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**THE EQUAL GAIN SPLITTING RULE FOR
SEQUENCING SITUATIONS AND THE
GENERAL NUCLEOLUS**

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THE EQUAL GAIN SPLITTING RULE FOR SEQUENCING SITUATIONS AND THE GENERAL NUCLEOLUS

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Abstract

The Equal Gain Splitting (EGS) rule is introduced by *Curiel, Pederzoli and Tijs (1989)* in the context of sequencing situations. This paper gives an alternative characterization of the EGS rule. For this characterization we consider the efficiency property and a fairness property on the set $\mathcal{H}$, consisting of the coalitions of the form $\{1, 2, \dots, i\}$, $1 \leq i \leq n-1$ and their complements. The set $\mathcal{H}$ also yields a generalized core for sequencing games. It is shown that this core is a convex hull of 2^{n-1} vectors and that the EGS rule is the average of these vectors. Moreover, it is shown that the EGS rule coincides with the general nucleolus with respect to $\mathcal{H}$ as introduced in *Maschler, Potters and Tijs (1992)*. Finally, necessary and sufficient conditions for the EGS rule to coincide with the nucleolus are given.

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1 Introduction

In one-machine sequencing situations each agent (player) has one job that has to be processed on a single machine. Each job is specified by its processing time, the time the machine takes to handle the job. We assume that the cost of a player depend linearly on the completion time of his job. Furthermore, there is an initial order on the jobs of the agents before the processing of the machine starts.

Each group of agents (coalition) is allowed to obtain cost savings by rearranging their jobs in a way that is admissible with respect to the initial order. By defining the worth of a coalition as the maximum cost savings a coalition can make by admissible rearrangements, we obtain a cooperative game called sequencing game, related to the one machine sequencing situation. This game theoretic approach has been taken in *Curiel, Pederzoli and Tijs (1989)*. They also defined the EGS (equal gain splitting) rule on the class of sequencing situations to obtain a division of the value of the grand coalition. It has been shown that the EGS rule is in the core of the corresponding sequencing game. *Curiel, Potters, Rajendra Prasad, Tijs and Veldman (1993)* considered the class of component additive games and introduced the β -rule as the average of two marginal vectors. They showed that the class of sequencing games is contained in the class of component additive games. Moreover, they showed that the β -rule coincides with the EGS rule in the class of sequencing games. Hence, the EGS rule can be regarded as an alternative to the Shapley value. It takes the average of two marginal vectors, while the Shapley value takes the average of all marginal vectors.

In this paper an alternative characterization for the EGS rule is presented. For this purpose we introduce the class $\mathcal{H}$ of coalitions of the form $\{1, \dots, i\}$ or $\{i + 1, \dots, n\}$, $1 \leq i \leq n - 1$. The EGS rule will be characterized by efficiency and the fair head-tail split property. A division rule satisfies the fair head-tail property if the marginal benefits of a union of each head and its complement is equally shared. Further, the set $\mathcal{H}$ yields a generalized core. It is shown that this core is the convex hull of 2^{n-1} vectors and that the EGS rule is the average of these vectors. It is shown that the general nucleolus with respect to $\mathcal{H}$, as introduced by *Maschler, Potters and Tijs (1992)*, coincides with the

EGS rule. Finally, necessary and sufficient conditions for the EGS rule to coincide with the nucleolus are given.

2 Sequencing situations and the EGS rule

This section describes sequencing situations and the corresponding sequencing games. Further, we recall the definition of the EGS rule and some well known facts from game theory. For the EGS rule we give a new characterization by efficiency and the fair head-tail split property.

In a one machine sequencing situation there is a queue of agents, each with one job, to be processed by one machine. The finite set of agents is denoted by $N = \{1, \dots, n\}$. The position of the agents in the queue is described by a bijection $\sigma : N \rightarrow \{1, \dots, n\}$. Specifically, $\sigma(i) = j$ means that player i is in position j . We assume that there is an initial order $\sigma_0 : N \rightarrow \{1, \dots, n\}$ on the jobs of the players before the processing of the machine starts. We number the agents in such way that the initial queue corresponds with σ_0 is defined by $\sigma_0(i) = i$ for all $i \in N$. The processing time p_i of the job of player i is the time the machine takes to handle this job. Further, it is assumed that every agent has an affine cost function $c_i : [0, \infty) \rightarrow \mathbf{R}$ defined by $c_i(t) = \alpha_i t + \beta_i$ with $\alpha_i > 0, \beta_i \in \mathbf{R}$. So $c_i(t)$ is the cost for agent i if he is t units of time in the system.

