \end{aligned} \quad (19)$$

for all $t_i \in T_i \setminus C_i$ and all $t'_i \in T_i$.

A multiple unit auction mechanism $\varphi(\cdot) \in M(T, P(Z \times D))$ is interim individually rational (i.e., is contained in the set Λ) if there are h sets $Q_1, \dots, Q_i, \dots, Q_h$, with $Q_i \in \Sigma_i$ and $\mu_i(Q_i) = 0$, such that for each buyer $i = 1, 2, \dots, h$,

$$\int_{T_{-i}} \int_{Z \times D} v_i(t_i, t_{-i}, (z, c)) \varphi(d(z, c) | t_i, t_{-i}) q_i(dt_{-i} | t_i) \geq r_i(t_i), \quad (20)$$

for all $t_i \in T_i \setminus Q_i$.

We shall assume that for auction outcome $(m, 0, \dots, 0, \bar{c}_1, \dots, \bar{c}_h)$ in which the seller keeps all units of the good and the i th buyer pays a positive amount $\bar{c}_i$, $(\bar{c}_1, \dots, \bar{c}_h) \in D(t)$ for all t and

$$\int_{T_{-i}} v_i(t_i, t_{-i}, (m, 0, \dots, 0, \bar{c}_1, \dots, \bar{c}_h)) q_i(dt_{-i} | t_i) \leq r_i(t_i) \text{ for all } t_i \in T_i.$$

Thus, $F(t)$ is nonempty for all t and Assumption 5(2) is satisfied. We shall also assume that there is a probability measure $\varphi'' \in P(Z \times D)$ such that $\varphi'' \in K(t)$ a.e. $[\mu]$ and for each buyer $i = 1, 2, \dots, h$,

$$\int_{T_{-i}} \int_{Z \times D} v_i(t_i, t_{-i}, (z, c)) \varphi''(d(z, c)) q_i(dt_{-i} | t_i) \geq r_i(t_i) \text{ a.e. } [\mu_i].$$

Thus, Assumption 5 (1) holds.

The buyer's underlying utility functions $v_i(\cdot, \cdot, \cdot): T \times Z \times D \rightarrow \mathbb{R}$ can take several different forms. For example, we might suppose that

$$v_i(t, (z, c)) = \rho_i(t, z_i) - c_i,$$

where $\rho_i(t, z_i)$ is the value to the i th buyer of z_i units of the good given information t and c_i is the total amount the i th buyer must pay to the seller to obtain z_i units. Specializing further, we might assume that

$$v_i(t, (z, c)) = \zeta_i(t) \cdot z_i - c_i,$$

where $\zeta_i(t)$ is the *per unit* value of the good to the i th buyer given information t .

To complete our description of the multiple unit auction let $\pi(\cdot, \cdot, \cdot): T \times Z \times D \rightarrow \mathbb{R}$ be the seller's utility function over type profiles and auction outcomes. We will assume that

- (a) $\pi(t, \cdot, \cdot)$ is upper semicontinuous on $Z \times D$ for each $t \in T$,
- (b) $\pi(\cdot, \cdot, \cdot)$ is $\Sigma \times B(Z) \times B(D)$ -measurable, and
- (c) there exists a p_0 -integrable function $\psi_0(\cdot): T \rightarrow \mathbb{R}$ such that $\pi(t, (z, c)) \leq \psi_0(t)$ for all $(t, (z, c)) \in T \times Z \times D$.

Given choice $\varphi \in P(Z \times D)$, the seller's (expected) utility is given by

$$u(t, \varphi) = \int_{Z \times D} \pi(t, (z, c)) \varphi(d(z, c)). \quad (21)$$

For each $t \in T$, $\varphi \rightarrow u(t, \varphi)$ is upper semicontinuous on $P(Z \times D)$ with respect to the narrow topology (Nowak (1984), Lemma 1.5). Moreover, $(t, \varphi) \rightarrow u(t, \varphi)$ is $\Sigma \times B(P(Z \times D))$ -measurable (Nowak (1984), Lemma 1.6). Given these observations and the fact that the seller's utility function, $\pi(\cdot, \cdot, \cdot)$, satisfies assumptions (a)-(c) above, it is easy to verify that the seller's expected utility function $u(\cdot, \cdot): T \times P(Z \times D) \rightarrow \mathbb{R}$ given in (21) satisfies Assumption 6.

Under auction mechanism $\varphi(\cdot) \in \Gamma \cap B \cap \Lambda$ the seller's expected utility is

$$\int_T u(t, \varphi(t)) p_0(dt). \quad (22)$$

The seller's underlying utility function $\pi(\cdot, \cdot): T \times Z \times D \rightarrow \mathbb{R}$ can also take several different forms. For example, we might suppose that

$$\pi(t, (z, c)) = p_0(t, z_0) + \sum_{i=1}^h (c_i - h_i(t, z_i)),$$

where $p_0(t, z_0)$ is the value to seller of z_0 units of the good (not sold at auction - but kept by the seller) given information t , $(c_i - h_i(t, z_i))$ is the profit earned by the seller from the sale of z_i units of the good to the i th buyer at a price of c_i given information t , and $\sum_{i=1}^h (c_i - h_i(t, z_i))$ is the total profit earned by the seller from the sale of $\sum_{i=1}^h z_i$ units to the buyers. Here the function $h_i(t, z_i)$ can be interpreted as the cost to the seller of supplying z_i units of the good to the i th buyer given information t .

It is easy to see that the seller's multiple unit auction problem is a version of the leader's problem (4). Moreover, because our multiple unit auction model satisfies Assumptions 1-6, it follows directly from Theorem 1 that the seller's problem has a solution.

Corollary 2 (*Existence of an Optimal Multiple Unit Auction Mechanism*)

Given the multiple unit auction model above, there exists an auction mechanism $\phi^*(\cdot) \in \Gamma \cap B \cap \Lambda$ solving the seller's problem

$$\max_{\phi(\cdot) \in \Gamma \cap B \cap \Lambda} \int_T \int_{Z \times D} \pi(t, (z, c)) \phi(d(z, c) | t) p_0(dt). \quad (23)$$

Single Unit Auctions with Risk Averse Players: A Special Case

The multiple unit auction model above can easily be specialized to the single unit case. Keeping all elements in the model above the same and letting $m = 1$ (recall m is the total number of units the seller puts up for auction) we obtain a single unit auction model with risk averse players satisfying Assumptions 1-6. Existence for the single unit case with risk aversion then follows immediately from Corollary 2. Note that with $m = 1$ the set Q becomes $Q = \{0, 1\}$ and a typical feasible auction outcome has the form

$$(z, c) = (0, \dots, 0, 1, 0, \dots, 0, c_1, c_2, \dots, c_h),$$

where the 1 in the i th component indicates that player i (for $i = 0, 1, \dots, h$) has won the auction and therefore receives the single unit being auctioned off.

3.3 Bayesian Contract Auctions

Now we consider the problem faced by a principal seeking to choose, via a contract auction, an agent or contractor from among h competing agents to carry out a particular project.

We begin by assuming that each player's probability beliefs concerning type profiles satisfies Assumption 1.

