Tilburg University

Characterizations of a Game Theoretical Cost Allocation Method

Otten, G.J.M.

Publication date:
1993

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Otten, G. J. M. (1993). *Characterizations of a Game Theoretical Cost Allocation Method*. (CentER Discussion Paper; Vol. 1993-37). CentER.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Center
for
Economic Research

Discussion paper

ECO
CBM
RR
8414
1993
37

* C I N O O 7 5 0 *

R20

R40

2000-2001

1000-2000

**Center
for
Economic Research**

No. 9337

**Characterizations of a Game
Theoretical Cost Allocation Method**

by G.-J. Otten

June 1993

ISSN 0924-7815

Characterizations of a Game Theoretical Cost Allocation Method

Gert-Jan Otten *

Tilburg University

P.O. Box 90153, 5000 LE Tilburg, The Netherlands

March, 1993

Abstract

In the 1930's the Tennessee Valley Authority developed several methods to allocate the costs of multipurpose water projects. One of these methods is the alternate cost avoided method. This paper provides two characterizations of the alternate cost avoided method, one on a class of cost games with a fixed player set, the other on a class of cost games with a variable player set using a reduced game property.

*The author is indebted to P. Borm, T. Driessen, Y. Funaki, S. Tijs and J. Zarzuelo for helpful discussions and useful suggestions.

K.U.B.
BIBLIOTHEEK
TILBURG

1 Introduction

Cost allocation problems occur in many practical situations, where individuals work together in a joint project. In these cases the problem arises of allocating the joint costs to the participants in the project in a "fair" way. A mathematical tool to analyse this type of problems is provided by cooperative game theory.

Examples of cost allocation problems studied in a game theoretical context are the setting of airport landing fees (e.g. Littlechild and Owen (1973), Littlechild and Thompson (1977)), the allocation of joint overhead costs of a firm among its different divisions (e.g. Shubik (1962), Jensen (1977), Hamlen et al. (1977)), and the apportioning of costs of multipurpose water development projects (e.g. Ransmeier (1942), Suzuki and Nakayama (1976), Loughlin (1977), Straffin and Heaney (1981), Young et al. (1982)).

Especially the last type of cost allocation problems has a rich history dating back to the 1930's in which the Tennessee Valley Authority (TVA) was established (see Ransmeier 1942, Parker (1943)). The problem TVA engineers were confronted with was the apportioning of costs of projects in the Tennessee River among the different 'purposes' to be served (mainly navigation, flood control, and hydro-electric power). TVA engineers made several proposals to allocate the costs of projects to these purposes. Almost all these methods begin by allocating the so-called separable cost, to each 'participant' (purpose), and then dividing the remaining nonseparable cost.

Two of the methods developed by the TVA are the egalitarian nonseparable cost (ENSC) method, which allocates the nonseparable cost equally among the participants, and the alternate cost avoided (ACA) method, which allocates the nonseparable cost among the participants in proportion to the 'cost savings' made by including a participant in the joint project instead of developing a separate project only to serve the purposes of that participant.

A modification of the ACA-method is the separable cost remaining benefit (SCRB) method. This has become the principal method used by civil engineers to allocate the costs of multipurpose water projects (see e.g. Inter-Agency Committee on Water Resources (1958)).

A game theoretical base for the ACA-method was established by Gately (1974). Gately proposed a new solution concept for cooperative games based on a player's "propensity

to disrupt" the solution. This solution concept has been further generalized by Fischer and Gately (1975), Littlechild and Vaidya (1976) and Charnes et al. (1978). It was shown by Straffin and Heaney (1981) that the allocation method proposed by Gately corresponds precisely to the ACA-method.

The purpose of this paper is to provide an axiomatic characterization of the ACA-method on a certain class of cost games with a fixed player set as well as on a class of cost games with a variable player set, using a reduced game property. This is the subject of section 3. First, in section 2 we discuss the cost allocation problem in a formal game theoretical context, and recall some of the cost allocation methods proposed by the TVA.

2 Game theory and cost allocation problems

To formulate a cost allocation problem in terms of cooperative game theory, it is modelled as a *cost game* (N, c) . Here, N represents a finite set of participants among which the costs of a joint project are allocated. For example, N can be a set of potential customers of a public facility, the divisions of a firm, or municipalities which share a joint water system, etc. The elements of N are called *players* and subsets of N are called *coalitions*. For any coalition $S \subset N$, the minimal costs of designing a project for the purposes of S only are denoted by $c(S)$. In particular, $c(\emptyset) := 0$, where $\emptyset$ denotes the empty set. The function $c : 2^N \rightarrow \mathbf{R}$ is called the (*joint*) *cost function*. Let CG^N denote the set of all cost games with player set N .

