Tilburg University

Forecast accuracy analysis

Hempenius, A.L.

Publication date:
1981

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Hempenius, A. L. (1981). *Forecast accuracy analysis: Applied to forecasts of the Dutch Central Planning Bureau, 1964-1978*. (Research memorandum / Tilburg University, Department of Economics; Vol. FEW 90). Unknown Publisher.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

CBM
R

7626
1981
90

Bestemming

TJDSCHRIFTENBUREAU
BIBLIOTHEEK
KATHOLIEKE
HOGESCHOOL
TILBURG

Nr.

subfaculteit der econometrie

RESEARCH MEMORANDUM

TILBURG UNIVERSITY
DEPARTMENT OF ECONOMICS

Postbus 90135 - 5000 LE Tilburg
Netherlands

B10

FEW
90

FORECAST ACCURACY ANALYSIS
Applied to Forecasts of the Dutch
Central Planning Bureau, 1964-1978.

By
A.L. Hempenius

KATHOLIEKE HOGESCHOOL TILBURG
SUBFACULTEIT DER ECONOMETRIE.

FORECAST ACCURACY ANALYSIS

Applied to Forecasts of the Dutch
Central Planning Bureau, 1964-1978.

By

A.L. Hempenius

1. Introduction

This paper has two objectives: the first one is to present, in Section 2, a methodology for the analysis of forecast errors and the second one is to apply, in Section 3, the methods presented to economic forecasts made by the Dutch Central Planning Bureau for the period 1964-1978.

In achieving the first objective several well-known inaccuracy measures will be reviewed. It will be endeavored to motivate the methodology as much as possible so that as little "ad-hockery" as possible remains. However, as one is dealing with descriptive measures, there is always, to some extent, room for individual preference for some particular measures.

As to the application to the Dutch forecasts, it might be asked why this particular period has been chosen. Several papers have analysed forecasts of previous years: Van de Panne (1959) analysed the period 1949-1956; there are Van den Beld (1965) and Sims (1967) analysing the period 1953-1963, which period was also covered by Theil (1966), except for the year 1963. The present paper's choice of period, 1964-1978, is largely motivated by the presence of the above analyses of previous periods.

It will be obvious that, although economic variables have been used in the application, the applicability of the measures presented is general.

2. Forecast Accuracy Analysis

2.1. Introduction

Suppose that there are two sequences of n numbers, one of forecasts for a certain variable and another of realized values for the same variable. Suppose further that it is desired to answer the question: "how close

do the forecasts approximate the realizations?" In Section 2 three different approaches to an answer will be distinguished. The first approach, to be called geometric analysis, considers the closeness of the two column vectors $F = (f_1, \dots, f_n)'$ and $R = (r_1, \dots, r_n)'$, where f_t and r_t are the t -th forecast and realization, respectively; see Section 2.2. The second approach, to be called the least squares analysis, considers the pairs (r_t, f_t) , $t = 1, \dots, n$, by analyzing the scatter of these points in a plane; see Section 2.3. The third approach starts from the concept of a loss function on the forecast error $f_t - r_t$; see Section 2.4.

2.2. Geometric Analysis

The obvious measure of closeness of the two n -dimensional vectors F and R , which have been defined in the previous section, is the distance d , with

$$(1) \quad d = \sqrt{\sum_{t=1}^n (f_t - r_t)^2} = \|F - R\|.$$

In order to be able to compare d -values, e.g. for two different variables, a measure is desired which is independent of the units of the variable considered and which corrects for differences in n .¹⁾ As d is the length of the vector of forecast errors, $F - R$, there is the possibility of using the lengths of F and/or R in order to arrive at a dimensionless measure which also corrects for n . One could, for example, investigate the virtue of the following measures:

$$(2) \quad U = d/\|R\|,$$

$$(3) \quad U' = d/(\|R\| + \|F\|),$$

$$(4) \quad U'' = d/\sqrt{\|R\| \cdot \|F\|},$$

$$(5) \quad U''' = d/\|F\|.$$

1) If one considers several different variables, all measured as percentage changes with respect to values of the previous period, as is done for many variables in the Dutch Central Planning Bureau models, then comparison of d -values for the different variables is not hampered by uncomparable dimensions.

U' and U'' use the arithmetic²⁾ and the geometric mean of $\|F\|$ and $\|R\|$, respectively, as denominator. Theil (1961) first proposed U' and later, in Theil (1966), he abandoned U' for U : the measure U' has the advantage of assuming values between 0 and 1, but the disadvantage of treating F and R symmetrically, whereas it might be argued, that comparisons of the forecast errors should be made with respect to the realizations only. This argument leaves $U = d/\|R\|$ as sole possibility of the four candidates (2), (3), (4) and (5).

Possibly not all forecast vectors at distance d from R would be valued equally by the decision maker who uses the forecasts. In Figure 1 all forecast vectors F with distance d to the vector of realizations R are represented by a circle; three forecast vectors have been drawn. The best

FIGURE 1

forecast vector of these three seems F_2 , as it has the smallest angle with R . More detailed information on the extent to which F differs from R , is thus given by the angle θ between the two vectors, with cosine c :

$$(6) \quad c = \cos \theta = \frac{R'F}{\|R\| \cdot \|F\|},$$

where it is assumed for uniqueness that $0 \leq \theta \leq \pi$. For this measure it holds that $-1 \leq c \leq 1$, with -1 "bad": the forecast vector is a negative multiple of the realization vector; and with 1 "possibly good": the forecast vector is a positive multiple of the realization vector.

For $n=2$ the vectors R and F are two-dimensional and may be drawn in a plane as in Figure 1. One might ask the question, for $n=2$, whether d and $\cos \theta$ uniquely determine F , for given R , i.e. whether every aspect of the

2) The factor $\frac{1}{2}$ has been omitted in the denominator of (3).

difference between F and R is completely covered by d and $\cos \theta$. The first thing to note then, is that the angle θ misses orientation, i.e. one does not know whether F is "under or above" R in Figure 1. This is caused by the fact that $\cos x = \cos -x$. Giving the value of the sine³⁾ gives the orientation of θ , where θ now ranges from $-\pi$ to π . Knowing the orientation still would not uniquely determine F; see also Figure 1, where F_1 and F_1^* have the same (oriented) angle and the same distance with respect to R. If, instead of the distance $\|F-R\|$, one knows the value of the ratio $\|F\|/\|R\|$:

$$(7) \quad \frac{\|F\|}{\|R\|} = \sqrt{\frac{\sum f_t^2}{\sum r_t^2}},$$

then, together with the values of $\cos \theta$ and $\sin \theta$, one can determine the vector F. The forecast error vector $F-R$ is, for given R, thus completely determined by these three values, which is a statement that is, of course, not true for $n > 2$. For $n > 2$ one may, however, use these values as descriptive measures, at least if their interpretation also makes sense, statistically.

The above quantities⁴⁾ will therefore be phrased in the terminology of descriptive statistics: d is, except for a factor $n^{-1/2}$, the root mean square forecast error; U has been called the inequality coefficient; c is the (noncentral) correlation coefficient; and $\|F\|/\|R\|$ is the root of the (quadratic) mean of the forecasts divided by the (quadratic) mean of the realizations, and is thus a measure for systematic bias, as will also follow from results in Section 2.3.

In d each squared forecast error $f_t - r_t$ has equal weight. A slight generalization of d is obtained by weighing the square of the t-th forecast error by a weight w_t , which may be seen as a particular case of the generalized distance:

$$(8) \quad \sqrt{(F-R)'W(F-R)},$$

where W is a positive definite matrix, so that interaction of forecast errors might be penalized. Economically it makes sense to use different

3) The sine is $\det [R, F] / (\|R\| \cdot \|F\|)$.

4) The interpretation of (the absolute value of) $\sin \theta$ may be found in Section 2.3.

weights for different periods as a correct forecast is more important in one period than in another and, when extending the measures to more variables simultaneously, to use different weights for different variables, because one variable is more important than another.⁵⁾

A useful method of checking the virtue of a proposed overall measure derived from d, like the inequality coefficient U, has been provided by Granger and Newbold (1973). Assume that $r_t = \alpha r_{t-1} + \varepsilon_t$ with the ε_t ($t = 1, \dots, n$) zero mean independent random variables and with $0 \leq \alpha \leq 1$. Assume further that the forecasting formula used is $f_t = \beta r_{t-1}$, with $0 \leq \beta \leq 1$. Then

$$(9) \quad \lim_{n \rightarrow \infty} \frac{1}{n} \sum (f_t - r_t)^2 = [(1-\alpha)^2 + (\beta-\alpha)^2] \text{ var}(r)$$

and $\text{var}(f) = \beta^2 \text{ var}(r)$. For the measures U, U', U'' and U''' one then has:

$$(10) \quad \lim_{n \rightarrow \infty} U^2 = (1-\alpha)^2 + (\beta-\alpha)^2,$$

$$(11) \quad \lim_{n \rightarrow \infty} (U')^2 = 1 - \frac{8\beta(1+\alpha)}{(1+\beta)^2},$$

$$(12) \quad \lim_{n \rightarrow \infty} (U'')^2 = \frac{1}{\beta} + \beta - 2\alpha,$$

$$(13) \quad \lim_{n \rightarrow \infty} (U''')^2 = \frac{1}{\beta} \left(\frac{1}{\beta} - 2\alpha \right).$$

so that (10) has its smallest value for $\beta = \alpha$, as it should be, if one assumes that a choice of forecast formula is made by looking at the smallest of the U-values. However, in (11), (12) and (13) $\beta = \alpha$ does not give the smallest value.