Working backward we first analyze the winning agent's action choice problem (i.e., the moral hazard stage). Let A be a compact metric space of actions available to agents and let Φ be a compact metric space of contracts available to the principal. For each agent $i = 1, 2, \dots, h$, let $v_i(\cdot, \cdot, \cdot): T_i \times \Phi \times A \rightarrow \mathbb{R}$ be the i th agent's utility defined over types T_i , contracts Φ , and actions A . Note that here the i th agent's utility depends only on his own type and not on the types of others. Finally for $i = 1, 2, \dots, h$ let $A_i(\cdot): T_i \rightarrow 2^A$ be a set-valued mapping specifying for agent i the set of actions available to the agent depending on his type. We shall assume that

- (a) $v_i(t_i, \cdot, \cdot)$ is continuous on $\Phi \times A$ for each $t_i \in T_i$,
- (b) $v_i(\cdot, s, a)$ is Σ_i -measurable on T_i for each contract/action pair $(s, a) \in \Phi \times A$, and
- (c) $A_i(\cdot)$ is Σ_i -measurable.

Suppose now that the i th agent wins the auction and is awarded contract $s \in \Phi$. If the i th agent is type $t_i \in T_i$, he will choose an action from the feasible set $A_i(t_i)$ so as to solve the problem

$$\max_{a \in A_i(t_i)} v_i(t_i, s, a). \quad (24)$$

Let

$$v_i^*(t_i, s) := \max_{a \in A_i(t_i)} v_i(t_i, s, a). \quad (25)$$

The quantity $v_i^*(t_i, s)$ is a type t_i i th agent's optimal expected utility under contract $s \in \Phi$. The function $v_i^*(\cdot, \cdot)$ is $\Sigma_i \times B(\Phi)$ -measurable on $T_i \times \Phi$ (see Schall (1974)), and $v_i^*(t_i, \cdot)$ is continuous on Φ for $t_i \in T_i$ (see Berge (1963)). Here $B(\Phi)$ is the Borel σ -field in the compact metric space of contracts Φ .

The i th agent's reaction correspondence given by

$$A_i^*(t_i, s) := \{a \in A_i(t_i) : v_i(t_i, s, a) \geq v_i^*(t_i, s)\}. \quad (26)$$

By Lemma 1.10 of Nowak (1984), $A_i^*(\cdot, \cdot)$ is $\Sigma_i \times B(\Phi)$ -measurable on $T_i \times \Phi$ and by Theorem 1 of Robinson and Day (1974), $A_i^*(t_i, \cdot)$ is upper semicontinuous on Φ for $t_i \in T_i$.

Suppose now that type profile t is reported by agents and that agent i wins the auction, is type t_i , and is awarded contract s . The principal's next problem then is to request that the winning agent take an action from the set $A_i^*(t_i, s)$ that is in the principal's best interest. Thus, the principal's problem is to solve

$$\max_{a \in A_i^*(t_i, s)} \pi(t, s, a). \quad (27)$$

Note that any action request by the principal from the set $A_i^*(t_i, s)$ is incentive compatible provided the winning agent is type t_i and contract s is awarded.

We shall assume that the principal's utility function $\pi(\cdot, \cdot, \cdot) : T \times \Phi \times A \rightarrow \mathbb{R}$ is such that

- (a) $\pi(t, \cdot, \cdot)$ is upper semicontinuous on $\Phi \times A$ for each $t \in T$,
- (b) $\pi(\cdot, \cdot, \cdot)$ is $\Sigma \times B(\Phi) \times B(A)$ -measurable, and
- (c) there exists a p_0 -integrable function $\psi_0(\cdot) : T \rightarrow \mathbb{R}$ such that $\pi(t, s, a) \leq \psi_0(t)$ for all $(t, s, a) \in T \times \Phi \times A$.

Now let

$$\pi^*(t, (i, s)) = \max_{a \in A_i^*(t_i, s)} \pi(t, s, a). \quad (28)$$

Thus $\pi^*(t, (i, s))$ is the principal's optimal utility if agent i wins the auction, is awarded contract s , and takes the action in $A_i^*(t_i, s)$ requested by the principal.

In order to take into account the possibility that no contracting occurs (i.e., that no agent wins the auction), we must extend the definition of the principal's utility function. To begin let

$$g_0^*(t, (j, s)) = \pi^*(t, (j, s))(1 - \eta_0(j)) + g_0(t)\eta_0(j), \quad (29)$$

where

$$\eta_0(j) = \begin{cases} 1 & \text{if } j = 0 \\ 0 & \text{if } j \neq 0. \end{cases}$$

Given truthfully reported types t , if agent j wins the auction ($j \neq 0$) and is awarded contract s , the principal's utility is given by $g_0^*(t, (j, s)) = \pi^*(t, (j, s))$, and if no contracting occurs ($j = 0$), the principal's utility is given by $g_0^*(t, (j, s)) = g_0(t)$. We shall assume that $g_0(\cdot): T \rightarrow \mathbb{R}$ is Σ -measurable and bounded from above by some p_0 -integrable function. Thus, the principal's utility function $g_0^*(\cdot, (\cdot, \cdot)): T \times I \times \Phi \rightarrow \mathbb{R}$ is such that

- (a) $g_0^*(t, \cdot, \cdot)$ is upper semicontinuous on $I \times \Phi$ for each $t \in T$,
- (b) $g_0^*(\cdot, \cdot, \cdot)$ is $\Sigma \times B(I) \times B(\Phi)$ -measurable, and
- (c) $g_0^*(\cdot, \cdot, \cdot)$ is bounded from above on $T \times I \times \Phi$ by some p_0 -integrable function.

In order to take into account the possibility that the i th agent loses the auction, we must also extend the definition of agents' utility functions. For agent $i = 1, \dots, h$, let

$$g_i^*(t_i, (j, s)) = v_i^*(t_i, s)\eta_i(j) + g_i(t_i)(1 - \eta_i(j)), \quad (30)$$

where

$$\eta_i(j) = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{if } j \neq i. \end{cases}$$

Given that agent i is type t_i , if agent i wins the auction ($j = i$) and is awarded contract s , agent i 's utility is given by $g_i^*(t_i, (j, s)) = v_i^*(t_i, s)$, and if agent i loses the auction ($j \neq i$), agent i 's utility is given by $g_i^*(t_i, (j, s)) = g_i(t_i)$. Note that for $i = 1, \dots, h$, the utility function $g_i^*(\cdot, (\cdot, \cdot)): T_i \times I \times \Phi \rightarrow \mathbb{R}$ is such that

$$g_i^*(t_i, \cdot, \cdot) \text{ is continuous on } I \times \Phi \text{ for each } t_i \in T_i \\ \text{(recall that } I = \{0, 1, \dots, h\} \text{)}.$$

For each agent $i = 1, \dots, h$, we shall assume that

$$g_i(t_i) \leq \bar{r}_i(t_i) \text{ for all } t_i \in T_i,$$

where $\bar{r}_i(\cdot): T_i \rightarrow \mathbb{R}$ is the i th agent's interim reservation utility function. Thus if agent i loses the auction, his utility level is at most his interim utility level.

For each type profile $t \in T$, let

$$F(t) = \{(j,s) \in I \times \Phi : j = 1, \dots, h \text{ and } v_j^*(t_j, s) \geq e_j(t_j)\} \cup \{(0,s) : s \in \Phi\}. \quad (31)$$

Here, $e_j(\cdot) : T_j \rightarrow \mathbb{R}$ is the j th agent's ex post reservation utility function, and $v_j^*(t_j, s)$ is the j th agent's optimal level of utility under contract s given type t_j (see expression (25)).