Example 1: As an example of a joint cost game based on a cost allocation problem, we consider the cost allocation problem for the TVA ten dam system. Here the purposes navigation, flood control and hydro-electric power are denoted as players 1,2, and 3 respectively. Table 1 is adapted from Ransmeier (1942, p. 329).

coalitions S	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
cost $c(S)$	0	163,520	140,826	250,096	301,607	378,821	367,370	412,584

Table 1. The cost game for the TVA ten dam project (costs in \$ 1000).

Given a cost game (N, c) , the cost allocation problem now becomes to choose a cost allocation in a “fair” manner. A *cost allocation* for (N, c) is a vector $x \in \mathbf{R}^N$ such that $\sum_{i \in N} x_i = c(N)$. Here, x_i is the cost allocated to player $i \in N$. The TVA engineers proposed several cost allocation methods. If A^N is a subset of CG^N , then a *(cost) allocation method* on A^N is a map $f : A^N \rightarrow \mathbf{R}^N$, which assigns to every cost game $(N, c) \in A^N$ a cost allocation $f(c) \in \mathbf{R}^N$.

Almost all cost allocation methods proposed by the TVA begin by charging every player a minimal cost, called separable cost, which are the additional cost of including the player in the project already designed for the other players. Thus, for a cost game (N, c) , the *separable cost* $SC_i(c)$ of player $i \in N$ are formally defined by

$$SC_i(c) := c(N) - c(N \setminus \{i\}).$$

To use methods based on the idea above it is reasonable to make the following two assumptions on the underlying cost game.

$$SC_i(c) \leq c(\{i\}) \quad \text{for all } i \in N, \tag{1}$$

$$\sum_{i \in N} SC_i(c) \leq c(N) \leq \sum_{i \in N} c(\{i\}). \tag{2}$$

Conditions (1) and (2) are well-known balancedness conditions for cost games. If $SC_i(c) > c(\{i\})$ for some $i \in N$, then it is not favourable to include player i in the joint project. Condition (2) implies that after each player is charged his minimal costs there is still a positive amount of cost remaining which should be allocated. These remaining cost are called the *nonseparable cost* and are given by

$$NSC(c) := c(N) - \sum_{i \in N} SC_i(c).$$

The easiest way to allocate the nonseparable cost is to divide these cost equally among the players. This method is called the *egalitarian nonseparable cost (ENSC) method*, and it is one of the first allocation methods proposed by the TVA. Thus, for a cost game (N, c) the cost allocated to player $i \in N$ by the ENSC-method are

$$ENSC_i(c) = SC_i(c) + \frac{1}{|N|} NSC(c).$$

An alternative allocation method is the *alternate cost avoided ACA) method*, which was first proposed by Martin Gleaser, a TVA consultant in 1938 (see Ransmeier (1942)). By this method the nonseparable cost are divided in proportion to $c(\{i\}) - SC_i(c)$. Hence,

$$ACA_i(c) := SC_i(c) + \frac{c(\{i\}) - SC_i(c)}{\sum_{j \in N} c(\{j\}) - SC_j(c)} NSC(c) \quad \text{for all } i \in N$$

The number $c(\{i\}) - SC_i(c)$ represents the *alternate cost avoided* by including player i in the joint project.

A modification of the ACA-method is the *separable cost remaining benefit (SCRB) method*. If $b(i)$ is the benefit of the project to player i , then i would not be willing to pay more than $\min\{b(i), c(\{i\})\}$. The remaining benefit to player i is defined by $\min\{b(i), c(\{i\})\} - SC_i(c)$. The SCRB-method allocates the nonseparable cost proportional to the remaining benefits. Since in many situations the benefits exceed the alternate costs, the SCRB-method often coincides with the ACA-method.

The mayor drawback of the cost allocation methods mentioned above is that they only take into account the values of the coalitions with 1, $|N| - 1$ and $|N|$ players. In particular, there is no guarantee that the corresponding allocations of these methods are core elements of the cost game, which means that there might be subcoalitions that have an incentive to split of from the grand coalition.

From a practical viewpoint however, the advantage of these methods is that in general they are much easier to compute than game theoretical solution concepts as the Shapley value (Shapley (1953)), the nucleolus (Schmeidler (1969)) and the cost gap method (Driessen en Tijs (1985), Tijs en Driessen (1986)), which take into account the values of *all* coalitions.

Moreover, as is shown in e.g. Suzuki and Nakayama (1976), Legros (1982) and Driessen and Tijs (1985) there are (large) classes of cost games for which some of the solution concepts mentioned above coincide with one (or more) of the game theoretical solution concepts.

Example 2: For the TVA cost game of example 1 the cost allocations of the ENSC- and ACA-method are given in table 2 together with the cost allocations corresponding to the game theoretical solutions mentioned above. Note that in this case the cost allocations by the ACA-method and the cost gap method coincide.

	1	2	3
ENSC-method	119,424	107,973	185,187
ACA-method	117,476	99,157	195,951
Shapley value	117,829	100,756	193,999
nucleolus	116,234	93,540	202,810
cost gap method	117,476	99,157	195,951

Table 2. Cost allocation for the TVA cost game by five methods (cost in \$ 1000).