5) In Van den Beld (1965) all variables are measured or transformed to percentage changes. Yet for each variable the individual forecast errors $f_t - r_t$ ($t = 1, \dots, n$) are divided by $\|R\|$, which is motivated as follows: "Standardizing in this way seems justified, since it is intuitively clear that a two percent error in predicting, say, consumption price changes is much more serious an error from the forecaster's point of view than a two percent error in the prediction of investment changes, the latter being normally subject to much larger rates of change than the former."

Another method by which to judge the virtue of a proposed overall measure derived from d , is the following. One may form a relative measure, by dividing d by the d -value obtained from a simple forecasting method. For example: define d_0 as the d (distance)-value for the no change forecasting method; now define⁶⁾:

$$(14) \quad U_0 = d/d_0 .$$

Another example: define d_k as the d -value for the forecasting method that uses some $AR(k)$ -scheme; now define:

$$(15) \quad U_k = d/d_k .$$

One "only" has to agree on some "simple" forecasting method, and a relative d-measure⁷⁾ has been obtained which is used as follows: if U_0 , U_k , etc. is smaller than 1, then the forecasting method used is on the average, for the sample used, better than the simple method. Of the measures formed from d in (2), (3), (4) and (5), only the one in (2) may be formed in this way, at least if the variable of interest is measured in terms of percentage changes (see footnote 6). In Appendix A another useful relative d -measure, the distance d divided by the standard deviation of the realizations, is discussed.

2.3. Least Squares Analysis

In this section the scatter of points (r_t, f_t) in the rf -plane will be used to derive measures for the accuracy of forecasts. Forecasts are measured along the vertical axis, which is motivated by the traditional argument of regression analysis, that one is interested in the accuracy of forecasts given the set of realizations. A less convincing argument for this choice may be found in the forecast errors $f_t - r_t = v_t$, rewritten as follows:

6) Note that, for a variable defined in terms of percentage changes with respect to the previous period, one has $\|R\| = d_0$, because the no change forecasting method then means $f_t = 0$ for all t , so that $\Sigma(f_t - r_t)^2 = \Sigma r_t^2$. This means that, for such a variable, one has $U = U_0$.

7) See also Sims (1967) and Mincer and Zarnowitz (1969).

$f_t = r_t + v_t$, an equation that may be seen as the simplest possible "re-gression model": $f_t = \beta r_t + \epsilon_t$ with $\beta = 1$ and ϵ_t the forecast error.⁸⁾ In Figure 2 dots denote the pairs (r_t, f_t) for the variable Commodity Exports of Section 3. Also denoted is the line of perfect forecast: $f = r$.

FIGURE 2

A simple description of the scatter of points is the least squares line $f = br$, with

$$(16) \quad b = \frac{\sum r_t f_t}{\sum r_t^2}$$

See Figure 2 for an example.

As the ideal value for b is 1, mean departure from the line of perfect forecast is described by $b - 1$. A negative (positive) value of $b - 1$ indicates that on the average there is underestimation (overestimation) of the r_t -values by the f_t -values. A test of the hypothesis, that the forecasts f_t are unbiased with respect to r_t , could be performed by testing whether $\beta = 1$ in the above mentioned regression model, although the relevant t -statistic is best considered as a descriptive measure.

8) It may be, that one is interested in the relative forecast errors $(f_t - r_t)/r_t = v'_t$, but there may also be written as $f_t = r_t + v_t$.

Another measure of interest is, of course, the dispersion around the line $f = br$. Defining $e_t = f_t - br_t$, this dispersion may be measured by $(\sum e_t^2)/n$ or rather by its traditionally normalized value:

$$(17) \quad \frac{\frac{1}{n} \sum e_t^2}{\frac{1}{n} \sum f_t^2} = 1 - c^2,$$

where c is $\cos \theta$ defined in (6). In order to have a measure with the same dimension as $b-1$, it is advisable to use $\sqrt{1-c^2} = \sqrt{\sin^2 \theta}$. The ideal value of the pair $(b-1, \sqrt{1-c^2})$ is $(0,0)$. The maximal and worst value of $\sqrt{1-c^2}$ is 1, in which case $b-1$ is -1.

An overall measure may be obtained by constructing a measure from the distance of the points (r_t, f_t) with respect to the line $f = r$. Measuring these distances vertically, in line with the remarks at the start of this section concerning the choice of variable for the horizontal axis, and applying the quadratic penalty of least squares to each distance separately, one obtains after summation: $\sum (f_t - r_t)^2 = d^2$.

The following relation between d^2 , $(b-1)^2$ and $\sum e_t^2$ also gives a decomposition of d^2 :

$$(18) \quad d^2 = (b-1)^2 \sum r_t^2 + \sum e_t^2,$$

or in terms of quadratic means:

$$(19) \quad \frac{1}{n} \sum (f_t - r_t)^2 = (b-1)^2 \left(\frac{1}{n} \sum r_t^2 \right) + \frac{1}{n} \sum e_t^2.$$

Relation (18) leads to the following decomposition of U^2 :

$$(20) \quad U^2 = (b-1)^2 + (1-c^2) \frac{\sum f_t^2}{\sum r_t^2}.$$

(Note that again there is reason for dividing d^2 by $\sum r_t^2$: in this way one isolates the systematic bias $b-1$.) In (18), (19) and (20) the first term after the equality sign is called the bias component, due to the systematic under- or overestimation measured by $b-1$. The second term after the equality sign is called the dispersion component, due to the dispersion around the line $f = br$. The relative contributions of these components

are, of course, the same in (18), (19) and (20). These relative components are denoted by d_b^2 and d_c^2 , respectively.

The factor $\Sigma f_t^2 / \Sigma r_t^2$ of $(1-c^2)$ in (20) describes under- or overestimation. This may be seen as follows:

$$(21) \quad \frac{\Sigma f_t^2}{\Sigma r_t^2} = \frac{\Sigma f_t r_t / \Sigma f_t r_t}{\Sigma r_t^2 / \Sigma f_t^2} = b/b^*$$

In (21) b^* is the least squares coefficient in $r_t = b^* f_t + e_t^*$, i.e. $1/b^*$ measures the same thing as b , viz. a kind of average proportionality factor between f and r . The (geometric) average of b and $1/b^*$, also written as⁹⁾ $\tilde{b}$:

$$(22) \quad \tilde{b} = \sqrt{b/b^*},$$

evidently indicates underestimation, if it is smaller than 1, or overestimation, if it is greater than 1. In (7) $\|F\|/\|R\|$ is the same as $\tilde{b}$.

Relation (20) may thus be written as:

$$(23) \quad U^2 = (b-1)^2 + (1-c^2)\tilde{b}^2.$$

A provisional conclusion from the above is that the triple $(b-1, \sqrt{1-c^2}, d)$, with d suitably normalized as, for example, in (14) or (15), gives a good description of the forecast quality of the forecasts $f_1, f_2, \dots, f_n$. The first element measures average under- or overestimation, the second measures dispersion around the line $f=br$ and the third measures the overall performance of the forecasting method.

The above approach of considering the least-squares line $f=br$ follows naturally from the arguments given at the start of this section. There is, however, an argument pleading for the reverse order, which considers the model that produces the forecasts as consisting of a systematic part disturbed by a disturbance term. If the systematic part is perfectly forecasted, then one has $r = f + \text{disturbance}$, so that the forecast error is

9) The value $\sqrt{b/b^*}$ is also known as estimator of β from the geometric regression model $f_t = \beta r_t + \epsilon_t$; see Frisch (1929).

minus the disturbance. For several reasons the systematic part is in general not perfectly forecasted, so that one has $r = f - (\text{forecast error of the systematic part}) + \text{disturbance}$. From such considerations and from the argument that forecasts are available before the realizations, some conclude that writing $r = f + u$, with u minus the forecast error, is a sensible thing to do, from which they go on to consider regressions with r as dependent variable.¹⁰⁾

In such a set-up testing the forecasts for bias may be done by testing whether, in $r_t = \beta^* f_t + \varepsilon_t^*$, the hypothesis $\beta^* = 1$ may be accepted. The least-squares estimate b^* of β^* is:

$$(24) \quad b^* = \frac{\sum f_t r_t}{\sum f_t^2},$$

where in general $1/b^* \neq b$ and $1/b^* > b$, if $b > 0$. The correlation coefficient c is the same for the set-up with f and the set-up with r as "dependent" variable. The overall measure, constructed from the distance of the points (f_t, r_t) with respect to the line $r = f$, is again $\sum (f_t - r_t)^2 = d^2$. A decomposition of d^2 is obtained from (18), by interchanging the role of r_t and f_t :

$$(25) \quad d^2 = (b^* - 1)^2 \sum f_t^2 + \sum e_t^{*2},$$

where $e_t^* = r_t - b^* f_t$. For U^2 the following decomposition follows from (25):

$$(26) \quad U^2 = (b^* - 1)^2 \tilde{b}^2 + (1 - c^2).$$

(Note that for isolating $(b^* - 1)^2$ one would have to divide d^2 in (25) by $\sum f_t^2$, which is rather undesirable.)

If one cannot make a definite choice for one of the set-ups¹¹⁾, then an obvious measure for systematic forecast bias is the geometric average $\sqrt{b/b^*} = \tilde{b}$. The measure of dispersion around the least squares line is $\sqrt{1 - c^2}$, in both set-ups. The overall measure is also the same (d^2) in both

10) Mincer and Zarnowitz (1969) define the forecast error as $r - f$. Theil (1966) calls it a matter of choice whether one takes r or f as "dependent" variable. He chooses r .