If agent j' wins the auction and is awarded contract s' and if

$$(j', s') \in \{(j,s) \in I \times \Phi : j = 1, \dots, h \text{ and } v_j^*(t_j, s) \geq e_j(t_j)\}$$

then agent j' will be able to achieve his reservation utility level under the awarded contract s' via an optimal action choice from $A_i(t_i)$. Thus from the perspective of the winning agent j' auction outcome (j', s') is ex post rational - that is, under contract s' it is rational for agent j' to enter into the contract with the principal. Alternatively if the auction outcome is $(0, s'')$ for some $s'' \in \Phi$ then no agent is selected and hence no contracting occurs. The set $F(t)$ is the feasible set of auction outcomes given truthfully reported types t . Moreover, for each t , $F(t)$ is a closed subset of the compact metric space $I \times \Phi$ of player/contract pairs. This follows from the definition of $F(t)$ and the fact that for each $t \in T$ the function $(j,s) \rightarrow v_j^*(t_j, s)$ is continuous on $\{1, 2, \dots, h\} \times \Phi$ and the function $j \rightarrow r_j(t_j)$ is continuous on $\{1, 2, \dots, h\}$.

We are now in a position to define the choice set K and the feasible choice correspondence, $K(\cdot) : T \rightarrow 2^K$ corresponding to the contract auction. To begin, let $P(I \times \Phi)$ denote the set of all probability measures defined on the product σ -field $\mathfrak{I}(I) \times B(\Phi)$ of the compact metric space $I \times \Phi$ of auction outcomes (i.e., player/contract pairs). Here, $B(\Phi)$ denotes the Borel σ -field generated by the metric on the contract space Φ and $\mathfrak{I}(I)$ denotes the σ -field consisting of all subsets of the finite set I . Also let the set of choices K be given by $P(I \times \Phi)$. Here the space E is taken to be the set of all bounded signed measures on $I \times \Phi$ equipped with the narrow topology. The set of choices $K = P(I \times \Phi)$ is clearly convex. By III.60 in Dellacherie and Meyer (1978), $K = P(I \times \Phi)$ satisfies Assumption 2. Denote by φ a typical element of the choice set $P(I \times \Phi)$ and by $\varphi(\cdot)$ or $\varphi(d(j,s) | \cdot)$ a typical element of the set of auction (revelation) mechanisms $M(T, P(I \times \Phi))$.

Now let the feasible choice correspondence, $K(\cdot): T \rightarrow 2^K$ be given by

$$K(t) := \{\varphi \in P(I \times \Phi) : \varphi(F(t)) = 1\}. \quad (32)$$

Given type profile t , probability measure $\varphi \in K(t)$ assigns probability 1 to the set of player/contract pairs $F(t)$. For each t , $K(t)$ is convex. Since each $F(t)$ is compact it follows from III.58 and III.60 in Dellacherie and Meyer (1978) that the feasible choice correspondence $K(\cdot): T \rightarrow 2^K$ defined via (32) above satisfies Assumption 3. The feasible set of auction mechanisms (i.e., the set of ex post individually rational auction mechanisms) is then given by

$$\Gamma = \{\varphi(\cdot) \in M(T, P(I \times \Phi)) : \varphi(t) \in K(t) \text{ a.e.}[\mu]\}. \quad (33)$$

Given choice $\varphi \in P(I \times \Phi)$, the i th agent's (expected) utility is given by

$$v_i(t_i, \varphi) = \int_{I \times \Phi} g_i^*(t_i, (j, s)) \varphi(d(j, s)). \quad (34)$$

Repeating arguments similar to those given after expression (17) above (defining buyers' expected utilities in the multiple unit auction case), we conclude that each agent's expected utility function $v_i(\cdot, \cdot): T_i \times P(I \times \Phi) \rightarrow \mathbb{R}$ given in (34) satisfies Assumption 4.

Under auction mechanism $\varphi(\cdot) \in M(T, P(I \times \Phi))$ the i th agent's interim expected utility given true type $t_i \in T_i$ and reported type $t'_i \in T_i$ is

$$\int_{T_{-i}} v_i(t_i, \varphi(t'_i, t_{-i})) q_i(dt_{-i} | t_i). \quad (35)$$

A contract auction mechanism $\varphi(\cdot) \in M(T, P(I \times \Phi))$ is Bayesian incentive compatible (i.e., is contained in the set B) if there are h sets $C_1, \dots, C_i, \dots, C_h$, with $C_i \in \Sigma_i$ and $\mu_i(C_i) = 0$, such that for each buyer $i = 1, 2, \dots, h$,

$$\begin{aligned} \int_{T_{-i}} \int_{I \times \Phi} g_i^*(t_i, (j, s)) \varphi(d(j, s) | t_i, t_{-i}) q_i(dt_{-i} | t_i) \\ \geq \int_{T_{-i}} \int_{I \times \Phi} g_i^*(t_i, (j, s)) \varphi(d(j, s) | t'_i, t_{-i}) q_i(dt_{-i} | t_i), \end{aligned} \quad (36)$$

for all $t_i \in T_i \setminus C_i$ and all $t'_i \in T_i$.

A contract auction mechanism $\varphi(\cdot) \in M(T, P(I \times \Phi))$ is interim individually rational (i.e., is contained in the set Λ) if there are h sets $Q_1, \dots, Q_i, \dots, Q_h$, with $Q_i \in \Sigma_i$ and $\mu_i(Q_i) = 0$, such that for each buyer $i = 1, 2, \dots, h$,

$$\int_{T_{-i}} \int_{I \times \Phi} g_i^*(t_i, (j, s)) \varphi(d(j, s) | t_i, t_{-i}) q_i(dt_{-i} | t_i) \geq r_i(t_i), \quad (37)$$

for all $t_i \in T_i \setminus Q_i$. As before $r_i(\cdot) : T_i \rightarrow \mathbb{R}$ is the i th agent's interim reservation utility function.

Now consider the auction mechanism $\bar{\varphi}(\cdot)$ that selects the principal (and hence selects no contracting) with probability 1 for all reported types. Thus, under mechanism $\bar{\varphi}(\cdot)$, $\bar{\varphi}(\{(0, s) : s \in \Phi\} | t) = 1$ for all t . Given the definition of the i th agent's utility function (see (30)), we have

$$\int_{T_{-i}} \int_{I \times \Phi} g_i^*(t_i, (j, s)) \bar{\varphi}(d(j, s) | t_i, t_{-i}) q_i(dt_{-i} | t_i) = g_i(t_i) \text{ for all } t_i \in T_i.$$

By assumption $g_i(t_i) \leq r_i(t_i)$ for all $t_i \in T_i$. Thus, we have

$$\int_{T_{-i}} \int_{I \times \Phi} g_i^*(t_i, (j, s)) \bar{\varphi}(d(j, s) | t_i, t_{-i}) q_i(dt_{-i} | t_i) \leq r_i(t_i) \text{ for all } t_i \in T_i.$$

and thus, our contract auction model satisfies Assumption 5(2). We shall also assume that there is a probability measure $\varphi'' \in P(I \times \Phi)$ such that $\varphi'' \in K(t)$ a.e. $[\mu]$ and for each agent $i = 1, 2, \dots, h$,

$$\int_{T_{-i}} \int_{I \times \Phi} g_i^*(t_i, (j, s)) \varphi''(d(j, s)) q_i(dt_{-i} | t_i) \geq r_i(t_i) \text{ a.e. } [\mu_i].$$

Thus, Assumption 5 (1) holds.