3 Characterizations of the ACA-method

This section further investigates the ACA-method. Attention is restricted to the class of cost games (N, c) for which (1) and (2) hold. This class is denoted by F^N and F_m denotes the class of cost games with m or more players satisfying (1) and (2).

Geometrically, for a cost game $(N, c) \in F^N$ the cost allocation $ACA(c)$ is the unique element in the hyperplane $\{x \in \mathbb{R}^N \mid \sum_{i \in N} x_i = c(N)\}$ which lies on the line segment with end points $(SC_i(c))_{i \in N}$ and $(c(\{i\}))_{i \in N}$ (see figure 1).

figure 1.

Let $A \subset F_1$. Clearly, the ACA-method satisfies *individually rationality on A*, i.e., $ACA_i(c) \leq c(\{i\})$ for all $i \in N$ and all $(N, c) \in A$.

Furthermore, the ACA-method satisfies the *symmetry property on A*, i.e., for all $(N, c) \in A$ and all players i and j that are symmetric in (N, c) , i.e., $c(S \cup \{i\}) = c(S \cup \{j\})$ for all $S \subset N \setminus \{i, j\}$, it holds that $ACA_i(c) = ACA_j(c)$.

The ACA-method also satisfies *invariance w.r.t. strategic equivalence on A*, i.e., for all $(N, c) \in A$, all $k > 0$ and all $a \in \mathbf{R}^N$, such that $(N, kc + a) \in A$, we have that $ACA(kc + a) = kACA(c) + a$. Here the game $(N, kc + a)$ is defined by $(kc + a)(S) := kc(S) + \sum_{i \in S} a_i$ for all $S \subset N$.

Another property of the ACA-method on A is *weak proportionality* which says that if $(N, c) \in A$ is such that $SC_i(c) = 0$ for all $i \in N$, then $ACA(c)$ is proportional to the vector $(c(\{i\}))_{i \in N}$ of individual costs.

This weak proportionality property shows great resemblance to the restricted proportionally property of the τ -value (Tijs (1981), (1987)). Cost games for which each player's separable cost are zero arise when the increase in the total costs of adding an extra player can be neglected compared to the total cost of the project.

Similar to the characterization of the τ -value by Tijs (1987) one can prove

Theorem 1: The ACA-method is the unique cost allocation method on F^N which satisfies invariance w.r.t. strategic equivalence and weak proportionality.

Proof: Suppose that $f : F^N \rightarrow \mathbf{R}^N$ satisfies the two mentioned properties. Let $(N, c) \in F^N$. It suffices to show that $f(c) = ACA(c)$.

Define the game $(N, \hat{c}) \in F^N$ by

$$\hat{c}(S) := c(S) - \sum_{i \in S} SC_i(c) \quad \text{for all } S \subset N.$$

Then $SC_i(\hat{c}) = 0$ for all $i \in N$. From the weak proportionality property it follows that there exists an $\alpha \in \mathbf{R}$ such that for all $i \in N$

$$f_i(\hat{c}) = \alpha \hat{c}(\{i\}) = \alpha(c(\{i\}) - SC_i(c)).$$

From the strategic equivalence property it follows that for all $i \in N$

$$f_i(c) = SC_i(c) + f_i(\tilde{c}) = SC_i(c) + \alpha(c(\{i\}) - SC_i(c)).$$

Using the fact that $\sum_{i \in N} f_i(c) = c(N)$, it easily follows that $f(c) = ACA(c)$. $\square$

The last part of this section provides a characterization of the ACA-method on the class F_1 using a reduced game property. In the literature several types of reduced games have been considered to provide a foundation of game theoretic solution concepts based on the consistency principle. We mention, Hart and Mas-Colell (1989) for the Shapley value, Sobolev (1975), Snijders (1991) for the (pre)nucleolus, Peleg (1986) for the core, and recently, Driessen (1992) for the τ -value. Also the ENSC-method has been characterized by means of a reduced game property (Moulin (1985), Driessen and Funaki (1993)). For a detailed survey on consistency see e.g. Driessen (1991).

The idea behind consistency is the following. Given a cost game, and a cost allocation for this game, determined by a cost allocation method, imagine that a coalition decides to renegotiate the allocation within their subgroup. The new situation is described by a reduced game. A cost allocation method is consistent w.r.t this reduced game if the new cost allocation within this subgroup is the same as in the original game.

Let (N, c) be a cost game, $k \in N$ and $x \in \mathbf{R}^N$ a cost allocation. The *reduced game* $(N \setminus \{k\}, c^{k,x})$ corresponding to (N, c) is defined as follows. For $S \subset N \setminus \{k\}$

$$c^{k,x}(S) := \begin{cases} c(S) & \text{if } |S| \leq 1 \\ c(S \cup \{k\}) - x_k & \text{if } 2 \leq |S| \leq |N| - 1. \end{cases}$$

It should be noted that the reduced game introduced here coincides with the reduced game of Moulin (1985) except for the 1-person coalitions.