A sequencing situation as described above is denoted by (N, p, α) , where $N = \{1, \dots, n\}$, $p = (p_i)_{i \in N}$ and $\alpha = (\alpha_i)_{i \in N}$. The vector $\beta = (\beta_i)_{i \in N}$ is omitted in the description of the sequencing situation since the fixed costs it represents are independent of the positions of the players in the queue.

For player $i \in \{1, \dots, n-1\}$ we define the following sets with respect to the initial order σ_0 . The head of player i is the set $\{1, 2, \dots, i\}$ and the tail of player i is the set $\{i, i+1, \dots, n\}$. Note that the head (tail) of player i contains the set of players that precedes (follows) i in the initial order σ_0 . The collection of coalitions consisting of all heads and all tails is denoted by $\mathcal{H}$.

If the processing order is given by $\sigma : N \rightarrow \{1, \dots, n\}$ then the completion time of player i is equal to $C(\sigma, i) := \sum_{j: \sigma(j) \leq \sigma(i)} p_j$. The total costs of $c_\sigma(S)$ of a coalition $S \subset N$,

is given by $c_\sigma(S) := \sum_{i \in S} \alpha_i(C(\sigma, i)) + \beta_i$.

The (maximal) cost savings of a coalitions S depend on the set of admissible rearrangements of this coalition. A bijection $\sigma : N \rightarrow \{1, \dots, n\}$ is called admissible for S if it satisfies the following two conditions:

(i) The completion time of each agent outside the coalition S is equal to his completion time in the initial order: $C(\sigma_0, i) = C(\sigma, i)$ for all $i \in N \setminus S$.

(ii) the players of S are not allowed to jump over players outside S :

$$\{1, \dots, i\} \cap N \setminus S = \{j \mid \sigma(j) \leq \sigma(i)\} \cap N \setminus S \text{ for all } i \in S.$$

The set of admissible rearrangements for a coalition S is denoted by Σ_S .

Before the cooperative sequencing game is given, we recall some well known facts concerning cooperative games. A *cooperative game* is a pair (N, v) where N is a finite set of players and v is a mapping $v : 2^N \rightarrow \mathbf{R}$ with $v(\emptyset) = 0$ and where 2^N is the collection of all subsets of N .

A game (N, v) is called *superadditive* if for all coalitions $S, T \in 2^N$ with $S \cap T = \emptyset$ we have

$$v(S \cup T) \geq v(S) + v(T)$$

Cooperative game theory focuses on 'fair' and/or 'stable' division rules for the worth $v(N)$ of the grand coalition. A core element $x = (x_i)_{i \in N} \in \mathbf{R}^N$ is such that no coalition has an incentive to split off, i.e.

$$\sum_{i \in N} x_i = v(N) \text{ and } x(S) \geq v(S) \text{ for all } S \in 2^N.$$

where $x(S) = \sum_{i \in S} x_i$. The core $C(v)$ consists of all core elements. A game is called balanced if its core is non-empty.

For the nucleolus of a game, introduced by *Schmeidler (1969)*, we need the following notation. Let $F := (F_T)_{T \in 2^N}$, where F_T is the excess function corresponding to T defined by $F_T(x) := v(S) - x(S)$. The function $\Theta : \mathbf{R}^{2^n} \rightarrow \mathbf{R}^{2^n}$ is the map that orders the coordinates in a weakly decreasing order. Then the nucleolus of a game (N, v) is defined by

$$\eta(N, v) = \{x \in I(v) \mid \Theta \circ F(x) \preceq_L \Theta \circ F(y) \text{ for all } y \in I(v)\}$$

where $I(v) := \{x \mid x(N) = v(N), x(i) \geq v(i) \text{ for all } i \in N\}$ is the imputation set of (N, v) .