To complete our description of the contract auction model, observe that given choice $\varphi \in P(I \times \Phi)$, the principal's (expected) utility is given by

$$u(t, \varphi) = \int_{I \times \Phi} g_0^*(t, (j, s)) \varphi(d(j, s)). \quad (38)$$

Repeating arguments similar to those given after expression (21) above (defining the seller's expected utility in the multiple unit auction case), we conclude that the principal's expected utility function $u(\cdot, \cdot) : T \times P(I \times \Phi) \rightarrow \mathbb{R}$ given in (38) satisfies Assumption 6.

Under contract auction mechanism $\varphi(\cdot) \in \Gamma \cap B \cap \Lambda$ the principal's expected utility is

$$\int_T u(t, \varphi(t)) p_0(dt). \quad (39)$$

As is true in the case of the multiple unit auction, it is easy to see that the principal's contract auction problem is a version of the leader's problem (4). Moreover, because our contract auction model satisfies Assumptions 1-6, it follows directly from Theorem 1 that the principal's problem has a solution.

Corollary 3 (*Existence of an Optimal Contract Auction Mechanism*)

Given the contract auction model above, there exists an auction mechanism $\varphi^*(\cdot) \in \Gamma \cap B \cap \Lambda$ solving the principal's problem

$$\max_{\varphi(\cdot) \in \Gamma \cap B \cap \Lambda} \int_T \int_{I \times \Phi} g_0^*(t, (j, s)) \varphi(d(j, s) | t) p_0(dt). \quad (40)$$

4. Proof of the Main Existence Result

Given the nature of the Bayesian incentive compatibility constraints, the proof of Theorem 1 is nonstandard. The proof given here rests on a new result showing that the feasible set of interim individually rational and Bayesian incentive compatible revelation mechanisms is K -compact (Theorem 2, below). As discussed in the introduction, this result refines and extends Theorem 4.1 in Page (1992). As in the proof of Theorem 4.1, the proof of Theorem 2 given here utilizes the notions of K -convergence and K -compactness, as well as the abstract Komlos Theorem due to Balder (1996, Theorem 4.1(i) - also see Theorem 2.1 in Balder (1990)). The proof of Theorem 2 also avoids a minor error in the proof of Theorem 4.1 in Page (1992).¹⁰ We begin with the definitions K -convergence and relative K -compactness and then proceed to a statement of Balder's (1996) Theorem 4.1(i) (with the notation and numbering of the assumptions modified to fit the notation and numbering of assumptions given here).

Let (T, Σ) be a type space equipped with a σ -finite measure μ .

¹⁰The basic steps in the proofs of Theorem 2 and Theorem 4.1 in Page (1992) are similar. However, because a minor error in the proof of Theorem 4.1 is avoided here in the proof of Theorem 2, the steps in the proof of Theorem 2 do not match precisely the steps in the proof of Theorem 4.1.

Definition 4.1 (*K-convergence*)

A sequence of mechanisms $\{f_n(\cdot)\}_n \subset M(T, K)$ is said to K -converge $[\mu]$ to a K -limit $\hat{f}(\cdot) \in M(T, K)$ if for each subsequence $\{f_{n_k}(\cdot)\}_k$ there is a μ -null set $N \in \Sigma$ (i.e., $\mu(N) = 0$) such that

$$\lim_{m \rightarrow \infty} \frac{1}{m} \sum_{k=1}^m f_{n_k}(t) = \hat{f}(t) \text{ for all } t \in T \setminus N.$$

Definition 4.2 (*Relative K-compactness*)

A subset $\Psi \subset M(T, K)$ is said to be relatively K -compact $[\mu]$ if every sequence in Ψ contains a subsequence K -converging $[\mu]$ to some mechanism $\hat{f}(\cdot) \in M(T, K)$.

A subset $\Psi \subset M(T, K)$ is said to be K -compact $[\mu]$ if every sequence in Ψ contains a subsequence K -converging $[\mu]$ to some mechanism $\hat{f}(\cdot) \in \Psi$.

Balder/Komlos Theorem (*K-compactness of Γ*)

Let $\{f_n(\cdot)\}_n$ be a sequence of revelation mechanisms in Γ . Under Assumptions 2 and 3 there exists a subsequence $\{f_{n_k}(\cdot)\}_k$ and a mechanism $\hat{f}(\cdot) \in \Gamma$ such that¹¹

$$\lim_{m \rightarrow \infty} \frac{1}{m} \sum_{k=1}^m f_{n_k}(t) = \hat{f}(t) \text{ a.e.}[\mu]. \quad (41)$$

In Theorem 2 we show that given type space (T, Σ, μ) with μ σ -finite, under Assumptions 1-4 and Assumption 5(2) the set of feasible, Bayesian incentive compatible, and interim individually rational mechanisms $\Gamma \cap B \cap \Lambda$ is K -compact $[\mu]$.

Theorem 2 (*K-compactness of $\Gamma \cap B \cap \Lambda$*)

Let $\{f_n(\cdot)\}_n$ be a sequence of revelation mechanisms in $\Gamma \cap B \cap \Lambda$. Under Assumptions 1-4 and Assumption 5(2) there exists a subsequence $\{f_{n_k}(\cdot)\}_k$ and a mechanism $f^*(\cdot) \in \Gamma \cap B \cap \Lambda$ such that

$$\lim_{m \rightarrow \infty} \frac{1}{m} \sum_{k=1}^m f_{n_k}(t) = f^*(t) \text{ a.e.}[\mu].$$

¹¹This pointwise convergence of averages continues to hold with the same $\hat{f}(\cdot)$ but with varying μ -null sets for every further subsequence of $\{f_{n_k}(\cdot)\}_k$.

Proof of Theorem 2

First note that $\Gamma \cap B \cap \Lambda$ is convex. The convexity of Γ follows from Assumptions 2 and 3. The convexity of $B \cap \Lambda$ follows from Assumption 4 and in particular from the affinity of $v_i(t, \cdot)$ on $K(t)$ for each i and t .