The interpretation of this reduced game is as follows. In the reduced game the cost of a 1-person coalition is the same as in the original game. However, if in the reduced situation the players want to cooperate in a coalition S , then player k should be involved and, therefore, the cost of coalition S in the reduced game is the cost of coalition $S \cup \{k\}$ in the original game minus the original cost x_k allocated to player k .

Let $A \subset F_1$ and let $m(A) := \min\{|N| \mid (N, c) \in A\}$. A cost allocation method f on A satisfies the *reduced game property* on A if for all $(N, c) \in A$ with $n > m(A)$ and all

$k \in N$ it holds that

$$(i) \ (N \setminus \{k\}, c^{k, f(c)}) \in A, \text{ and}$$

$$(ii) \ f_i(c^{k, f(c)}) = f_i(c) \quad \text{for all } i \in N \setminus \{k\}.$$

The ACA-method satisfies the reduced game property on the class F_3 . This is shown in

Lemma 2: The ACA-method satisfies the reduced game property on F_3 .

Proof: Let $(N, c) \in F_3$ with $|N| \geq 4$, and let $k \in N$. We first show that the reduced game $(N \setminus \{k\}, c^{k, ACA(c)})$ is an element of F_3 . Herefore note that for all $i \in N \setminus \{k\}$

$$c^{k, ACA(c)}(\{i\}) = c(\{i\}) \quad (3)$$

and since $|N| \geq 4$ also

$$SC_i(c^{k, ACA(c)}) = c(N) - ACA_k(c) - (c(N \setminus \{i\}) - ACA_k(c)) = SC_i(c). \quad (4)$$

Since $(N, c) \in F_3$, it follows that $SC_i(c^{k, ACA(c)}) \leq c^{k, ACA(c)}(\{i\})$ for all $i \in N \setminus \{k\}$.

It remains to show that

$$\sum_{i \in N \setminus \{k\}} SC_i(c^{k, ACA(c)}) \leq c^{k, ACA(c)}(N \setminus \{k\}) \leq \sum_{i \in N \setminus \{k\}} c^{k, ACA(c)}(\{i\}). \quad (5)$$

Note that for $i \in N \setminus \{k\}$

$$SC_i(c) \leq ACA(c) \leq c(\{i\}).$$

Then, using (3), (4), and the fact that $c^{k, ACA(c)}(N \setminus \{k\}) = \sum_{i \in N \setminus \{k\}} ACA_i(c)$ the required inequality (5) is easily obtained.

Now we show that $ACA_i(c^{k, ACA(c)}) = ACA_i(c)$ for all $i \in N \setminus \{k\}$.

Since $ACA_i(c) = SC_i(c) + \alpha(c(\{i\}) - SC_i(c))$ for all $i \in N$, where α is such that

$$c(N) = \sum_{i \in N} SC_i(c) + \alpha \sum_{i \in N} (c(\{i\}) - SC_i(c)). \quad (6)$$

Similarly, using (3) and (4), we obtain that $ACA_i(c^{k,ACA(c)}) = SC_i(c) + \beta(c(\{i\}) - SC_i(c))$ for all $i \in N \setminus \{k\}$, where β is such that

$$c(N) - ACA_k(c) = \sum_{i \in N \setminus \{k\}} SC_i(c) + \beta \sum_{i \in N \setminus \{k\}} (c(\{i\}) - SC_i(c)). \quad (7)$$

Subtracting (7) from (6) we obtain

$$ACA_k(c) = SC_k(c) + \alpha(c(\{k\}) - SC_k(c)) + (\alpha - \beta) \sum_{i \in N \setminus \{k\}} (c(\{i\}) - SC_i(c)).$$

Hence,

$$(\alpha - \beta) \sum_{i \in N \setminus \{k\}} (c(\{i\}) - SC_i(c)) = 0. \quad (8)$$

We now distinguish two cases.

If $\sum_{i \in N \setminus \{k\}} (c(\{i\}) - SC_i(c)) = 0$, then $ACA_i(c) = SC_i(c)$ for all $i \in N \setminus \{k\}$. Since, in this case,

$$c^{k,ACA(c)}(N \setminus \{k\}) = \sum_{i \in N \setminus \{k\}} ACA_i(c) = \sum_{i \in N \setminus \{k\}} SC_i(c) = \sum_{i \in N \setminus \{k\}} SC_i(c^{k,ACA(c)})$$

it easily follows that $ACA_i(c^{k,ACA(c)}) = ACA_i(c)$ for all $i \in N \setminus \{k\}$.

If $\sum_{i \in N \setminus \{k\}} (c(\{i\}) - SC_i(c)) \neq 0$, then by (8) $\alpha - \beta = 0$. Hence, $ACA_i(c^{k,ACA(c)}) = ACA_i(c)$ for all $i \in N \setminus \{k\}$. $\square$

Example 3 illustrates that the ACA-method does not satisfy the reduced game property on the set F_2 . This is due to the fact that by reducing a 3-person game to a 2-person game the separable costs of the players may change.