11) Van de Panne (1959), for example, calculates both b and $1/b^*$.

set-ups. For this case of uncommitment to a particular set-up there is no natural decomposition.

The most convincing case for the choice of starting point for the least squares analysis is for the set-up which considers f_t as "dependent" variable. There remains to investigate the possibility of including a constant term in the "regression" model, which then would read: $f_t = \alpha + \beta_c r_t + u_t$; in "estimated" form: $f_t = a + b_c r_t + \hat{u}_t$. In such a set-up there is no forecast bias if $\alpha = 0$ and $\beta_c = 1$, which might be investigated by an F-statistic.¹²⁾

An argument pleading for the inclusion of a constant term, is the fact that a better description of the scatter is provided by a least squares line that includes a constant. This follows directly from the fact that the position of the scatter in the rf -plane is succinctly described by the point $(\bar{r}, \bar{f})$, the slope and the dispersion of the scatter. Figure 3 illustrates, rather extremely, the possibility that a least squares line without constant term, gives an inadequate description of the scatter. If no constant term is included, then the least squares slope does not give the

FIGURE 3

12) See e.g. Mincer and Zarnowitz (1969).

direction of the scatter. With respect to the extreme case of Figure 3, in which f and r show extreme negative correlation, it should be remarked that on the average there is neither under- nor overestimation, so that the value $b = 1$ provides the correct answer in this respect.

Unfortunately, the pair (a, b_c) has not the natural interpretation of b of (16). Stated in geometrical terms, this is due to the fact that only lines through the origin can be easily compared to the line of perfect fit. The description of the scatter's position in the plane in terms of the two coefficients a and b_c is, of course, superior to the description by means of the one coefficient b . However, the primary objective is not this description. If one still prefers this set-up with a constant term, one may consult Appendix A for decompositions of d^2 .

2.4. Starting from a Loss Function

In the previous two subsections descriptive statistics for measuring the accuracy of forecasts have been developed, firstly, starting from the comparison of the two vectors R and F in $\mathbb{R}^n$, and secondly, starting from the n points (f_t, r_t) in $\mathbb{R}^2$. In the present section a more fundamental starting point will be chosen, by considering the loss a decisionmaker attaches to an erroneous forecast.

Suppose then that for period t a loss function ℓ_t , defined on the forecast error $v_t = f_t - r_t$, can be formulated¹³⁾:

$$(27) \quad \ell_t = \ell_t(v_t | v_{t-1}, v_{t-2}, \dots),$$

which evaluates the loss a decisionmaker experiences, due to an imperfect forecast of r_t , given the knowledge of previous forecast errors. It is assumed that $\ell_t = 0$ if and only if $v_t = 0$; otherwise ℓ_t is positive. Instead of the arguments v_t , some decisionmakers might prefer the use of the arguments $v_t' = (f_t - r_t)/r_t$, the relative forecast errors.

As to the specification of (27), it will be assumed that ℓ_t is of the form $\ell_{1t}(v_t) + \ell_{2t}(v_t, v_{t-1}, \dots)$, where ℓ_{2t} takes care of possible interaction in the evaluation of forecast errors. Well-known examples for ℓ_{1t} are $c_{1t} v_t^2$ and $c_{1t}' |v_t|$. For ℓ_{2t} one might think of $c_{2t} v_{t-1}^2 v_t^2$ and

13) See also Theil (1964) and (1966) and Granger (1969) for the concept of a loss function in the forecast errors.

$c'_{2t}|v_{t-1}||v_t|$. For the sequel the following quadratic specification of ℓ_t will be used:

$$(28) \quad \ell_t(v_t|v_{t-1}) = c_{1t} v_t^2 + c_{2t} v_{t-1}^2 v_t^2,$$

where c_{1t}, c_{2t} are positive and where it is assumed that $c_{1t} + c_{2t} = 1$. One may thus see (28) as a weighted sum of v_t^2 and the interaction quantity $v_{t-1}^2 v_t^2$.

An interesting parameter is the (unconditional) expected loss, $E(\ell_t)$, to which the forecast f_t gives rise. For the example in (28) one has

$$(29) \quad E(\ell_t) = c_{1t} E(v^2) + c_{2t} E[(v_{-1} v)^2],$$

where it is assumed that the v_i ($i=1,2,\dots$) are observations from a wide stationary stochastic process of at least order 4. If c_{2t} is "very small", then $E(\ell_t) \approx E(v^2)$. Also, if $E[(v_{-1} v)^2]$ is "very small", then $E(\ell_t) \approx c_{1t} E(v^2)$. So measuring the expected loss by means of d^2/n , the mean square forecast error, which unbiasedly estimates $E(v^2)$, is a sensible thing to do, if c_{2t} or $E[(v_{-1} v)^2]$ are very small and c_{1t} is independent of t .

Assuming that the loss function used is of the type specified in (28), it is interesting to calculate estimates of $E[(v_{-1} v)^2]$, which may be, unbiasedly, done by means of:

$$(30) \quad \frac{1}{n-1} \sum_{t=i+1}^n (f_t - r_t)^2 (f_{t-i} - r_{t-i})^2,$$

where it is assumed, that the v_i ($i=1,2,\dots$) are observations from a wide stationary proces of appropriate order. Standardization of $E[(v_{-1} v)^2]$, of course, takes place by division through $E(v^4)$, which results in a measure that assumes values between 0 and 1. Standardization of (30) may thus be done as follows:

$$(31) \quad \frac{\frac{1}{n-1} \sum_{t=i+1}^n (f_t - r_t)^2 (f_{t-i} - r_{t-i})^2}{\frac{1}{n} \sum_{t=1}^n (f_t - r_t)^4} \equiv \hat{\theta}.$$

In the above the stochastic process of the forecasting errors has been mentioned. An interesting aspect of the v_t -process is its serial correlation. The correlation coefficient between v_t and v_{t-i} , say γ_i , may be consistently estimated as follows:

$$(32) \quad \hat{\gamma}_i = \frac{\sum_{t=i+1}^n (v_t - \bar{v})(v_{t-i} - \bar{v})}{\sum_{t=1}^n (v_t - \bar{v})^2} .$$

If there seems to be negligible serial correlation, then, assuming normality of the v_t , one may test unbiasedness of the forecasts, i.e. $E(v) = 0$, by means of the usual t-ratio:

$$(33) \quad t = \frac{\bar{v} \sqrt{n}}{\sqrt{\sum (v_t - \bar{v})^2 / (n-1)}} .$$

If one concludes that $E(v) = 0$, then the mean square forecast error unbiasedly estimates the variance of the forecast errors.

A somewhat more general approach to the question of serial correlation and unbiasedness is obtained, if one assumes, that the v_t -process may be modelled by means of an ARMA-scheme. If one concludes, for example, that $v_t = 0,25 v_{t-1} + \epsilon_t$, where ϵ_t is white noise, then the forecasts are unbiased, as there is a constant term of zero in this ARMA-scheme, and the forecast errors are serially correlated.

A special case of a quadratic loss function occurs, if there is no interaction with previous forecast errors:

$$(34) \quad \ell_t(v_t) = c_t v_t^2 ,$$

where, if c_t is a constant, it stands to reason to take $c = 1$. If $c_t = 1$, the unbiased estimator of $E(\ell)$ is $d^2/n = (1/n) \sum v_t^2$. This second moment may be decomposed as follows:

$$(35) \quad \frac{d^2}{n} = \bar{v}^2 + s_v^2 ,$$

where $s_v^2 = \sum (v_t - \bar{v})^2 / n$. Decomposing $s_v^2 = s_{f-r}^2$ further, and using c' to

denote the (central) correlation coefficient of the f_t and r_t , one gets:

$$(36) \quad \frac{d^2}{n} = (\bar{f} - \bar{r})^2 + (s_f - s_r)^2 + 2s_f s_r (1 - c') \quad ,$$

which is a well-known decomposition used by Theil [(1961) and (1966)], with relative components denoted by Theil as U^M , U^S and U^C , respectively. In (36) characteristics of the empirical distribution (joint and marginal) of the f_t and r_t are used for comparison. Decomposition (35) seems adequate enough: the first component is due to forecast error bias and the second component to the dispersion of the forecast error around the bias $\bar{v}$. In what way this dispersion may be decomposed, is not particularly interesting. The relative components of (35) are denoted by $d_m^2 (= U^M)$ and $d_v^2 (= U^S + U^C)$, respectively.

Another special case of (27) is the loss function without interaction, that penalizes the absolute values of the forecast errors. This case is treated in some detail in Appendix B, along with the loss function (34) for v_t' , the relative forecast error.

3. Application to Forecasts of the Dutch CPB, 1964-1978

The methods developed in Section 2 will now be applied to forecasts of the Dutch Central Planning Bureau (CPB) for the years 1964 up to and including 1978. From the yearly Central Economic Plans forecasts and realizations for twenty-eight variables have been taken. In Table 1 these variables, together with the dimension of the forecast (and realization), are stated. Also stated are the number of forecasts and the standard deviations of realizations as a first indication of forecast difficulty. These standard deviations have been calculated over the years for which there are forecasts. Some variables occurred only in a very small number of years and were therefore not included in the list of variables.

In Appendix C the forecasts, realizations and forecast errors for the variables and years are listed, the symbol "N" denoting that no forecast has been made or, in a few cases, that the forecast is of a dimension which differs from that of most years. Variables which the CPB considered exogenous in a certain year, have the symbol "E" after their forecast value.