Let $\{f_n(\cdot)\}_n$ be a sequence of mechanisms in $\Gamma \cap B \cap \Lambda$. By the Balder/Komlos Theorem we can assume without loss of generality that for some μ -null set $N \in \Sigma$ (i.e., $\mu(N) = 0$),

$$\lim_{n \rightarrow \infty} f^n(t) = \hat{f}(t) \text{ for all } t \in T \setminus N,$$

where $\hat{f}(\cdot) \in \Gamma$ and where for all n

$$f^n(\cdot) = \frac{f_1(\cdot) + \dots + f_n(\cdot)}{n}. \quad (42)$$

We will show that there exists a mechanism $\bar{f}^*(\cdot)$ with $\bar{f}^*(t) = \hat{f}(t)$ a.e. $[\mu]$ such that $\bar{f}^*(\cdot) \in \Gamma \cap B \cap \Lambda$. To begin, let

$$N(t_i) = \{t_{-i} \in T_{-i} : (t_i, t_{-i}) \in N\}. \quad (43)$$

For each i , we have (see Ash (1972), section 2.6)

$$\mu(N) = \int_{T_i} \mu_{-i}(N(t_i)) \mu_i(dt_i) = 0, \quad (44)$$

so that for some $N_i \in \Sigma_i$ with $\mu_i(N_i) = 0$,

$$\mu_{-i}(N(t_i)) = 0 \text{ for all } t_i \in T_i \setminus N_i. \quad (45)$$

Now define

$$h(t) = \begin{cases} 1 & \text{if } t \in \cup_i N_i \times T_{-i}, \\ 0 & \text{if } t \in T \setminus (\cup_i N_i \times T_{-i}), \end{cases} \quad (46)$$

and consider the auction mechanism

$$\bar{f}^*(t) = \hat{f}(t) \cdot (1 - h(t)) + \bar{f} \cdot h(t), \quad (47)$$

where $\bar{f} \in K$ is the "penalty" choice given in Assumption 5(2).

Since $h(\cdot)$ is Σ -measurable, $f^*(\cdot) \in M(T, K)$. Moreover, since $f^*(t) = \hat{f}(t)$ for all $t \in T \setminus (\cup_i N_i \times T_{-i})$, where $\mu(\cup_{i=1}^h (N_i \times T_{-i})) = 0$, $f^*(t) = \hat{f}(t)$ a.e. $[\mu]$. Recall that the N_i are the μ_i -null sets given in (44) and (45). Thus $f^*(\cdot)$ is contained in Γ and $f^*(\cdot)$ is a K-limit (with respect to the dominating measure μ) of the sequence $\{f_n(\cdot)\}_n$.

From the definition of $f^*(\cdot)$ (see (46) and (47)), it follows that if $t \in T$ is such that for some follower i , $t = (t_i, t_{-i}) \in N_i \times T_{-i}$, then $h(t) = h(t_i, t_{-i}) = 1$ and $f^*(t) = \bar{f}$ (i.e., the mechanism chooses the penalty $\bar{f}$). Note also that because $t_i \in N_i$ implies that $\mu_{-i}(N(t_i)) > 0$ (see (44) and (45)), $f^n(t_i, t_{-i})$ may fail to converge to $\hat{f}(t_i, t_{-i})$. Thus, $\{f_n(\cdot)\}_n$ may fail to K-converge to $\hat{f}(\cdot)$ for $t \in \cup_i N_i \times T_{-i}$.

Now fix $t_i \in T_i$ and let

$$t_{-i} \in T_{-i} \setminus [(\cup_{j,j \neq i} N_j \times T_{-i,j}) \cup N(t_i)],$$

where

$$N_j \times T_{-i,j} = T_1 \times \dots \times T_{i-1} \times T_{i+1} \times \dots \times T_{j-1} \times N_j \times T_{j+1} \times \dots \times T_h.$$

From the perspective of follower i , $T_{-i} \setminus [(\cup_{j,j \neq i} N_j \times T_{-i,j}) \cup N(t_i)]$ is the set of other follower type profiles t_{-i} such that no other follower has type $t_j \in N_j$ ($j \neq i$) triggering the penalty choice and such that no type profile t_{-i} causes a failure of K-convergence (recall that K-convergence of the sequence $\{f_n(\cdot)\}_n$ may fail to hold at (t_i, t_{-i}) if $t_{-i} \in N(t_i)$, see (43)).

For $i = 1, \dots, h$ let $C_{in} \cup Q_{in} \in \Sigma_i$ be the μ_i -null set of i th follower types where for types $t_i \in C_{in} \cup Q_{in}$ interim individual rationality and/or Bayesian incentive compatibility may fail to hold for the i th follower under the mechanism $f_n(\cdot)$ (see (2) and (3)). Now let

$$F_{i\infty} = [\cup_{n=1}^{\infty} (C_{in} \cup Q_{in})] \cup N_i, \quad (48)$$

where again the sets N_i are the μ_i -null sets given in (44) and (45). Note that $\mu_i(F_{i\infty}) = 0$ and for $t_i \in T_i \setminus F_{i\infty}$, $\mu_{-i}(N(t_i)) = 0$. Observe also that if

$$t_i \in T_i \setminus F_{i\infty} \subset T_i \setminus N_i$$

and if

$$t_{-i} \in T_{-i} \setminus [(\cup_{j,j \neq i} N_j \times T_{-i,j}) \cup N(t_i)],$$

then

$$(t_i, t_{-i}) \in T \setminus (\cup_{i=1}^h (N_i \times T_{-i})).$$

We can now conclude the following:

$$\text{for } i = 1, \dots, h, \quad t_i \in T_i \setminus F_{i\infty}, \text{ and } t_{-i} \in T_{-i} \setminus [(\cup_{j,j \neq i} N_j \times T_{-i,j}) \cup N(t_i)],$$

$$(a) \quad f^*(t_i, t_{-i}) = \hat{f}(t_i, t_{-i}).$$

$$(b) \quad \lim_{n \rightarrow \infty} f^n(t_i, t_{-i}) = \hat{f}(t_i, t_{-i}),$$

$$(c) \quad q_i((\cup_{j,j \neq i} N_j \times T_{-i,j}) \cup N(t_i) | t_i) = 0.$$

Conclusion (c) is an immediate consequence of Assumption 1(2) and the fact that for $t_i \in T_i \setminus F_{i\infty} \subset T_i \setminus N_i$, $\mu_{-i}((\cup_{j,j \neq i} N_j \times T_{-i,j}) \cup N(t_i)) = 0$.

It now follows from observations (b) and (c), Assumption 4 concerning followers' utility functions, and the Dominated Convergence Theorem (see Ash (1972)) that for $i = 1, \dots, h$ and $t_i \in T_i \setminus F_{i\infty}$

$$\lim_{n \rightarrow \infty} \int_{T_{-i}} v_i(t_i, t_{-i}, f^n(t_i, t_{-i})) q_i(dt_{-i} | t_i) = \int_{T_{-i}} v_i(t_i, t_{-i}, \hat{f}(t_i, t_{-i})) q_i(dt_{-i} | t_i). \quad (49)$$

Given observations (a) and (c),

$$\int_{T_{-i}} v_i(t_i, t_{-i}, \hat{f}(t_i, t_{-i})) q_i(dt_{-i} | t_i) = \int_{T_{-i}} v_i(t_i, t_{-i}, f^*(t_i, t_{-i})) q_i(dt_{-i} | t_i). \quad (50)$$

Given the definition of the μ_i -null set $F_{i\infty}$ (see (48)), the convexity of $\Gamma \cap B \cap \Lambda$ and the fact that $\{f_n(\cdot)\} \subset \Gamma \cap B \cap \Lambda$, we have for all n , $i = 1, \dots, h$, and $t_i \in T_i \setminus F_{i\infty}$

$$\int_{T_{-i}} v_i(t_i, t_{-i}, f^n(t_i, t_{-i})) q_i(dt_{-i} | t_i) \geq r_i(t_i).$$

By (49) and (50) we have for all $i = 1, \dots, h$, and $t_i \in T_i \setminus F_{i\infty}$

$$\int_{T_{-i}} v_i(t_i, t_{-i}, f^*(t_i, t_{-i})) q_i(dt_{-i} | t_i) \geq r_i(t_i). \quad (51)$$

Thus, $f^*(\cdot) \in \Lambda$.