Example 3: Let $N := \{1, 2, 3\}$ and define (N, c) as follows. For $S \subset N$

$$c(S) = \begin{cases} 2 & \text{if } \{2, 3\} \not\subset S \\ 4 & \text{if } \{2, 3\} \subset S. \end{cases}$$

Clearly, $(N, c) \in F_2$ and $ACA(c) = (0, 2, 2)$. The reduced game $(\{1, 2\}, c^{3, ACA(c)}) \in F_2$ is given by $c^{3, ACA(c)}(\{1\}) = c^{3, ACA(c)}(\{2\}) = 2$ and $c^{3, ACA(c)}(\{1, 2\}) = 2$.

Hence, $ACA(c^{3, ACA(c)}) = (1, 1) \neq (0, 2) = (ACA_1(c), ACA_2(c))$.

Lemma 3: Let f be a cost allocation method on F_3 which satisfies weak proportionality on $F_3 \setminus F_4$ and the reduced game property on F_3 . Then f satisfies weak proportionality on F_3 .

Proof: Let $(N, c) \in F_3$ with $|N| \geq 4$ be such that $SC_i(c) = 0$ for all $i \in N$ and suppose that f satisfies the weak proportionality property on $F_3 \setminus F_{|N|}$.

Let $k \in N$ and let $(N \setminus \{k\}, c^{k, f(c)})$ be the $(|N| - 1)$ -person reduced game of (N, c) . Then $(N \setminus \{k\}, c^{k, f(c)}) \in F_3 \setminus F_{|N|}$. Since $SC_i(c^{k, f(c)}) = 0$ for all $i \in N \setminus \{k\}$ (cf. (4)), there exists an $\alpha \in \mathbf{R}$ such that

$$f_i(c^{k, f(c)}) = \alpha c^{k, f(c)}(\{i\}) = \alpha c(\{i\}) \quad \text{for all } i \in N \setminus \{k\}.$$

Since f satisfies the reduced game property on F_3 it follows that

$$f_i(c) = f_i(c^{k, f(c)}) = \alpha c(\{i\}) \quad \text{for all } i \in N \setminus \{k\}.$$

Varying $k \in N$ leads to

$$f(c) = (\alpha c(\{1\}), \dots, \alpha c(\{n\})).$$

□

Now we can formulate our main theorem which characterizes the ACA-method on F_1 .

Theorem 4: The ACA-method is the unique cost allocation method on F_1 which satisfies

- (i) symmetry on F_1 ,
- (ii) invariance w.r.t. strategic equivalence on F_1 ,
- (iii) weak proportionality on $F_3 \setminus F_4$,

(iv) the reduced game property on F_3 .

Proof: Clearly, the ACA-method satisfies (i)-(iv).

Let f be a cost allocation method, defined on F_1 , satisfying (i)-(iv). Let $(N, c) \in F_1$. We show that $f(c) = ACA(c)$. Herefore we distinguish three cases.

If $|N| = 1$, then $f(c) = c(\{1\}) = ACA(c)$.

If $|N| = 2$, then (i) and (ii) imply that $f_i(c) = c(\{i\}) + \frac{1}{2}(c(N) - c(\{i\}) - c(N \setminus \{i\})) = ACA_i(c)$ for $i = 1, 2$.

If $|N| \geq 3$, then theorem 1 and lemma 3 imply that $f(c) = ACA(c)$. □

It may be noted that also the the ENSC-method satisfies symmetry, invariance w.r.t. strategic equivalence and the reduced game property on the set F_3 . However, this cost allocation method does not satisfy weak proportionality.

For a cost game $(N, c) \in F_1$, the *center of imputation set (CIS) value* is defined by

$$CIS_i(c) := c(\{i\}) + \frac{1}{|N|}(c(N) - \sum_{j \in N} c(\{j\})) \quad \text{for all } i \in N.$$

If in theorem 4 condition (iii) is omitted and condition (iv) is replaced by the reduced game property on F_2 then a characterization of the CIS-value on F_1 is obtained. It is left to the reader to show that the CIS-value is indeed the unique cost allocation method on F_1 which satisfies symmetry, invariance w.r.t. strategic equivalence, and the reduced game property on F_2 .

References

- CHARNES, A., ROUSSEAU, J., AND SEIFORD, L. (1978). "Complements, mollifiers and the propensity to disrupt," *International Journal of Game Theory*, **7**, 37-50.
- DRIESSEN, T.S.H. (1991). "A survey of consistency properties in cooperative game theory," *SIAM Review*, **33**, 43-59.
- DRIESSEN, T.S.H. (1992). *On the reduced game property for and the axiomatization of the τ -value*. Discussion Paper, University of Twente, The Netherlands.