TABLE 1: Variables with dimension, number of forecasts and standard deviation

Variable No.	Name	Dimension	Number of forecasts	Standard deviation
1.	World trade	Percentage change of volume	15	4.5
2.	Price imports	Percentage change of price	15	8.7
3.	Competitive price abroad	Percentage change of price	13	5.1
4.	Wages per unit industrial output abroad	Percentage change of price	9	5.2
5.	Industrial wage level	Percentage change of price	15	2.6
6.	Residential construction	Percentage change of volume	14	8.0
7.	Labour time reduction	Percentage change of time	8	0.8
8.	Liquid funds percentage	Ultimo percentage of net national income	13	2.0*
9.	Public consumption	Percentage change of value	11	5.6
10.	Public investment	Percentage change of value	11	8.5
11.	Private consumption	Percentage change of volume	15	1.8
12.	Private investment	Percentage change of volume	15	8.1
13.	Commodity exports	Percentage change of volume	15	6.2
14.	Commodity imports	Percentage change of volume	15	5.8
15.	Gross national product	Percentage change of volume	14	2.4
16.	Industrial production	Percentage change of volume	15	2.6
17.	Price private consumption	Percentage change of price	15	2.6
18.	Price commodity exports	Percentage change of price	15	6.9
19.	Labour productivity industry	Percentage change of volume	15	2.0
20.	Wages per unit industrial output	Percentage change of price	10	4.1
21.	Real disposable income	Percentage change of volume	8	1.0
22.	Current account balance of payments	Billions of guilders	15	3.0*
23.	Inventories	Billions of guilders	15	1.4*
24.	Tax pressure	Percentage of net national income	12	1.7*
25.	Labour income percentage	Percentage of net national income	15	2.3*
26.	Savings percentage	Percentage of net national income	15	2.3*
27.	Unemployment	Thousands of persons	15	63.6*
28.	Premium pressure	Percentage of net national income	9	1.6*

* The standard deviations with a star have for convenience been so denoted, as they are not comparable to the other ones.

The econometric models used by the CPB have as dimension for most of the variables percentage change with respect to the previous year; see also Table 1. For our forecast accuracy analysis the dimension of interest to the decisionmaker, i.e. the dimension of the published forecasts, will be used, which for 20 out of 28 variables means percentage change, for 5 variables percentage of national income, for 2 variables billions of guilders and for 1 variable thousands of persons.

Before presenting the results of the accuracy analysis for all twenty-eight variables, the application of the methods of Section 2 is illustrated for two percentage change variables: variable no. 1, World Trade, an exogenous variable with average forecast difficulty¹⁴⁾ as measured by its standard deviation of 4.5 and variable no. 12, Private Investment, an endogenous variable with high forecast difficulty (standard deviation 8.1). The forecasts, realizations, and forecast errors for these two variables may be found in Appendix C. In Figure 4.a. the pairs (r_t, f_t) are represented in a scatter diagram, together with the line $f = br$ of Section 2.3 and the line of perfect forecast. Figure 4.b. shows the forecast errors. In Table 2 the values of selected statistics for the three types of analysis, to be found in Sections 2.2, 2.3 and 2.4, are shown. The geometrical and the least squares analysis are complementary to each other. The analysis of the forecast errors v_t is of a different character: it does not analyse the relation of the vector F with respect to the vector R , but concentrates on the univariate error process v_t . An analysis of the relative forecast errors $v'_t = (f_t - r_t)/r_t$, instead of v_t , is more or less of the same type as the least squares analysis; see Appendix B. For this reason this analysis is omitted.

The values of $U_0 = U$ in Table 2 indicate that both variables were forecasted successfully in comparison to the no-change forecasts. The values of U_1 show that, in comparison to the somewhat more sophisticated "forecasts" of the simple autoregressive scheme $r_t = ar_{t-1}$, the performance of

14) For percentage change variables the standard deviation of r is a reasonable first indication of forecast difficulty, as percentage variables do not suffer much from the presence of trend, which contributes to the standard deviation, but might be easy to forecast. A better general measure is, of course, U_1 , and to a lesser degree U_0 .

FIGURE 4.a.

FIGURE 4.b.

TABLE 2: Values of statistics for the variables World Trade and Private Investment

	Formule number	World Trade	Private Investment
Geometrical analysis	U_0	(2)	0.41
	U_1	(15)	0.64
	U_s	(A.5) in Appendix A	0.85
	c	(6)	0.94
	$\ F\ /\ R\ $	(7)	0.73
Least squares analysis	b	(16)	0.68
	d_b^2	(20)	0.61
	d_e^2	(20)	0.39
	t_b	t-statistic for $H_0: \beta = 1$	- 4.68
	$\sqrt{1-c^2}$	(17)	0.35
	$1/b^*$	(24)	0.78
Forecast error analysis	d^2/n	(1)	14.48
	$\bar{v}$	mean $v_t = f_t - r_t$	- 1.89
	d_m^2	(35)	0.25
	d_v^2	(35)	0.75
	t_v	(33)	- 2.15
	$\hat{\gamma}_1$	(32)	- 0.24
	a	normality test statistic	0.78
	$\hat{\theta}$	(31)	0.36

World Trade deteriorates much more than that of Private Investment¹⁵⁾.
The value of U_s , which is the root mean square forecast error normalized

15) The constant α has been "estimated" by ordinary least squares:

$$\hat{\alpha} = \left(\sum_{t=2}^n r_t r_{t-1} \right) / \sum_{t=2}^n r_{t-1}^2$$
 The forecast for period t then is $\hat{\alpha} r_{t-1}$.
The CPB had, of course, no knowledge of this, kind of average, value over the period. The alternative of using only realizations known at the time of forecast, adapts the value of α at each forecasting point, and was considered to complex to be used in an inaccuracy measure.
These autoregressive "forecasts" require the values of r_0 , the 1963-realizations. If there is a time gap in the forecasts over the period 1964-1978, then the realization directly previous to the gap is also provided in Appendix C. These realizations are given between brackets: (...).

by means of the standard deviation of realizations, is again much higher than that of U_1 for World Trade, and remains at about the same level for Private Investment. Although Private Investment is more difficult to forecast, as witnessed by the standard deviation of r , its normalized performance is somewhat better than that of World Trade. For World Trade c is close to 1, and so the angle between the vectors R and F is small. For Private Investment c is not close to 1. The quotient $\|F\|/\|R\|$ indicates underestimation, which is approximately 30 percent for both variables.

$\|F\|/\|R\| = \tilde{b}$, the slope of geometric regression, is always greater than b , for positive b . The differences in the values of $\|F\|/\|R\|$ and b are due to different values of b and $1/b^*$. For World Trade $\|F\|/\|R\|$ and b do not differ much, but for Private Investment the large difference between b and $1/b^*$ leads to an assessment of underestimation by means of $b-1$ in the order of 45 percent. For the assessment of underestimation $b-1$ will be used (and not $\|F\|/\|R\| - 1$) and will be rounded to the nearest higher multiple of 5 percent. The relative bias component, due to $b \neq 1$, in d^2 is $d_b^2 = 0.61$ and 0.56 , respectively, and the relative dispersion component, due to $\sum e_t^2 \neq 0$, is $d_e^2 = 1 - d_b^2 = 0.39$ and 0.44 , respectively. The value of $t_b = (b-1)/s_b$, with s_b the square root of $(\sum e_t^2)/(n-1)\sum r_t^2$, may be used as an indication for the significance of under- or overestimation. For both variables the underestimation seems significant. The dispersion of the points (r_t, f_t) around the line $f=br$ is measured by $\sqrt{1-c^2}$, which is 0.35 and 0.60 , respectively.

From the forecast error analysis it is seen, that the mean square errors are 14.5 and 34.3 , respectively; see, however, the values of U_s . Underestimation, in the units of f and r (percentage points) is 1.9 and 1.5 , respectively, with only 1.9 significantly different from zero. (No appreciable first order autocorrelation seems present and both error processes seem to be normal¹⁶⁾.) The relative bias components, due to $\bar{v} \neq 0$, in d^2 are $d_m^2 = 0.25$ and 0.06 , respectively, and the corresponding relative dispersion components, due to $s_v^2 \neq 0$, are $d_v^2 = 1 - d_m^2 = 0.75$ and 0.94 . The

16) As the number of observations is small, the statistic used for detecting non-normality is $a = \text{mean deviation} / \text{standard deviation} = (\sum |v_t - \bar{v}|) / \sqrt{n \sum (v_t - \bar{v})^2}$; see Biometrika Tables for Statisticians, Vol I, pp. 68 and 207. For $n = 11$ the mean and standard deviation of this statistic are 0.818 and 0.058 , respectively, and for $n = 16$ the corresponding values are 0.811 and 0.050 , for a normal population.

value of $\hat{\theta}$ for Private Investment indicates that a quadratic loss function of the type in (28) should be one without interaction in time (i.e. with $c_{2t} = 0$), for the above analysis, based only on d^2 , to be valid.

For the rest of the analysis one need not employ all the above used measures. The measure c provides the same information as $\sqrt{1-c^2}$, so that one may follow one's preference. In the following $\sqrt{1-c^2}$ will be used, because most values of c in Table 3 are so close to 1, that the differences between the values of $\sqrt{1-c^2}$ are much larger than the differences between the values of c . The measure $\|F\|/\|R\|$ is a less natural choice for measuring relative forecast bias than the measure b , together with the "statistic" t_b . For the decomposition in relative components the value of d_b^2 is sufficient. As to the analysis of the forecast errors v_t , the measures d_v^2 and $\hat{\theta}$ will not be presented, d_v^2 not because $d_m^2 + d_v^2 = 1$, and $\hat{\theta}$ not because it will be assumed, that the simplest quadratic loss function is relevant.