In order to show that $f^*(\cdot) \in B$, we will show that for all $i = 1, \dots, h$, and $t_i \in T_i \setminus F_{i\infty}$

$$\int_{T_{-i}} v_i(t_i, t_{-i}, f^*(t_i, t_{-i})) q_i(dt_{-i} | t_i) \geq \int_{T_{-i}} v_i(t_i, t_{-i}, f^*(t'_i, t_{-i})) q_i(dt_{-i} | t_i)$$

for all t'_i in T_i . (52)

There are two cases to consider.

Case 1: Follower i is type $t_i \in T_i \setminus F_{i\infty}$ and reports his type as $t'_i \in N_i$ implying that $\mu_{-i}(N_i(t'_i)) > 0$.

Under case 1, $f^*(t'_i, t_{-i}) = \bar{f}$ for all $t_{-i} \in T_{-i}$ (see (47)). Thus, on the right-hand-side of (52) we have

$$\int_{T_{-i}} v_i(t_i, t_{-i}, f^*(t'_i, t_{-i})) q_i(dt_{-i} | t_i) = \int_{T_{-i}} v_i(t_i, t_{-i}, \bar{f}) q_i(dt_{-i} | t_i) \leq r_i(t_i).$$

The latter inequality holds by Assumption 5(2). Since $t_i \in T_i \setminus F_{i\infty}$, it follows from (51) above that inequality (52) holds for case 1.

Case 2: Follower i is type $t_i \in T_i \setminus F_{i\infty}$ and reports his type as $t'_i \in T_i \setminus N_i$ implying that $\mu_{-i}(N_i(t'_i)) = 0$.

Under case 2, $f^*(t'_i, t_{-i}) = \hat{f}(t'_i, t_{-i})$ for all $t_{-i} \in T_{-i} \setminus [(\cup_{j \neq i} N_j \times T_{-i,j}) \cup N(t'_i)]$. More importantly since $\mu_{-i}(N_i(t'_i)) = 0$ we have $\mu_{-i}((\cup_{j \neq i} N_j \times T_{-i,j}) \cup N(t'_i)) = 0$. Thus by Assumption 1(2), $q_i((\cup_{j \neq i} N_j \times T_{-i,j}) \cup N(t'_i) | t_i) = 0$ for all $t_i \in T_i$ and in particular for $t_i \in T_i \setminus F_{i\infty}$. Finally given the definition of the set $N_i(t'_i)$ (again see (43)) we have for all $t_{-i} \in T_{-i} \setminus [(\cup_{j \neq i} N_j \times T_{-i,j}) \cup N(t'_i)]$, $\lim_{n \rightarrow \infty} f^n(t'_i, t_{-i}) = \hat{f}(t'_i, t_{-i})$.

Given these observations and Assumption 4 concerning followers' utility functions, it follows from the Dominated Convergence Theorem (see Ash (1972)) that

$$\lim_{n \rightarrow \infty} \int_{T_{-i}} v_i(t_i, t_{-i}, f^n(t'_i, t_{-i})) q_i(dt_{-i} | t_i) = \int_{T_{-i}} v_i(t_i, t_{-i}, \hat{f}(t'_i, t_{-i})) q_i(dt_{-i} | t_i). \quad (53)$$

Moreover,

$$\int_{T_{-i}} v_i(t_i, t_{-i}, \hat{f}(t'_i, t_{-i})) q_i(dt_{-i} | t_i) = \int_{T_{-i}} v_i(t_i, t_{-i}, f^*(t'_i, t_{-i})) q_i(dt_{-i} | t_i). \quad (54)$$

Since $\{f^n(\cdot)\} \subset \Gamma \cap B \cap \Lambda$ and $t_i \in T_i \setminus F_{i\infty}$, by the above we have for each n

$$\int_{T_{-i}} v_i(t_i, t_{-i}, f^n(t_i, t_{-i})) q_i(dt_{-i} | t_i) \geq \int_{T_{-i}} v_i(t_i, t_{-i}, f^n(t'_i, t_{-i})) q_i(dt_{-i} | t_i). \quad (55)$$

Taking limits on both sides of (55) it follows from (49)-(50), and (53)-(54) that

$$\int_{T_{-i}} v_i(t_i, t_{-i}, f^*(t_i, t_{-i})) q_i(dt_{-i} | t_i) \geq \int_{T_{-i}} v_i(t_i, t_{-i}, f^*(t'_i, t_{-i})) q_i(dt_{-i} | t_i).$$

Thus, inequality (52) holds for case 2.

Having established that inequality (52) holds cases 1 and 2, we conclude that $f^*(\cdot) \in B$
Q.E.D.

Remark

Because strategic misreporting by followers can cause a failure of K-convergence on a set of positive measure (i.e., $\mu_{-i}(N(t'_i)) > 0$), the K-limit $\hat{f}(\cdot)$ of a sequence $\{f_n(\cdot)\}_n$ of Bayesian incentive compatible mechanisms is not necessarily Bayesian incentive compatible. In the proof above, we use the dominating measure μ to identify mathematically inconvenient lies (or misreports) by followers and we use the penalty choice $\bar{f}$ to punish followers for such misreports. Following this line of reasoning we are able to construct a new mechanism $f^*(\cdot)$ contained in the μ -equivalence class determined by the K-limit $\hat{f}(\cdot)$ and show that this mechanism is Bayesian incentive compatible.

Proof of Theorem 1

Let

$$U^* = \sup_{f(\cdot) \in \Gamma \cap B \cap \Lambda} \int_T u(t, f(t)) p_0(dt),$$

and let $\{f_n(\cdot)\}_n$ be a sequence of revelation mechanisms in $\Gamma \cap B \cap \Lambda$ such that

$$\int_T u(t, f_n(t)) p_0(dt) \rightarrow U^*.$$

By Theorem 2 we can assume without loss of generality that the sequence $\{f_n(\cdot)\}_n$ K-converges to K-limit $f^*(\cdot) \in \Gamma \cap B \cap \Lambda$. By Theorem 4.1(ii) in Balder (1996),

$$\lim_n \int_T u(t, f_n(t)) p_0(dt) = U^* \leq \int_T u(t, f^*(t)) p_0(dt).$$

Since $f^*(\cdot) \in \Gamma \cap B \cap \Lambda$, we have

$$\int_T u(t, f^*(t)) p_0(dt) = \max_{f(\cdot) \in \Gamma \cap B \cap \Lambda} \int_T u(t, f(t)) p_0(dt).$$

Q.E.D.