- DRIESSEN, T.S.H., AND FUNAKI, Y. (1993). Working Paper, University of Twente, The Netherlands.
- DRIESSEN, T.S.H., AND TIJS, S.H. (1985). "The cost gap method and other cost allocation methods for multipurpose water projects," *Water Resources Research*, **21**, 1469-1475.
- FISCHER, D., AND GATELY, D. (1975). *A comparison of various solution concepts for three-person cooperative games with non-empty cores*. Center for Applied Economics, New York University.
- GATELY, D. (1974). "Sharing the gains from regional cooperation: a game theoretic application to planning investment in electric power," *International Economic Review*, **15**, 195-208.
- HAMLEN, S., HAMLEN, W., AND TSCHIRHART, J. (1977). "The use of core theory in evaluating joint cost allocation schemes," *Accounting Review*, **52**, 616-627.
- HART, S., AND MAS-COLELL, A. (1989). "Potential, value and consistency," *Econometrica*, **57**, 589-614.
- Inter-Agency Committee on Water Resources (1958), Proposed practices for economic analysis of river basin projects. Report, Washington, D.C.
- JENSEN, D.L. (1977). "A class of mutually satisfactory allocations," *Accounting Review*, **52**, 842-856.
- LEGROS, P. (1982). *The nucleolus and the cost allocation problem*. Report, Evanston, III.
- LITTLECHILD, S.C., AND OWEN, G. (1973). "A simple expression for the Shapley value in a special case," *Management Science*, **20**, 370-372.
- LITTLECHILD, S.C., AND THOMPSON, G.F. (1977). "Aircraft landing fees: a game theory approach," *Bell Journal of Economics*, **8**, 186-204.
- LITTLECHILD, S.C., AND VAIDYA, K.G. (1976). "The propensity to disrupt and the disruption nucleolus of a characteristic function game," *International Journal of Game Theory*, **5**, 151-161.
- LOEHMAN, E., ORLANDO, E., TSCHIRHART, J., AND WHINSTON, A. (1979). "Cost allocation for a regional waste-water treatment system," *Water Resources Research*, **15**, 193-202.

- LOUGHLIN, J.C. (1977). "The efficiency and equity of cost allocation methods for multipurpose water projects," *Water Resources Research*, **13**, 8-14.
- MOULIN, H. (1985). "The separability axiom and equal sharing methods," *Journal of Economic Theory*, **36**, 120-148.
- PARKER, T. (1943). "Allocation of the Tennessee Valley Authority projects," *Transactions of the American Society of Civil Engineers*, **108**, 174-187.
- PELEG, B. (1986). "On the reduced game property and its converse," *International Journal of Game Theory*, **15**, 187-200. A correction (1987), *International Journal of Game Theory*, **16**, 290.
- RANSMEIER, J.S. (1942). *The Tennessee Valley Authority: A Case Study in the Economics of Multiple Purpose Stream Planning*. Nashville, TN: Vanderbilt University Press.
- SCHMEIDLER, D. (1969). "The nucleolus of a characteristic function game," *SIAM Journal on Applied Mathematics*, **17**, 1163-1170.
- SHAPLEY, L. (1953). "A value for n -person games," *Contributions to the theory of games II* (Eds. A. Tucker and H. Kuhn), 307-317.
- SHUBIK, M. (1962). "Incentives, decentralized control, the assignment of joint costs and internal pricing," *Management Science*, **8**, 325-343.
- SNIJDERS, C. (1991). *Axiomatization of the nucleolus*. Preprint No. 676, Department of Mathematics, University of Utrecht, The Netherlands.
- SOBOLEV, A.I. (1975). "The characterization of optimality principles in cooperative games by functional equations," in: *Mathematical Methods in the Social Sciences* (N.N. Vorobev, ed.), **6**, 94-151. (Vilnius, in Russian).
- STRAFFIN, P., AND HEANEY, J.P. (1981). "Game theory and the Tennessee Valley Authority," *International Journal of Game Theory*, **10**, 35-43.
- SUZUKI, M., AND NAKAYAMA, M. (1976). "The cost assignment of cooperative water resource development: a game theoretical approach," *Management Science*, **22**, 1081-1086.
- TIJS, S.H. (1981). "Bounds for the core and the τ -value," in: *Game Theory and Mathematical Economics* (Eds. O. Moeschlin and P. Pallaschke), North Holland, Amsterdam, The Netherlands, 123-132.

- TIJS, S.H. (1987). "An axiomatization of the τ -value," *Mathematical Social Sciences*, **13**, 177-181.
- TIJS, S.H., AND DRIESSEN, T.S.H. (1986). "Game theory and cost allocation problems," *Management Science*, **32**, 1015-1028.
- YOUNG, H.P., OKADA, N., AND HASHIMOTO, T. (1980). "Cost allocation in water resources development," *Water Resources Research*, **18**, 463-475.

Discussion Paper Series, CentER, Tilburg University, The Netherlands:

(For previous papers please consult previous discussion papers.)