In Table 3 the values of the just chosen statistics are presented for all twenty-eight variables. First the results for the ten variables under the heading "Assumptions" will be discussed. These are variables which in almost all years have been treated as exogenous variables: of 124 forecasts for these ten variables only 9 are for endogenous variables; see Appendix C. The overall impression, as obtained from the values of U_0 , U_1 and U_s , is not unfavorable, although four out of ten have a U_1 -value close to 1 and thus do not compare favorably to the simple autoregressive scheme. The important variable number 5, Industrial Wage Level, might have been predicted almost as well by this simple scheme. The ranking of the variables 1 to 10 with respect to forecasting performance on the basis of U_0 is rather different than this ranking done on the basis of either U_1 or U_s , whereas U_1 and U_s induce approximately the same ranking. Underestimation is predominant and ranges from zero for Labour Time Reduction to fifty-five percent for Residential Construction. The proportion in d^2 due to $b \neq 1$ ranges from 0.0 to 0.9, where it must be noted that the more important variables 1 to 6 have half or more of their d^2 , or their estimated expected loss, d^2/n , due to $b \neq 1$. Dispersion around the line $f=br$ is rather low, except for Residential Construction and Public Investment; see also the graphs in Appendix D for variables 1 to 6. There one may see a clear difference in approach for the scatter analysis and the error analysis illustrated by variable 2, Price Imports: the extreme pair (34.9, 35.0) is very close to the line $f=br=0.69r$, as $25.0/34.9 = 0.72$, so that it hardly influences the

TABLE 3: Accuracy analysis of twenty-eight variables from Central Economic Plans, 1964-1978

Assumptions	Geometrical and least squares analysis							Error analysis					
	U_0	U_1	U_s	b	d_b^2	$\sqrt{1-c^2}$	t_b	$\frac{d^2}{n}$	$\bar{v}$	d_m^2	t_v	$\hat{\gamma}_1$	a
1. World trade	0.41	0.64	0.85	0.68	0.61	0.35	-4.68	14.48	-1.89	0.25	-2.15	-0.24	0.78
2. Price imports	0.33	0.35	0.37	0.69	0.88	0.16	-9.92	10.23	-1.29	0.16	-1.64	0.27	0.73
3. Competitive price abroad	0.48	0.59	0.60	0.55	0.89	0.28	-9.75	9.31	-1.39	0.21	-1.76	0.31	0.72
4. Wages per unit industrial output abroad	0.51	0.62	0.76	0.55	0.80	0.39	-5.56	15.58	-1.85	0.22	-1.50	-0.12	0.77
5. Industrial wage level	0.20	0.91	0.92	0.86	0.49	0.16	-3.64	5.81	-1.69	0.49	-3.69	0.33	0.79
6. Residential construction	0.77	0.93	0.89	0.44	0.54	0.77	-3.92	50.88	-3.21	0.20	-1.82	0.04	0.76
7. Labour time reduction	0.30	0.55	0.57	1.00	0.00	0.29	0.00	0.22	0.19	0.16	1.16	-0.18	0.84
8*. Liquid funds percentage	0.91	0.91	1.00	1.02	0.11	0.05	1.24	3.84	0.70	0.13	1.32	-0.06	0.81
9. Public consumption	0.26	0.64	0.59	0.92	0.09	0.26	-1.01	10.61	-1.11	0.12	-1.15	-0.32	0.85
10. Public investment	0.73	0.96	1.14	0.64	0.24	0.71	-1.80	93.02	0.77	0.01	0.25	0.11	0.80
Results													
11. Private consumption	0.36	1.00	1.05	0.76	0.45	0.33	-3.40	3.42	-0.83	0.20	-1.89	0.27	0.86
12. Private investment	0.62	0.62	0.73	0.54	0.56	0.60	-4.25	34.27	-1.47	0.06	-0.97	-0.21	0.83
13. Commodity exports	0.44	0.67	0.76	0.66	0.60	0.39	-4.62	22.21	-1.61	0.12	-1.36	0.16	0.87
14. Commodity imports	0.45	0.63	0.74	0.65	0.64	0.38	-4.96	17.99	-1.73	0.17	-1.67	0.14	0.83
15. Gross national product	0.43	0.70	0.82	0.76	0.31	0.42	-2.43	3.98	-0.25	0.02	-0.46	0.03	0.79
16. Industrial production	0.38	0.66	0.80	0.73	0.49	0.35	-3.69	4.32	-0.75	0.13	-1.44	0.08	0.82
17. Price private consumption	0.17	0.57	0.48	0.91	0.25	0.16	-2.16	1.49	-0.45	0.14	-1.48	0.12	0.81
18. Price commodity exports	0.39	0.42	0.44	0.66	0.79	0.26	-7.25	9.36	-0.75	0.06	-0.94	0.13	0.62
19. Labour productivity ind.	0.40	0.70	1.00	0.71	0.54	0.35	-4.02	4.00	-0.88	0.19	-1.83	0.07	0.81
20. Wages per unit ind. outp.	0.33	0.49	0.61	0.73	0.64	0.26	-3.98	6.25	-1.70	0.46	-2.78	-0.31	0.82
21. Real disposable income	0.38	0.41	1.01	0.89	0.08	0.38	-0.78	0.93	-0.33	0.11	-0.95	0.06	0.86
22*. Current account balance of payments	0.91	0.98	0.77	0.63	0.29	0.68	-2.38	5.47	0.19	0.01	0.31	0.52	0.71
23*. Inventories	0.48	0.51	0.66	0.85	0.15	0.39	-1.54	0.87	0.08	0.01	0.31	-0.36	0.78
24*. Tax pressure	0.88	1.05	0.43	1.00	0.03	0.03	-0.59	0.56	-0.12	0.02	-0.52	0.03	0.74
25*. Labour income percentage	1.10	1.10	0.75	0.99	0.12	0.01	-1.38	3.05	-0.60	0.12	-1.37	-0.07	0.81
26*. Savings percentage	0.79	0.79	0.58	1.00	0.00	0.07	-0.10	1.84	0.06	0.00	0.17	0.11	0.89
27*. Unemployment	0.30	0.33	0.11	1.00	0.00	0.06	0.25	50.32	-0.63	0.01	-0.34	0.28	0.89
28*. Premium pressure	0.75	0.95	0.37	1.02	0.29	0.03	1.79	0.33	0.32	0.32	1.93	-0.02	0.67

* The dimension of variables numbers 8 and 22 up to 28 included may be found in Table 1. All other variables are measured as percentage changes with respect to the previous year.

position of the line, whereas the forecast error $25.0 - 34.9 = -9.9$ has a strong influence on $\bar{v} = 1.29$. Underestimation, as measured by $\bar{v}$, is not as significant as underestimation as measured by $b-1$, which is witnessed by the values of t_v as compared with the values of t_b . The forecast error may be described by a normal process independent in time, which is born out by the values of a and $\hat{\gamma}_1$. Except for Industrial Wage Level, the proportion in d^2 due to $\bar{v} \neq 0$ is low.

In comparing the results for these first ten variables one should note the presence of variable 8, which is the only one among the first ten variables which has not the dimension "percentage change with respect to the previous year". This means that U_0 is not equal to U for variable 8. More important is, that d^2/n and $\bar{v}$ for variable 8 cannot be compared to the respective values for the other variables. As all other statistics are dimensionless, the analysis is hardly hampered by the presence of variables with different dimensions.

Under the heading "Results" in Table 3 the accuracy analysis for eighteen endogenous variables is presented, the first ten of which are in percentage change form. For the important variables 11 to 15, 17, 22 and 27 one may also consult the graphs in Appendix D. Only for these variables the results of the accuracy analysis will be discussed.

First the results of variable 11, (Private) Consumption, and variable 12, (Private) Investment will be compared. Both outperform the no-change forecasts, with Consumption doing best. However, Consumption is more easily forecasted: it has the (much) smaller standard deviation and the simple AR-scheme is not beaten. Investment seems significantly better forecasted than the AR-scheme. The same relative performance is shown by U_g , the standardized value of the root-mean-square-error. Both variables are significantly underestimated: by 25 percent for Consumption and by 45 percent for Investment. Dispersion around the line $f = br$ is of course sizeable for Investment. For both variables approximately 50 percent of the value of d^2 is due to $b \neq 1$. (Somewhat more precise: 45 percent and 55 percent, respectively.) In terms of percentage change points, the units of both variables, underestimation is about 0.8 (significantly) for Consumption and about 1.5 (insignificantly) for Investment. The variance of the forecast errors is the dominant source (80 and 95 percent) of the estimated loss, d^2/n .

Variables 13 and 14, (Commodity) Exports and Imports, show almost the same results. Both have been forecasted considerably better than the no-

change-forecasts and reasonably better than the autoregressive forecasts. The forecasts underestimate reality by a significant 35 percent, leading to a bias component of d_b^2 of about 60 percent. Underestimation in the units of Exports is 1.6 percentage change points (1.7 for Imports, both hardly significant or insignificant). The forecast error variance is again the main source of d^2/n (about 85 percent).

Variable 15, Gross National Product (growth), of course has a performance that depends on that of the previous variables: considerably better than the no-change-forecasts, and reasonably better than the autoregressive forecasts. Underestimation is a significant 25 percent, resulting in a relatively small bias component of d_b^2 of 30 percent of d^2 . Underestimation in terms of percentage change points is an insignificant 0.25 points.