Given a sequencing situation (N, p, α) the worth of a coalition S of the corresponding sequencing game (Curiel *et al.* (1989)) is defined as the maximal cost savings the coalition can achieve by means of an admissible rearrangement. Formally,

$$v(S) = \max_{\sigma \in \Sigma_S} \left\{ \sum_{i \in S} (\alpha_i C(\sigma_0, i) + \beta_i) - \sum_{i \in S} (\alpha_i C(\sigma, i) + \beta_i) \right\} \quad (1)$$

Curiel *et al.* (1993) showed that (1) is equivalent to the following expression:

$$v(S) = \sum_{i < j} g_{ij} u_{\{i, i+1, \dots, j\}}(S)$$

Here $g_{ij} := \max\{\alpha_j p_i - \alpha_i p_j, 0\}$ represents the gain attainable for player i and j in case player i is directly in front of player j . The game $u_{\{i, i+1, \dots, j\}}$ is the simple game defined by

$$u_{\{i, i+1, \dots, j\}}(S) := \begin{cases} 1 & \text{if } \{i, i+1, \dots, j\} \subset S \\ 0 & \text{otherwise} \end{cases}$$

The Equal Gain Splitting (EGS) rule in a sequencing situation (N, p, α) is defined for all $i \in N$ by

$$EGS_i(N, p, \alpha) := \frac{1}{2} \sum_{j < i} g_{ji} + \frac{1}{2} \sum_{j > i} g_{ij}$$

Note that the optimal order of a queue can be obtained from the initial order by switching neighbours only. In the EGS rule a player obtains half of the gains of all neighbour switches he is actually involved in, to obtain the optimal order.

Example 1 Let $N = \{1, 2, 3\}$, $p = (2, 2, 1)$ and $\alpha = (4, 6, 5)$. Then $g_{12} = 4$, $g_{23} = 4$ and $g_{13} = 6$. This implies that $EGS_1(N, p, \alpha) = \frac{1}{2}(4 + 6) = 5$, $EGS_2(N, p, \alpha) = \frac{1}{2}(4 + 4) = 4$, $EGS_3(N, p, \alpha) = \frac{1}{2}(6 + 4) = 5$. $\square$

Curiel *et al.* (1989) showed that a sequencing game is superadditive (even convex) and that the EGS rule is in the core of the sequencing game. Moreover, they provided a characterization of the EGS rule by using efficiency, dummy property, equivalence property and switch property. We will give an other characterization in terms of efficiency and the fair head-tail split property.

Definition 1 Let (N, v) be a sequencing game. Then a vector $x \in R^n$ satisfies the fair head-tail split property if

$$x(S) - v(S) = x(S^c) - v(S^c) = \frac{1}{2} \Delta_S \text{ for all } S \in \mathcal{H}$$

where $\Delta_S = v(N) - v(S) - v(S^c)$.

Note that from the fair head-tail property it follows that the excesses $F_S(x)$ and $F_{S^c}(x)$ are equal for all $S \in \mathcal{H}$. Before the characterization of the EGS rule is given we recall the definition of a σ_0 component additive game and of the β -rule, both introduced by *Curiel, Potters, Rajendra Prasad, Tijs and Veltman (1993)*. A coalition S is called connected with respect to σ_0 if for all $i, j \in S$ and $k \in N$, $\sigma_0(i) < \sigma_0(k) < \sigma_0(j)$ implies $k \in S$. Let T be a coalition that is not connected. A coalition S is a component of T if $S \subset T$, S is connected and for every $i \in T \setminus S$, $T \cup \{i\}$ is not connected. The components of T form a partition of T which we denote by $T \setminus \sigma_0$. A game (N, v) is called a σ_0 component additive games if it satisfies the following three conditions:

- (i) $v(i) = 0$ for all $i \in N$.
- (ii) (N, v) is superadditive.
- (iii) $v(T) = \sum_{S \in T \setminus \sigma_0} v(S)$

Let (N, v) be a σ_0 component additive game then

$$\beta_i(v) = \frac{1}{2} \{v(\{1, \dots, i\}) - v(\{1, \dots, i-1\}) + v(\{i, \dots, n\}) - v(\{i+1, \dots, n\})\} \quad (2)$$

Curiel et al. (1993) showed that the class of sequencing games is contained in the class of σ_0 component additive games and that the β rule restricted to the class of sequencing games coincides with the EGS rule. It is immediately clear that the β -rule satisfies efficiency and fair head-tail split property.