References

- Ash, R. B. (1972). *Real Analysis and Probability*. Academic Press, New York.
- Baye, M. R., Kovenock, D. and de Vries, C. (1996). The All-Pay Auction with Complete Information. *Economic Theory*. 8: 291-305.
- Berge, C. (1963). *Topological Spaces*. Macmillan, New York.
- Balder, E. J. (1990). New Sequential Compactness Results for Spaces of Scalarly Integrable Functions. *Journal of Mathematical Analysis and Applications*. 151: 1-16.
- _____ (1996). On the Existence of Optimal Contract Mechanisms for Incomplete Information Principal-Agent Models. *Journal of Economic Theory*. 68: 133-148.
- Branco, F. (1996). Multiple Unit Auctions of an Indivisible Good. *Economic Theory*. 8: 77-101.
- Castaing, C. and Valadier M. (1977). *Convex Analysis and Measurable Multifunctions*. Lecture Notes in Mathematics. No. 580. Springer-Verlag, Berlin.
- Cremer, J. and McLean, R. P. (1985). Optimal Selling Strategies Under Uncertainty for a Discriminating Monopolist When Demands Are Interdependent. *Econometrica* 53: 345-361.
- _____ (1988). Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions. *Econometrica* 56: 1247-1257.
- Dellacherie, C. and Meyer, P. A. (1975). *Probability and Potentials*. North-Holland, Amsterdam.
- Funk, P. (1993). Auctions with Interdependent Valuations. Working Paper, University of Bonn.
- Harris, M. and Raviv, A. (1981). Allocation Mechanisms and the Design of Auctions. *Econometrica*. 49: 1477-1499.
- Harsanyi, J. C. (1967/68). Games with Incomplete Information Played by Bayesian Players. *Management Science*. 14: 159-189, 320-334, 486-502.
- Jehiel, P., Moldovanu, B. and Stacchetti, E. (1994). Multidimensional Mechanism Design for Auction with Externalities. Mimeo.
- Komlos, J. (1967). A Generalization of a Problem of Steinhaus. *Acta Mathematica Academiae Scientiarum Hungaricae*. 18: 217-229.
- Laffont, J.-J. and Tirole, J. (1987). Auctioning Incentive Contracts. *Journal of Political Economy*. 95: 921-937.
- _____ (1993). *A Theory of Incentives in Procurement and Regulation*. MIT Press, Cambridge.

- Lebrun, B. (1996). Existence of an Equilibrium in First Price Auctions. *Economic Theory*. 7: 421-443.
- Maskin, E. and Riley, J. (1984). Optimal Auctions with Risk Averse Buyers. *Econometrica* 52: 1473-1518.
- Matthews, S. (1979). Risk Aversion and the Efficiency of First and Second Price Auctions. Working Paper #586, Department of Economics, University of Illinois.
- _____ (1983). Selling to Risk Averse Buyers with Unobservable Tastes. *Journal of Economic Theory*. 30: 370-400.
- McAfee, R. P. and Reny, P. (1992). Correlated Information and Mechanism Design. *Econometrica* 60: 395-421.
- Menezes, F. M. and Monteiro, P. K. (1995). Existence of Equilibrium in a Discriminatory Price Auction. Forthcoming in *Mathematical Social Sciences*.
- _____ (1996). Simultaneous Pooled Auctions. Working Paper, IMPA.
- Milgrom, P. R. and Weber, R. J. (1982). A Theory of Auctions and Competitive Bidding. *Econometrica*. 50: 1089-1122.
- Myerson, R. B. (1979). Incentive Compatibility and the Bargaining Problem. *Econometrica*. 47: 61-73
- _____ (1981). Optimal Auction Design. *Mathematics of Operations Research*. 6: 58-73.
- _____ (1982). Optimal Coordination Mechanisms in Generalized Principal-Agent Problems. *Journal of Mathematical Economics*. 10:67-81.
- Nowak, A. S. (1984). On Zero-Sum Stochastic Games with General State Space I. *Probability and Mathematical Statistics*. IV: 13-32.
- Page, F. H. Jr. (1989). Incentive Compatible Strategies for General Stackelberg Games with Incomplete Information. *International Journal of Game Theory*. 18: 409-421.
- _____ (1991a). Optimal Contract Mechanisms for Principal-Agent Problems with Moral Hazard and Adverse Selection. *Economic Theory*. 1: 323-338.
- _____ (1991b). Dominant Strategy Auction Mechanisms. Department of Finance, Working Paper, University of Alabama.
- _____ (1992). Bayesian Incentive Compatible Mechanisms. *Economic Theory*. 2: 509-524.

- _____ (1994). Dominant Strategy Mechanisms for Contract Auctions with Risk Aversion and Moral Hazard. *International Journal of Game Theory*. 23: 25-42.
- Riley, J. and Samuelson, W. F. (1981). Optimal Auctions. *American Economic Review*. 71: 381-392.
- Robinson, S. M., and Day, R. (1974). A Sufficient Condition for Continuity of Optimal Sets in Mathematical Programming. *Journal of Mathematical Analysis and Applications* 45: 506-511.
- Schal, M. (1974). A Selection Theorem for Optimization Problems. *Arch. Math.* 25: 219-224.
- Vickrey, W. (1961). Counterspeculation, Auctions and Competitive Sealed Tenders. *Journal of Finance*. 16: 8-37.

No.	Author(s)	Title
9661	U. Gneezy and J. Potters	An Experiment on Risk Taking and Evaluation Periods
9662	H.J. Bierens	Nonparametric Nonlinear Co-Trending Analysis, with an Application to Interest and Inflation in the U.S.
9663	J.P.C. Blanc	Optimization of Periodic Polling Systems with Non-Preemptive, Time-Limited Service
9664	M.J. Lee	A Root-N Consistent Semiparametric Estimator for Fixed Effect Binary Response Panel Data
9665	C. Fernández, J. Osiewalski and M.F.J. Steel	Robust Bayesian Inference on Scale Parameters
9666	X. Han and H. Webers	A Comment on Shaked and Sutton's Model of Vertical Product Differentiation
9667	R. Kollmann	The Exchange Rate in a Dynamic-Optimizing Current Account Model with Nominal Rigidities: A Quantitative Investigation
9668	R.C.H. Cheng and J.P.C. Kleijnen	Improved Design of Queueing Simulation Experiments with Highly Heteroscedastic Responses
9669	E. van Heck and P.M.A. Ribbers	Economic Effects of Electronic Markets
9670	F.Y. Kumah	The Effect of Monetary Policy on Exchange Rates: How to Solve the Puzzles
9671	J. Jansen	On the First Entrance Time Distribution of the M/D/ ∞ Queue: a Combinatorial Approach
9672	Y.H. Farzin, K.J.M. Huisman and P.M. Kort	Optimal Timing of Technology Adoption
9673	J.R. Magnus and F.J.G.M. Klaassen	Testing Some Common Tennis Hypotheses: Four Years at Wimbledon
9674	J. Fidrmuc	Political Sustainability of Economic Reforms: Dynamics and Analysis of Regional Economic Factors
9675	M. Das and A. van Soest	A Panel Data Model for Subjective Information on Household Income Growth
9676	A.M. Lejour and H.A.A. Verbon	Fiscal Policies and Endogenous Growth in Integrated Capital Markets
9677	B. van Aarle and S.-E. Hougaard Jensen	Output Stabilization in EMU: Is There a Case for an EFTS?
9678	Th.E. Nijman, F.A. de Roon and C. Veld	Pricing Term Structure Risk in Futures Markets