Inflation, as measured by variable 17, Price Private Consumption, has been forecasted very successfully as compared to the no-change forecasts and successfully with respect to the autoregressive forecasts. Percentage underestimation is small: a (significant) 10 percent, leading to a small bias component in d^2 . In percentage change points, the units of Price Private Consumption, underestimation is an insignificant 0.45.

Variable 22, Current Account (Balance of Payments), has billions of guilders as unit of measurement, which should be remembered in interpreting d^2/n and $\bar{v}$. The no-change and the autoregressive forecasts are nearly as good as the actual forecasts¹⁷⁾. Current Account is difficult to forecast. The performance relative to its standard deviation is as good as Investment, Exports, Imports and Gross National Product. (All have a U_g -value of approximately 0.75.) Percentage underestimation is serious and significant: 35 percent. As dispersion around the line $f=br$ is large (see also Appendix D), this underestimation does not lead to a large bias component: $d_b^2 \approx 0.3$. Underestimation in terms of $\bar{v}$ is small and probably insignificant (there is some autocorrelation in the v_t).

The last endogenous variable to be discussed is variable 27, Unemployment. It has been very successfully forecasted with respect to the no-change and autoregressive forecasts. The performance relative to the stan-

17) The no-change forecasts $f_t^{(0)}$ for a variable not having the dimension of percentage change, are $f_t^{(0)} = r_{t-1}$, so that $U_0 = d^2 / \sum_{t=1}^n (r_{t-1} - r_t)^2$. A time gap in the forecasting period is treated as $t=1$, i.e. by providing the realization directly previous to the gap; see in Appendix C the values between brackets.

dard deviation is the best of all variables. Percentage underestimation is nil (on the average) with a very small dispersion around $f=r$. Underestimation in terms of $\bar{v}$ is small and insignificant.

4. Conclusion

The summary measures U_0 , U_1 and U_s , which are all normalized root mean square forecast errors, are very useful. They are dimensionless and their use does not need the questionable trick of converting all variables to percentage change variables. Percentage under- or overestimation and its significance are best measured by the pair (b, t_b) . The relative contribution of (percentage or absolute) under- or overestimation to the mean square forecast error (or to U_0^2 , U_1^2 and U_s^2) is measured by d_b^2 or d_m^2 . Absolute under- or overestimation is measured by the pair $(\bar{v}, t_v)$.

The main findings of the accuracy analysis of the CPB forecasts for the principal endogenous variables are presented in rounded form in Table 4. Underestimation as measured by the average forecast error, $\bar{v}$, is generally

TABLE 4: Main results accuracy analysis for principal endogenous variables

	U_0	U_1	U_s	$b-1$	d_b^2	t_b
Private consumption	0.35	1.00	1.05	-0.25	0.45	-3.40
Private investment	0.60	0.60	0.75	-0.45	0.55	-4.25
Commodity exports	0.45	0.65	0.75	-0.35	0.60	-4.60
Commodity imports	0.45	0.65	0.75	-0.35	0.65	-4.95
Gross national product	0.45	0.70	0.80	-0.25	0.30	-2.45
Price private consumption	0.15	0.55	0.50	-0.10	0.25	-2.15
Current account balance of payments	0.90	1.00	0.75	-0.35	0.30	-2.40
Unemployment	0.30	0.35	0.10	0.00	0.00	0.25

insignificant and is therefore not stated. For the first six variables comparable values for U_0 are available for the period 1953 to 1963 from Van den Beld (1965); they are 0.38, 0.78, 0.56, 0.42, 0.45 and 0.44. So the principal variable, Gross National Product, was forecasted with exactly the same success, at least according to U_0 . The other U_0 -values for

1964-1978 are about the same (for Private Consumption, Commodity Exports and Imports) or lower (Private Investment, and especially Price Private Consumption), as compared to the corresponding values for 1953-1963. Percentage underestimation for the first six variables for 1949-1956 may be found in Van de Panne (1959)¹⁸: 0.40, 0.55, 0.25., 0.35, 0.35 and 0.30. Evidently percentage underestimation has become somewhat lower, about 0.10, at least compared to the period 1949-1956.

One other comment on Table 4: the results show clearly that the value of percentage underestimation, $b-1$, is only one determinant of d_b^2 , the relative bias component of d^2 ; the two other determinants are Σr_t^2 and Σe_t^2 , see (18). Detailed comments on the results of the accuracy analysis for the variables of Table 4 have been given in Section 3. The reader may also consult that section for the other endogenous and the exogenous variables.

18) Van de Panne (1959) uses weighted least squares in order to calculate b of $f = br$.

APPENDIX A

For the set-up $f_t = \alpha + \beta_c r_t + u_t$, with "estimated" form $f_t = \alpha + b_c r_t + \hat{u}_t$, the following decomposition of d^2 may be obtained:

$$(A.1) \quad d^2 = n(\bar{f} - \bar{r})^2 + (b_c - 1)^2 \Sigma (r_t - \bar{r})^2 + \Sigma \hat{u}_t^2.$$

Note that all three elements necessary for a compact description of the scatter's position are present: the center point $(\bar{r}, \bar{f})$, the direction (b_c) of the scatter and the scatter's dispersion $(\Sigma \hat{u}_t^2)$. Choosing the set-up with r as dependent variable, which is mostly done, i.e. $r_t = \alpha^* + \beta_c^* f_t + u_t^*$, with $r_t = a^* + b_c^* f_t + \hat{u}_t^*$ as least-squares fit, the decomposition of d^2 is:

$$(A.2) \quad d^2 = n(\bar{f} - \bar{r})^2 + (b_c^* - 1)^2 \Sigma (f_t - \bar{f})^2 + \Sigma \hat{u}_t^{*2}.$$

Both in (A.1) and (A.2) the dispersion component may be written in terms of the (central) correlation coefficient, say c' :

$$(A.3) \quad \Sigma \hat{u}_t^2 = (1 - c'^2) \Sigma (f_t - \bar{f})^2$$

$$(A.4) \quad \Sigma \hat{u}_t^{*2} = (1 - c'^2) \Sigma (r_t - \bar{r})^2.$$

The relative values of the three components in (A.2) have been named by Theil (1961) bias, regression and disturbance proportion, respectively, denoted by U^M , U^R and U^D , respectively. This same terminology could be applied to the relative components in (A.1).

In the set-up leading to (A.1) it seems natural to divide d^2 by $\Sigma (r_t - \bar{r})^2$, which is just another way of normalizing d^2 . The interpretation of $\Sigma (r_t - \bar{r})^2$ is interesting in two respects: (i) firstly it may be used to adapt the individual forecast errors for differences in the dispersion of the realizations of different variables¹⁹⁾; (ii) secondly²⁰⁾ it may be seen as the value of d^2 that a forecaster, who is able to forecast without bias, can minimally reach, viz. by using $f_t = \bar{r}$ as forecasts for all

19) See Van den Beld (1965) and footnote 5. Note, however, that Van den Beld uses the secondmoment and not the variance of realizations.

20) See also Theil (1966, p. 59).

$t = 1, \dots, n$. Both interpretations may be used to arrive at the following normalized d^2 :

$$(A.5) \quad U_s^2 = \frac{d^2}{\Sigma (r_t - \bar{r})^2} .$$

APPENDIX B

For the (time independent) quadratic loss function without interaction in time, that has the relative forecast error v_t' as argument, one has as estimator of the expected loss:

$$(B.1) \quad \frac{1}{n} \Sigma (v_t')^2 = \frac{1}{n} \Sigma \left(\frac{f_t}{r_t} - 1 \right)^2.$$

Decomposing this second moment, one has:

$$(B.2) \quad \frac{1}{n} \Sigma \left(\frac{f_t}{r_t} - 1 \right)^2 = \{ (\overline{f/r}) - 1 \}^2 + \frac{1}{n} \Sigma \left\{ \frac{f_t}{r_t} - (\overline{f/r}) \right\}^2$$

The first component again measures bias and may be compared to $(b-1)^2$ of Section 2.3, as b measures the same as $(\overline{f/r})$. The second component measures the dispersion of the f_t/r_t (or the $(f_t/r_t)-1$). Another well-known loss function penalises in terms of absolute values and thus penalises less severely large errors than does the quadratic loss function. For the (time independent) loss function without interaction in time one has as estimators of the expected loss, for v_t as argument:

$$(B.3) \quad \frac{1}{n} \Sigma |f_t - r_t|,$$

and for v_t' as argument:

$$(B.4) \quad \frac{1}{n} \Sigma \left| \frac{f_t}{r_t} - 1 \right|.$$

APPENDIX C

Forecasts, realizations and forecast errors for the variables of Table 1

	1. World trade			2. Price imports			3. Competitive price abroad			4. Wages per unit industrial output abroad [†]		
	Fore-cast	Realization	Error	Fore-cast	Realization	Error	Fore-cast	Realization	Error	Fore-cast	Realization	Error
(1963)		(8.1)			(1.3)							
1964	9.0 E	10.7	- 1.7	1.5 E	2.3	- 0.8	N			N		
1965	7.0 E	9.4	- 2.4	1.0 E	0.5	+ 0.5	N			N		
1966	6.0 E	6.7	- 0.7	0.0 E	0.7	- 0.7	2.0 E			N		
1967	5.0 E	3.0	+ 2.0	0.5 E	-0.8	+ 1.3	1.0 E			N		
1968	5.0 E	13.7	- 8.7	-1.0 E	-2.9	+ 1.9	-1.0 E			N		
1969	9.0 E	15.1	- 6.1	1.0 E	3.4	- 2.4	1.0 E			N		
1970	10.0 E	10.3	- 0.3	4.0 E	6.5	- 2.5	3.5 E			2.5 E		- 0.5
1971	6.0 E	7.0	- 1.0	1.0 E	4.4	- 3.4	1.5 E			5.5 E		- 5.0
1972	6.0 E	9.5	- 3.5	-0.5 E	-0.7	+ 0.2	0.0 E			N	(0.6*)	
1973	10.0 E	11.3	- 1.3	3.5 E	7.6	- 4.1	2.5 E			2.5 E	1.5	+ 1.0
1974	3.0 E	4.1	- 1.1	25.0 E	34.9	- 9.9	9.5 E			3.5 E	2.0	+ 1.5
1975	0.0 E	-1.4	+ 1.4	3.0 E	4.1	- 1.1	5.0 E			8.7 E	10.5	- 1.8
1976	6.0 E	14.0	- 8.0	4.0 E	6.4	- 1.4	5.0 E			7.0 E	17.0	-10.0
1977	7.0 E	4.0	+ 3.0	4.0 E	2.9	+ 1.1	3.0 E			3.0 E	2.0	+ 1.0
1978	5.0 E	5.0	0.0	0.0 E	-2.0	+ 2.0	0.5 E			2.0 E	2.0	0.0
										1.0 E	3.5	- 2.5

* This star denotes realizations according to Appendix C of Central Economic Plan 1979. Realizations not denoted by a star, have been taken from the last CEP in which they were recorded, which, except for the years 1977 and 1978, means the CEP of three years later than the corresponding forecast.

† For variable no. 4 there is the following dimension problem. In 1969, 1970 and 1971 the forecasts seem to have the dimension "percentage change of costs in dollars" and in all following years the forecast dimension is "percentage change of costs in guilders". For the years 1969 and 1970 this does not matter very much, as the exchange rate of guilders for dollars is almost constant over the years 1962 up to and included 1970. From 1971 on the exchange rate shows considerable yearly changes. For this reason the year 1971 has been excluded from the analysis.

	5. Industrial wage level			6. Residential construction			7. Labour time reduction			8. Liquid funds percentage		
	Fore-cast	Realization*	Error	Fore-cast	Realization*	Error	Fore-cast	Realization	Error	Fore-cast	Realization*	Error
(1963)		(9.0)										
1964	16.0 E	14.9	+ 1.1	N	(31.6)		N			40.0	(39.2)	+ 2.8
1965	9.0 E	11.1	- 2.1	9.0	11.8	- 2.8	N			N	37.2	
1966	9.5	11.0	- 1.5	7.0	7.0	0.0	N			N	(34.8)	
1967	8.0 E	8.8	- 0.8	3.0 E	12.1	- 9.1	N			36.0	35.1	+ 0.9
1968	5.0 E	8.9	- 3.9	-5.5 E	9.4	-14.9	N			37.5	36.2	+ 1.3
1969	8.5 E	13.4	- 4.9	2.0 E	- 3.0	+ 5.0	N	(1.2)		36.0 E	34.8	+ 1.2
1970	8.0 E	13.2	- 5.2	5.0 E	1.8	+ 3.2	2.0 E	2.0	0.0	36.5 E	34.6	+ 1.9
1971	12.0 E	13.1	- 1.1	6.5 E	8.7	- 2.2	1.5 E	1.5	0.0	34.0 E	33.5	+ 0.5
1972	12.0	12.6	- 0.6	-1.0 E	14.7	-15.7	N	(1.0)		35.0 E	32.6	+ 2.4
1973	13.5	15.6	- 2.1	-5.0 E	2.0	- 7.0	2.0 E	1.5	+ 0.5	32.0 E	34.3	- 2.3
1974	14.5 E	15.7	- 1.2	-6.0 E	-12.5	+ 6.5	2.0 E	2.0	0.0	35.5 E	38.7	- 3.2
1975	12.5 E	12.8	- 0.3	-9.5 E	- 7.5	- 2.0	2.0 E	2.5	- 0.5	38.0 E	36.0	+ 2.0
1976	8.7 E	10.9	- 2.2	0.0 E	2.5	- 2.5	1.5 E	0.5	+ 1.0	38.0 E	39.8	- 1.8
1977	7.5	7.8	- 0.3	12.5 E	15.9	- 3.4	0.5 E	0.0	+ 0.5	39.0 E	37.4	+ 1.6
1978	7.0 E	7.3	- 0.3	2.5 E	2.5	0.0	0.5 E	0.5	0.0	38.0 E	36.2	+ 1.8

	9. Public consumption			10. Public Investment			11. Private consumption			12. Private Investment		
	Fore- cast	Real- zation	Error	Fore- cast	Real- zation	Error	Fore- cast	Real- zation*	Error	Fore- cast	Real- zation*	Error
(1963)	4.0 E	10.5	- 6.5	8.5 E	26.0	-17.5	6.5	5.9	+ 0.6	13.0	(-2.1)	- 4.3
1964	7.0 E	4.5	+ 2.5	10.5 E	8.5	+ 2.0	5.0	7.5	- 0.8	9.0	4.0	+ 0.1
1966	6.5 E	7.2	- 0.7	15.0 E	7.0	+ 8.0	4.0	3.2	- 0.8	10.1	6.2	- 7.2
1967	5.5 E	8.5	- 3.0	5.0 E	16.5	-11.5	3.0	5.4	- 2.4	-1.0	10.1	- 1.1
1968	8.5 E	6.5	+ 2.0	11.0 E	16.5	- 5.5	3.5	6.6	- 3.1	0.0	10.5	-10.5
1969	N	N		N	N		4.0	7.9	- 3.9	6.0	-1.2	+ 7.2
1970	N	N		N	N		5.5	7.7	- 2.2	6.0	15.7	- 9.7
1971	N	N		N	N		5.0	3.0	+ 2.0	7.0	0.8	+ 6.2
1972	N	(7.0)		(-1.5)			4.0	3.2	+ 0.8	-6.0	-8.9	+ 2.9
1973	8.0 E	9.5	- 1.5	17.5 E	-0.5	+18.0	4.0	3.9	+ 0.1	8.5	10.7	- 2.2
1974	20.0 E	18.5	+ 1.5	16.0 E	5.5	+10.5	2.5	2.7	- 0.2	2.0	-0.3	+ 2.3
1975	22.5 E	24.5	- 2.0	18.0 E	22.0	- 4.0	3.0	3.4	+ 0.6	0.0	-7.5	+ 7.5
1976	18.0 E	14.5	+ 3.5	7.0 E	7.0	0.0	3.0	3.8	- 0.8	-6.5	-3.7	- 2.8
1977	7.0 E	10.0	- 3.0	4.5 E	0.5	+ 4.0	3.5	4.3	- 0.8	8.0	15.5	- 7.5
1978	4.5 E	9.5	- 5.0	6.5 E	2.0	+ 4.5	3.0	4.5	- 1.5	3.0	6.0	- 3.0

†† For the variables no. 9 and 10 there is also a dimension problem. In 1970, 1971 and 1972 "percentage change of volume" has been forecasted, whereas in the other years this has been "percentage change of value". The years 1970, 1971 and 1972 have therefore been excluded, as well as the year 1969, for which no realization is available which has the dimension percentage change of value. The realizations for 1967 and 1968 have been taken from the CEP 1969.

	13. Commodity exports			14. Commodity imports			15. Gross national product			16. Industrial production		
	Fore-cast	Realization*	Error	Fore-cast	Realization*	Error	Fore-cast	Realization	Error	Fore-cast	Realization*	Error
(1963)		(6.4)			(9.8)			(3.6)			(4.3)	
1964	6.0	12.8	- 6.8	12.0	15.6	- 3.6	5.0	8.5	- 3.5	6.0	9.9	- 3.9
1965	8.0	8.7	- 0.7	5.0	6.1	- 1.1	4.0	5.0	- 1.0	4.5	5.9	- 1.4
1966	10.0	6.4	+ 3.6	10.0	6.3	+ 3.7	5.5	2.0	+ 3.5	6.0	2.8	+ 3.2
1967	8.0	7.4	+ 0.6	3.0	6.4	- 3.4	N	(6.0)		4.0	6.0	- 2.0
1968	8.0	14.6	- 6.4	6.0	12.7	- 6.7	3.5	6.5	- 3.0	4.0	7.0	- 3.0
1969	10.0	16.7	- 6.7	8.5	15.0	- 6.5	4.5	5.5	- 1.0	5.0	6.7	- 1.7
1970	11.0	14.4	- 3.4	7.5	15.2	- 7.7	4.5	6.0	- 1.5	5.0	7.4	- 2.4
1971	7.5	11.0	- 3.5	6.0	5.6	+ 0.4	4.0	3.5	+ 0.5	4.5	4.5	0.0
1972	7.0	12.8	- 5.8	3.5	6.0	- 2.5	2.0	4.0	- 2.0	2.5	3.5	- 1.0
1973	9.0	14.0	- 5.0	8.5	11.8	- 3.3	4.0	4.5	- 0.5	4.5	6.4	- 1.9
1974	4.5	2.9	+ 1.6	0.5	-0.9	+ 1.4	2.5	2.5	0.0	2.5	3.8	- 1.3
1975	0.0	-4.6	+ 4.6	1.0	-4.7	+ 5.7	1.0	-2.0	+ 3.0	1.0	-1.5	+ 2.5
1976	8.5	12.5	- 4.0	7.5	11.0	- 3.5	4.5	5.0	- 0.5	4.5	4.6	- 0.1
1977	6.0	-1.9	+ 7.9	6.0	2.3	+ 3.7	4.5	2.5	+ 2.0	4.5	2.7	+ 1.8
1978	3.0	3.0	0.0	3.0	5.5	- 2.5	2.5	2.0	+ 0.5	2.5	2.5	0.0