Theorem 1 *Let (N, v) be the sequencing game corresponding to the sequencing situation (N, p, α) . Then $EGS(N, p, \alpha)$ is the unique rule that satisfies efficiency and the fair head-tail split property.*

PROOF: From (2) and the fact that the EGS rule coincides with the β -rule on the class of sequencing games it follows that the EGS rule satisfies the two properties. Conversely, assume that a vector x has both properties. Then it follows from the fair head-tail split property that for $k \in N \setminus \{n\}$

$$x(\{1, \dots, k\}) - v(\{1, \dots, k\}) = x(\{k+1, \dots, n\}) - v(\{k+1, \dots, n\}) \quad (3)$$

and

$$x(\{1, \dots, k-1\}) - v(\{1, \dots, k-1\}) = x(\{k, \dots, n\}) - v(\{k, \dots, n\}) \quad (4)$$

From (3) and (4) it follows by subtracting that

$$x(\{k\}) - v(\{1, \dots, k\}) + v(\{1, \dots, k-1\}) = -x(\{k\}) - v(\{k+1, \dots, n\}) + v(\{k, \dots, n\})$$

Hence, $x(\{k\}) = \beta_k(v) = EGS_k(N, p, \alpha)$ for all $k \in N \setminus \{n\}$. From the efficiency it follows immediately that $x(\{n\}) = EGS_n(N, p, \alpha)$. $\square$

3 The head tail core

In this section the head tail core is introduced. We characterize the extreme points of this set and show that the EGS rule is the average of these extreme points.

The head tail core $C^H(v)$ of a sequencing game (N, v) is defined by

$$C^H(v) = \{x \in \mathbf{R}^n \mid x(N) = v(N), x(S) \geq v(S) \text{ for all } S \in \mathcal{H}\}. \quad (5)$$

Obviously, we have that the core $C(v)$ of the game (N, v) is contained in the head tail core $C^H(v)$. The following example shows that the core of a game can be a strict subset of the head tail core.

Example 2 Consider the sequencing situation of example 1. Then the corresponding sequencing game is given by $v(\{1, 2\}) = v(\{2, 3\}) = 4$, $v(\{1, 2, 3\}) = 14$ and $v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{1, 3\}) = 0$. An extreme point of $C^H(v)$ is the vector $(10, -6, 10)$. Obviously, this vector is not an element of $C(v)$ (see figure 1). $\square$

The dual game (N, v^*) of (N, v) is defined by $v^*(S) = v(N) - v(S^c)$ for all $S \subset N$. Note that $v(N) = v^*(N)$ and that $v(S) \leq v^*(S)$ for all S by the superadditivity of (N, v) .

Now (5) is equivalent to

$$\begin{aligned} C^H(v) &= \{x \in \mathbf{R}^n \mid v(\{1, \dots, i\}) \leq \sum_{k=1}^i x_k \leq v^*(\{1, \dots, i\}) \text{ for all } i \in N\} \\ &= \{x \in \mathbf{R}^n \mid x = L^{-1}y, v(\{1, \dots, i\}) \leq y_i \leq v^*(\{1, \dots, i\}) \text{ for all } i \in N\} \end{aligned}$$

where L is the $n \times n$ non-singular lower triangular matrix with ones on and below the diagonal, and zeros above the diagonal. It is easy to see that the linear map L gives a 1-1 correspondence between C^H and the set D defined by

$$D := \{y \in R^n \mid v(\{1, \dots, i\}) \leq y_i \leq v^*(\{1, \dots, i\}) \text{ for all } i \in N\}$$

Consequently, there is a 1-1 correspondence between the extreme points of both sets.