No.	Author(s)	Title
9201	M. Verbeek and Th. Nijman	Minimum MSE Estimation of a Regression Model with Fixed Effects from a Series of Cross Sections
9202	E. Bomhoff	Monetary Policy and Inflation
9203	J. Quiggin and P. Wakker	The Axiomatic Basis of Anticipated Utility; A Clarification
9204	Th. van de Klundert and S. Smulders	Strategies for Growth in a Macroeconomic Setting
9205	E. Siandra	Money and Specialization in Production
9206	W. Härdle	Applied Nonparametric Models
9207	M. Verbeek and Th. Nijman	Incomplete Panels and Selection Bias: A Survey
9208	W. Härdle and A.B. Tsybakov	How Sensitive Are Average Derivatives?
9209	S. Albæk and P.B. Overgaard	Upstream Pricing and Advertising Signal Downstream Demand
9210	M. Cripps and J. Thomas	Reputation and Commitment in Two-Person Repeated Games
9211	S. Albæk	Endogenous Timing in a Game with Incomplete Information
9212	T.J.A. Storcken and P.H.M. Ruys	Extensions of Choice Behaviour
9213	R.M.W.J. Beetsma and F. van der Ploeg	Exchange Rate Bands and Optimal Monetary Accommodation under a Dirty Float
9214	A. van Soest	Discrete Choice Models of Family Labour Supply
9215	W. Güth and K. Ritzberger	On Durable Goods Monopolies and the (Anti-) Coase- Conjecture
9216	A. Simonovits	Indexation of Pensions in Hungary: A Simple Cohort Model
9217	J.-L. Ferreira, I. Gilboa and M. Maschler	Credible Equilibria in Games with Utilities Changing During the Play
9218	P. Borm, H. Keiding, R. Mclean, S. Oortwijn and S. Tijs	The Compromise Value for NTU-Games

No.	Author(s)	Title
9219	J.L. Horowitz and W. Härdle	Testing a Parametric Model against a Semiparametric Alternative
9220	A.L. Bovenberg	Investment-Promoting Policies in Open Economies: The Importance of Intergenerational and International Distributional Effects
9221	S. Smulders and Th. van de Klundert	Monopolistic Competition, Product Variety and Growth: Chamberlin vs. Schumpeter
9222	H. Bester and E. Petrakis	Price Competition and Advertising in Oligopoly
9223	A. van den Nouweland, M. Maschler and S. Tijs	Monotonic Games are Spanning Network Games
9224	H. Suehiro	A "Mistaken Theories" Refinement
9225	H. Suehiro	Robust Selection of Equilibria
9226	D. Friedman	Economically Applicable Evolutionary Games
9227	E. Bomhoff	Four Econometric Fashions and the Kalman Filter Alternative - A Simulation Study
9228	P. Borm, G.-J. Otten and H. Peters	Core Implementation in Modified Strong and Coalition Proof Nash Equilibria
9229	H.G. Bloemen and A. Kapteyn	The Joint Estimation of a Non-Linear Labour Supply Function and a Wage Equation Using Simulated Response Probabilities
9230	R. Beetsma and F. van der Ploeg	Does Inequality Cause Inflation? - The Political Economy of Inflation, Taxation and Government Debt
9231	G. Almekinders and S. Eijffinger	Daily Bundesbank and Federal Reserve Interventions - Do they Affect the Level and Unexpected Volatility of the DM/\$-Rate?
9232	F. Vella and M. Verbeek	Estimating the Impact of Endogenous Union Choice on Wages Using Panel Data
9233	P. de Bijl and S. Goyal	Technological Change in Markets with Network Externalities
9234	J. Angrist and G. Imbens	Average Causal Response with Variable Treatment Intensity
9235	L. Meijdam, M. van de Ven and H. Verbon	Strategic Decision Making and the Dynamics of Government Debt
9236	H. Houba and A. de Zeeuw	Strategic Bargaining for the Control of a Dynamic System in State-Space Form

No.	Author(s)	Title
9237	A. Cameron and P. Trivedi	Tests of Independence in Parametric Models: With Applications and Illustrations
9238	J.-S. Pischke	Individual Income, Incomplete Information, and Aggregate Consumption
9239	H. Bloemen	A Model of Labour Supply with Job Offer Restrictions
9240	F. Drost and Th. Nijman	Temporal Aggregation of GARCH Processes
9241	R. Gilles, P. Ruys and J. Shou	Coalition Formation in Large Network Economies
9242	P. Kort	The Effects of Marketable Pollution Permits on the Firm's Optimal Investment Policies
9243	A.L. Bovenberg and F. van der Ploeg	Environmental Policy, Public Finance and the Labour Market in a Second-Best World
9244	W.G. Gale and J.K. Scholz	IRAs and Household Saving
9245	A. Bera and P. Ng	Robust Tests for Heteroskedasticity and Autocorrelation Using Score Function
9246	R.T. Baillie, C.F. Chung and M.A. Tieslau	The Long Memory and Variability of Inflation: A Reappraisal of the Friedman Hypothesis
9247	M.A. Tieslau, P. Schmidt and R.T. Baillie	A Generalized Method of Moments Estimator for Long- Memory Processes
9248	K. Wärneryd	Partisanship as Information
9249	H. Huizinga	The Welfare Effects of Individual Retirement Accounts
9250	H.G. Bloemen	Job Search Theory, Labour Supply and Unemployment Duration
9251	S. Eijffinger and E. Schaling	Central Bank Independence: Searching for the Philosophers' Stone
9252	A.L. Bovenberg and R.A. de Mooij	Environmental Taxation and Labor-Market Distortions
9253	A. Lusardi	Permanent Income, Current Income and Consumption: Evidence from Panel Data
9254	R. Beetsma	Imperfect Credibility of the Band and Risk Premia in the European Monetary System