No.	Author(s)	Title
9679	M. Dufwenberg and U. Gneezy	Efficiency, Reciprocity, and Expectations in an Experimental Game
9680	P. Bolton and E.-L. von Thadden	Blocks, Liquidity, and Corporate Control
9681	T. ten Raa and P. Mohnen	The Location of Comparative Advantages on the Basis of Fundamentals only
9682	S. Hochguertel and van Soest	The Relation between Financial and Housing Wealth of Dutch A. Households
9683	F.A. de Roon, Th.E. Nijman and B.J.M. Werker	Testing for Spanning with Futures Contracts and Nontraded Assets: A General Approach
9684	F.Y. Kumah	Common Stochastic Trends in the Current Account
9685	U.Gneezy and M. Das	Experimental Investigation of Perceived Risk in Finite Random Walk Processes
9686	B. von Stengel, A. van den Elzen and D. Talman	Tracing Equilibria in Extensive Games by Complementary Pivoting
9687	S.Tijds and M. Koster	General Aggregation of Demand and Cost Sharing Methods
9688	S.C.W. Eijffinger, H.P. Huizinga and J.J.G. Lemmen	Short-Term and Long-Term Government Debt and Nonresident Interest Withholding Taxes
9689	T. ten Raa and E.N. Wolff	Outsourcing of Services and the Productivity Recovery in U.S. Manufacturing in the 1980s
9690	J. Suijs	A Nucleolus for Stochastic Cooperative Games
9691	C. Seidl and S.Traub	Rational Choice and the Relevance of Irrelevant Alternatives
9692	C. Seidl and S.Traub	Testing Decision Rules for Multiattribute Decision Making
9693	R.M.W.J. Beetsma and H. Jensen	Inflation Targets and Contracts with Uncertain Central Banker Preferences
9694	M. Voorneveld	Equilibria and Approximate Equilibria in Infinite Potential Games
9695	F.B.S.L.P. Janssen and A.G. de Kok	A Two-Supplier Inventory Model
9696	L. Ljungqvist and H. Uhlig	Catching up with the Keynesians
9697	A. Rustichini	Dynamic Programming Solution of Incentive Constrained Problems

No.	Author(s)	Title
9698	G.Gürkan and A.Y. Özge	Sample-Path Optimization of Buffer Allocations in a Tandem Queue - Part I: Theoretical Issues
9699	H. Huizinga	The Dual Role of Money and Optimal Financial Taxes
96100	H. Huizinga	The Taxation Implicit in Two-Tiered Exchange Rate Systems
96101	H. Norde, F. Patrone and S. Tijs	Characterizing Properties of Approximate Solutions for Optimization Problems
96102	M. Berg, A. De Waegenare and J. Wielhouwer	Optimal Tax Reduction by Depreciation: A Stochastic Model
96103	G. van der Laan, D. Talman and Z. Yang	Existence and Approximation of Robust Stationary Points on Polytopes
96104	H. Huizinga and S.B. Nielsen	The Coordination of Capital Income and Profit Taxation with Cross-Ownership of Firms
96105	H. Degryse	The Total Cost of Trading Belgian Shares: Brussels Versus London
96106	H. Huizinga and S.B. Nielsen	The Political Economy of Capital Income and Profit Taxation in a Small Open Economy
96107	T. Dieckmann	The Evolution of Conventions with Endogenous Interactions
96108	F. de Jong and M.W.M. Donders	Intraday Lead-Lag Relationships Between the Futures-, Options and Stock Market
96109	F. Verboven	Brand Rivalry, Market Segmentation, and the Pricing of Optional Engine Power on Automobiles
96110	D. Granot, H. Hamers and S. Tijs	Weakly Cyclic Graphs and Delivery Games
96111	P. Aghion, P. Bolton and S. Fries	Financial Restructuring in Transition Economies
96112	A. De Waegenare, R. Kast and A. Lapied	Non-linear Asset Valuation on Markets with Frictions
96113	R. van den Brink and P.H.M. Ruys	The Internal Organization of the Firm and its External Environment
96114	F. Palomino	Conflicting Trading Objectives and Market Efficiency
96115	E. van Damme and S. Hurkens	Endogenous Stackelberg Leadership
96116	E. Canton	Business Cycles in a Two-Sector Model of Endogenous Growth
9701	J.P.J.F. Scheepens	Collusion and Hierarchy in Banking

No.	Author(s)	Title
9702	H.G. Bloemen and E.G.F. Stancanelli	Individual Wealth, Reservation Wages and Transitions into Employment
9703	P.J.J. Herings and V.J. Vannetelbosch	Refinements of Rationalizability for Normal-Form Games
9704	F. de Jong, F.C. Drost and B.J.M. Werker	Exchange Rate Target Zones: A New Approach
9705	C. Fernández and M.F.J. Steel	On the Dangers of Modelling Through Continuous Distributions: A Bayesian Perspective
9706	M.A. Odijk, P.J. Zwaneveld, J.S. Hooghiemstra, L.G. Kroon and M. Salomon	Decision Support Systems Help Railed to Search for 'Win-Win' Solutions in Railway Network Design
9707	G. Bekaert, R.J. Hodrick and D.A. Marshall	The Implications of First-Order Risk Aversion for Asset Market Risk Premiums
9708	C. Fernández and M.F.J. Steel	Multivariate Student- <i>t</i> Regression Models: Pitfalls and Inference
9709	H. Huizinga and S.B. Nielsen	Privatization, Public Investment, and Capital Income Taxation
9710	S. Eijffinger, E. Schaling and M. Hoeberichts	Central Bank Independence: a Sensitivity Analysis
9711	H. Uhlig	Capital Income Taxation and the Sustainability of Permanent Primary Deficits
9712	M. Dufwenberg and W. Güth	Indirect Evolution Versus Strategic Delegation: A Comparison of Two Approaches to Explaining Economic Institutions
9713	H. Uhlig	Long Term Debt and the Political Support for a Monetary Union
9714	E. Charlier, B. Melenberg and A. van Soest	An Analysis of Housing Expenditure Using Semiparametric Models and Panel Data
9715	E. Charlier, B. Melenberg and A. van Soest	An Analysis of Housing Expenditure Using Semiparametric Cross-Section Models
9716	J.P. Choi and S.-S. Yi	Vertical Foreclosure with the Choice of Input Specifications
9717	J.P. Choi	Patent Litigation as an Information Transmission Mechanism
9718	H.Degryse and A. Irmen	Attribute Dependence and the Provision of Quality
9719	A. Possajennikov	An Analysis of a Simple Reinforcing Dynamics: Learning to Play an "Egalitarian" Equilibrium
9720	J. Jansen	Regulating Complementary Input Supply: Cost Correlation and Limited Liability
9721	J. ter Horst and M. Verbeek	Estimating Short-Run Persistence in Mutual Fund Performance

No.	Author(s)	Title
9722	G. Bekaert and S.F. Gray	Target Zones and Exchange Rates: An Empirical Investigation
9723	M. Slikker and A. van den Nouweland	A One-Stage Model of Link Formation and Payoff Division
9724	T. ten Raai	Club Efficiency and Lindahl Equilibrium
9725	R. Euwals, B. Melenberg and A. van Soest	Testing the Predictive Value of Subjective Labour Supply Data
9726	C. Fershtman and U. Gneezy	Strategic Delegation: An Experiment
9727	J. Potters, R. Sloof and F. van Winden	Campaign Expenditures, Contributions and Direct Endorsements: The Strategic Use of Information and Money to Influence Voter Behavior
9728	F.H. Page, Jr.	Existence of Optimal Auctions in General Environments

P.O. BOX 00152 5000 LE TILBURG THE NETHERLANDS

Bibliotheek K. U. Brabant

17 000 01380004 1