	17. Price private consumption			18. Price commodity exports			19. Labour productivity industry			20. Wages per unit industrial output		
	Fore-cast	Realization*	Error	Fore-cast	Realization*	Error	Fore-cast	Realization*	Error	Fore-cast	Realization	Error
(1963)		(3.8)			(2.3)			(2.8)				
1964	7.0	6.8	+ 0.2	1.0	2.1	- 1.1	4.5	7.8	- 3.3	N		
1965	4.5	4.0	+ 0.5	1.5	2.1	- 0.6	3.5	5.0	- 1.5	N		
1966	4.5	5.4	- 0.9	1.0	0.2	+ 0.8	5.0	2.2	+ 2.8	N		
1967	4.5	3.0	+ 1.5	0.5	-0.6	+ 1.1	4.0	6.6	- 2.6	N		
1968	3.0	2.6	+ 0.4	-1.0	-0.9	- 0.1	3.5	6.1	- 2.6	N	(0.0)	
1969	5.0	6.1	- 1.1	1.0	1.9	- 0.9	3.5	5.1	- 1.6	4.0	3.0	+ 1.0
1970	4.0	4.4	- 0.4	2.5	3.5	- 1.0	4.0	6.2	- 2.2	2.5	5.5	- 3.0
1971	6.0	8.3	- 2.3	1.0	2.4	- 1.4	3.5	4.2	- 0.7	7.0	8.0	- 1.0
1972	6.5	8.8	- 2.3	1.0	0.7	+ 0.3	3.5	5.0	- 1.5	5.5	5.0	+ 0.5
1973	7.5	9.3	- 1.8	3.0	6.5	- 3.5	4.0	6.5	- 2.5	6.5	8.0	- 1.5
1974	11.5	10.1	+ 1.4	18.0	28.3	-10.3	3.0	4.0	- 1.0	9.5	11.5	- 2.0
1975	9.5	10.7	- 1.2	6.0	4.7	+ 1.3	2.0	-0.4	+ 2.4	9.5	14.5	- 5.0
1976	8.7	9.2	- 0.5	6.0	6.2	- 0.2	5.0	5.4	- 0.4	0.0	0.0	0.0
1977	6.5	6.8	- 0.3	4.0	3.1	+ 0.9	4.5	3.0	+ 1.5	0.5	4.5	- 4.0
1978	4.5	4.5	0.0	2.0	-1.5	+ 3.5	3.0	3.0	0.0	0.5	2.5	- 2.0

	21. Real disposable income			22. Current account balance of payments			23. Inventories			24. Tax pressure		
	Fore-cast	Real-ization*	Error	Fore-cast	Real-ization	Error	Fore-cast	Real-ization	Error	Fore-cast	Real-ization*	Error
(1963)	N			-1.0	-0.7	-0.3	1.1	1.7	-0.6	N		-0.5
1964	N			-0.1	0.1	-0.2	0.9	0.9	0.0	N		-0.3
1965	N			0.4	-0.7	+1.1	1.5	0.9	+0.6	N		+1.8
1966	N			0.5	-0.3	+0.8	1.3	1.1	+0.2	N		-0.5
1967	N			0.5	0.3	+0.2	0.9	1.1	-0.2	26.5	27.0	-0.5
1968	N			0.3	0.3	+0.0	2.6	2.6	-0.3	28.5	27.8	-0.3
1969	N			0.0	-1.6	+1.6	2.3	2.6	-0.3	28.5	26.7	+0.1
1970	N	(8.5)		-1.3	-0.9	-0.4	2.5	1.9	+0.6	28.6	29.5	-0.9
1971	2.5	2.0	+0.5	1.0	3.7	-2.7	2.0	1.5	+0.5	29.5	30.4	-0.9
1972	4.0	2.9	+1.1	2.5	6.3	-3.8	3.2	1.5	+1.7	29.6	29.8	+0.2
1973	1.0	0.9	+0.1	2.2	5.5	-3.3	2.2	3.5	-1.3	30.9	30.4	+0.5
1974	1.0	2.7	-1.7	5.0	4.4	+0.6	0.8	-1.5	+2.3	30.3	30.6	-0.3
1975	3.5	3.8	-0.3	7.0	7.5	-0.5	3.7	3.2	-0.5	31.1	31.6	+0.4
1976	-0.5	0.9	-1.4	5.5	1.1	+4.4	3.9	3.9	-0.2	32.0	31.6	+0.4
1977	2.5	2.4	+0.1	2.5			4.2			32.3	32.4	-0.1
1978	2.0	3.0	-1.0	2.5								

	25. Labour income percentage			26. Savings percentage			27. Unemployment			28. Premium pressure		
	Fore-cast	Real-ization*	Error	Fore-cast	Real-ization	Error	Fore-cast	Real-ization	Error	Fore-cast	Real-ization*	Error
(1963)	75.8	(75.3)	+0.9	18.3	(17.4)	-1.6	37.5	(34.0)	+7.5	N		+7.5
1964	75.6	75.5	-0.6	19.5	19.7	-0.2	37.5	35.0	+2.5	N		+2.5
1965	74.5	78.5	-4.0	21.0	19.1	+1.9	37.5	45.0	-7.5	N		-7.5
1966	76.0	77.7	-1.7	19.2	19.5	-0.3	85.0	86.0	-1.0	N		-1.0
1967	76.5	77.1	-0.1	19.0	20.7	-1.7	90.0	84.0	+6.0	N		+6.0
1968	77.0	77.4	-0.9	21.5	20.3	+1.2	55.0	66.0	-11.0	N		-11.0
1969	76.5	79.3	-2.8	21.0	19.9	+1.1	56.0	56.0	-3.5	16.0	15.9	+0.1
1970	76.5	79.9	-2.1	19.5	20.2	-0.7	115.0	115.0	-4.0	17.0	16.8	+0.2
1971	82.0	81.0	+0.0	21.0	19.7	+1.3	69.0	69.0	-7.0	18.3	18.3	+0.5
1972	82.5	79.6	+1.1	19.5	21.9	-2.4	117.0	117.0	-7.0	18.8	16.9	+1.4
1973	78.5	79.6	-1.1	17.0	20.5	-1.5	143.0	143.0	-11.5	19.7	19.4	+0.3
1974	82.5	81.4	+1.1	16.0	15.2	+0.8	211.0	211.0	-7.5	20.3	20.3	-0.5
1975	81.0	81.1	+0.4	16.0	16.1	-0.1	220.0	220.0	-7.5	20.4	19.9	+0.5
1976	81.5	81.1	+0.6	16.0	15.1	+0.9	212.5	204.0	+8.5	20.0	19.7	+0.3
1977	81.0	80.4	+0.2	15.3	14.1	+1.2	215.0	206.0	+9.0	20.0	19.9	+0.1

APPENDIX D

Scatter diagrams of forecast-realization pairs and
forecast errors diagrams for selected variables

The uninterrupted lines in the scatter diagrams are the lines of perfect forecast $f=r$. The interrupted lines are the least squares lines $f=br$. Unless otherwise stated the units are percentage changes of levels.

(Billions of Guilders)

(Thousands of Persons)

REFERENCES

- Beld, C.A. van den, Forecasts and Realization: The Forecasts by the Netherlands Central Planning Bureau, 1953-1963, Monograph no.10, Central Planning Bureau, Staatsdrukkerij, The Hague, 1965.
- Central Planning Bureau, Central Economic Plan 1964, 1965,..., 1979, Staatsuitgeverij, The Hague. (In Dutch.)
- Frisch, R., "Correlation and Scatter in Statistical Variables", Nordisk Statistisk Tidsskrift, Vol.8, 1929.
- Granger, C.W.J., "Prediction with a Generalized Cost of Error Function", Operational Research Quarterly, Vol.20, 1969.
- Granger, C.W.J. and P. Newbold, "Some Comments on the Evaluation of Economic Forecasts", Applied Economics, Vol.5, 1973.
- Mincer, J. and V. Zarnowitz, "The Evaluation of Economic Forecasts", in J. Mincer, ed., Economic Forecasts and Expectations: Analysis of Forecasting Behavior and Performance, National Bureau of Economic Research, New York, 1969.
- Panne, C. van de, "The Predictive Quality of the Central Economic Plans, 1949-1956", De Economist, 1959. (In Dutch.)
- Pearson, E.S. and L.O. Hartley (ed.), Biometrika Tables for Statisticians, Vol.I, Cambridge University Press, Cambridge (1970).
- Sims, C.A., "Evaluating Short-Term Macro-Economic Forecasts: The Dutch Performance", Review of Economics and Statistics, Vol.49, 1967.
- Theil, H., Economic Forecasts and Policy, North-Holland Publishing Company, Amsterdam, 1961.

Theil, H., Optimal Decision Rules for Government and Industry, North-Holland Publishing Company, Amsterdam, 1964.

Theil, H., Applied Economic Forecasting, North-Holland Publishing Company, Amsterdam, 1966.

Bibliotheek K. U. Brabant

17 000 01059870 5