Clearly, the extreme points of D correspond to a system of n equations of the form

$$y_i = a_i, \quad i = 1, \dots, n \quad \text{where } a_i \in \{v(\{1, \dots, i\}), v^*(\{1, \dots, i\})\} \text{ for all } i \in N$$

Let $J \subset N$. We put $h^J = L^{-1}(y^J)$ where y^J is the extreme point of D corresponding to the system of equations given by

$$y_i^J = \begin{cases} v(\{1, \dots, i\}) & i \in N \setminus J \\ v^*(\{1, \dots, i\}) & i \in J \end{cases}$$

Then $E = \{h^J \mid J \subset N\}$ is the set of extreme points of $C^H(v)$. Consequently, we have

Theorem 2 $C^H(v) = \text{conv}\{h^J \mid h^J \in E\}$.

where h^J corresponds to the solutions of the set of equations

$$\begin{aligned} \sum_{k=1}^i x_k &= v^*(\{1, \dots, i\}) & i \in J \\ \sum_{k=1}^i x_k &= v(\{1, \dots, i\}) & i \in N \setminus J \end{aligned} \tag{6}$$

The number of extreme points is $2^{|I|}$ where $I = \{i \in N \mid v(\{1, \dots, i\}) \neq v^*(\{1, \dots, i\})\}$.

Hence, in the generic case we have 2^{n-1} different extreme points since $v^*(N) = v(N)$ and consequently $n \notin I$.

The following theorem shows that the average of the extreme points of the head tail core generated by (6) is the EGS rule.

Theorem 3 Let (N, p, α) be a sequencing situation and (N, v) the corresponding sequencing game. Then $EGS(N, p, \alpha) = \frac{1}{2^{n-1}} \sum_{J \subset N} h^J$.

PROOF: Note that for all $i \in N$ and all $J \subset N$ we have $y_i^J + y_i^{N \setminus J} = v(\{1, \dots, i\}) + v^*(\{1, \dots, i\})$. Consequently, for all $J \subset N$ it follows that $L(h^J + h^{N \setminus J}) = y^J + y^{N \setminus J} = y^\emptyset + y^N = L(h^\emptyset + h^N)$. Since $h^\emptyset + h^N = 2EGS(N, p, \alpha)$ we have

$$\frac{1}{2^{n-1}} \sum_{J \subset N} h^J = \frac{1}{2^n} \sum_{J \subset N} (h^J + h^{N \setminus J}) = \frac{1}{2^n} 2^n EGS(N, p, \alpha) = EGS(N, p, \alpha).$$

□

4 The EGS rule and the general nucleolus

In this section it is shown that the EGS rule has some similarity with the nucleolus in case the set of coalitions is restricted to the set $\mathcal{H}$. In particular, it is shown that the general nucleolus, introduced by *Maschler et al. (1993)*, coincides with the EGS rule.

The following example shows that the EGS rule is not necessarily equal to the nucleolus of a sequencing game.

Example 3 Let $N = \{1, 2, 3\}$, $p = (2, 2, 1)$ and $\alpha = (4, 6, 5)$. Then $v(\{1, 2\}) = v(\{2, 3\}) = 4$, $v(\{1, 2, 3\}) = 14$ and $v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{1, 3\}) = 0$. The nucleolus of the game is $(4\frac{2}{3}, 4\frac{2}{3}, 4\frac{2}{3})$, whereas the EGS rule equals $(5, 4, 5)$. $\square$

In *Maschler et al. (1993)* a general nucleolus is defined on the class of truncated games, an extension of the class of cooperative games. A truncated game with respect to the cooperative game (N, v) is a quadruplet $(N, \mathcal{S}, v, \Pi)$, where N is the player set, $\mathcal{S}$ is a subset of $2^N \setminus \{\emptyset, N\}$ called the set of permissible coalitions, $v : \mathcal{S} \rightarrow \mathbf{R}$ is the characteristic function and Π a set of permissible pre-imputations.