No.	Author(s)	Title
9301	N. Kahana and S. Nitzan	Credibility and Duration of Political Contests and the Extent of Rent Dissipation
9302	W. Güth and S. Nitzan	Are Moral Objections to Free Riding Evolutionarily Stable?
9303	D. Karotkin and S. Nitzan	Some Peculiarities of Group Decision Making in Teams
9304	A. Lusardi	Euler Equations in Micro Data: Merging Data from Two Samples
9305	W. Güth	A Simple Justification of Quantity Competition and the Cournot-Oligopoly Solution
9306	B. Peleg and S. Tijs	The Consistency Principle For Games in Strategic Form
9307	G. Imbens and A. Lancaster	Case Control Studies with Contaminated Controls
9308	T. Ellingsen and K. Wärneryd	Foreign Direct Investment and the Political Economy of Protection
9309	H. Bester	Price Commitment in Search Markets
9310	T. Callan and A. van Soest	Female Labour Supply in Farm Households: Farm and Off-Farm Participation
9311	M. Pradhan and A. van Soest	Formal and Informal Sector Employment in Urban Areas of Bolivia
9312	Th. Nijman and E. Sentana	Marginalization and Contemporaneous Aggregation in Multivariate GARCH Processes
9313	K. Wärneryd	Communication, Complexity, and Evolutionary Stability
9314	O.P. Attanasio and M. Browning	Consumption over the Life Cycle and over the Business Cycle
9315	F. C. Drost and B. J. M. Werker	A Note on Robinson's Test of Independence
9316	H. Hamers, P. Borm and S. Tijs	On Games Corresponding to Sequencing Situations with Ready Times
9317	W. Güth	On Ultimatum Bargaining Experiments - A Personal Review
9318	M.J.G. van Eijss	On the Determination of the Control Parameters of the Optimal Can-order Policy
9319	S. Hurkens	Multi-sided Pre-play Communication by Burning Money

No.	Author(s)	Title
9320	J.J.G. Lemmen and S.C.W. Eijffinger	The Quantity Approach to Financial Integration: The Feldstein-Horioka Criterion Revisited
9321	A.L. Bovenberg and S. Smulders	Environmental Quality and Pollution-saving Technological Change in a Two-sector Endogenous Growth Model
9322	K.-E. Wärneryd	The Will to Save Money: an Essay on Economic Psychology
9323	D. Talman, Y. Yamamoto and Z. Yang	The $(2^{n+m+1} - 2)$ -Ray Algorithm: A New Variable Dimension Simplicial Algorithm For Computing Economic Equilibria on $S^n \times R^m$
9324	H. Huizinga	The Financing and Taxation of U.S. Direct Investment Abroad
9325	S.C.W. Eijffinger and E. Schaling	Central Bank Independence: Theory and Evidence
9326	T.C. To	Infant Industry Protection with Learning-by-Doing
9327	J.P.J.F. Scheepens	Bankruptcy Litigation and Optimal Debt Contracts
9328	T.C. To	Tariffs, Rent Extraction and Manipulation of Competition
9329	F. de Jong, T. Nijman and A. Röell	A Comparison of the Cost of Trading French Shares on the Paris Bourse and on SEAQ International
9330	H. Huizinga	The Welfare Effects of Individual Retirement Accounts
9331	H. Huizinga	Time Preference and International Tax Competition
9332	V. Feltkamp, A. Koster, A. van den Nouweland, P. Borm and S. Tijs	Linear Production with Transport of Products, Resources and Technology
9333	B. Lauterbach and U. Ben-Zion	Panic Behavior and the Performance of Circuit Breakers: Empirical Evidence
9334	B. Melenberg and A. van Soest	Semi-parametric Estimation of the Sample Selection Model
9335	A.L. Bovenberg and F. van der Ploeg	Green Policies and Public Finance in a Small Open Economy
9336	E. Schaling	On the Economic Independence of the Central Bank and the Persistence of Inflation
9337	G.-J. Otten	Characterizations of a Game Theoretical Cost Allocation Method

Bibliotheek K. U. Brabant

17 000 01163442 6