Let $F := (F_T)_{T \in \mathcal{S}}$, where F_T are the excess functions defined by $F_T(x) := v(T) - x(T)$. The function $\Theta : \mathbf{R}^{|\mathcal{S}|} \rightarrow \mathbf{R}^{|\mathcal{S}|}$ is the map that orders the coordinates in a weakly decreasing order. Then the general nucleolus of the truncated game $(N, \mathcal{S}, v, \Pi)$ is defined by

$$\mathcal{N}(N, \mathcal{S}, v, \Pi) = \{x \in \Pi \mid \Theta \circ F(x) \preceq_L \Theta \circ F(y) \text{ for all } y \in \Pi\}.$$

If (N, p, α) is a sequencing situation and (N, v) the corresponding sequencing game, we introduce the truncated game $(N, \mathcal{H}, \bar{v}, \Pi)$ where $\bar{v}(S) = v(S)$ for all $S \in \mathcal{H}$ and $\Pi := \{x \in \mathbf{R}^n \mid x(N) = v(N)\}$, the pre-imputation set of (N, v) . The following theorem states that the EGS rule is the general nucleolus of the truncated game $(N, \mathcal{H}, \bar{v}, \Pi)$.

Theorem 4 *Let $(N, \mathcal{H}, v, \Pi)$ be the truncated game of the sequencing game (N, v) corresponding to (N, p, α) . Then $EGS(N, p, \alpha)$ coincides with $\mathcal{N}(N, \mathcal{H}, v, \Pi)$.*

PROOF: First it is shown that $\mathcal{N}(N, \mathcal{S}, v, \Pi)$ is a non-empty set.

Let $y \in \Pi \setminus C^{\mathcal{H}}(v)$, then there exists an $S \in \mathcal{H}$ such that $y(S) < v(S)$. Since for any $x \in C^{\mathcal{H}}(v)$ it holds that $x(T) \geq v(T)$ for all $T \in \mathcal{H}$ we have that $\Theta \circ F(x) \preceq_L \Theta \circ F(y)$. Hence, $\mathcal{N}(N, \mathcal{S}, v, \Pi) = \{x \in C^{\mathcal{H}} \mid \Theta \circ F(x) \preceq_L \Theta \circ F(y) \text{ for all } y \in C^{\mathcal{H}}(v)\}$. Since $C^{\mathcal{H}}(v)$ is a compact set and $\Theta \circ F$ is a continuous map it follows that $\mathcal{N}(N, \mathcal{S}, v, \Pi) \neq \emptyset$.

Let $x \in \mathcal{N}(N, \mathcal{S}, v, \Pi)$. Suppose there exists a $k \in N$ and an $\epsilon > 0$ such that

$$[v(\{1, \dots, k\}) - x(\{1, \dots, k\})] - [v(\{k+1, \dots, n\}) - x(\{k+1, \dots, n\})] > \epsilon$$

Take $y \in \Pi$ such that $y_i = x_i$ for all $i \in N \setminus \{k, k+1\}$, $y_k = x_k + \epsilon$ and $y_{k+1} = x_{k+1} - \epsilon$.

Then

$$v(T) - x(T) = v(T) - y(T) \quad \text{for all } T \in \mathcal{H} \setminus \{\{1, \dots, k\}, \{k+1, \dots, n\}\},$$

$$v(\{1, \dots, k\}) - x(\{1, \dots, k\}) = v(\{1, \dots, k\}) - y(\{1, \dots, k\}) + \epsilon$$

and

$$v(\{k+1, \dots, n\}) - x(\{k+1, \dots, n\}) = v(\{k+1, \dots, n\}) - y(\{k+1, \dots, n\}) - \epsilon.$$

Hence, $\Theta \circ F(y) \prec_L \Theta \circ F(x)$. This is in contradiction with the definition of $\mathcal{N}$. In a similar way we can show that there exists no $k \in N$ such that

$$[v(\{1, \dots, k\}) - x(\{1, \dots, k\})] - [v(\{k+1, \dots, n\}) - x(\{k+1, \dots, n\})] < 0.$$

This implies that for all $S \in \mathcal{H}$ we have $F_S(x) = F_{S^c}(x)$. Then it follows from theorem 1 that $x = EGS(N, p, \alpha)$. $\square$

5 Final remarks

In section 2 we stated that the class of σ_0 component additive games contains the class of sequencing games and that the β -rule restricted on the class of sequencing games coincides with the EGS rule. In the proofs of the previous results of this paper we only used the properties of a σ_0 component additive game and the expression of the β value. This implies that all results can be extended to the β rule with respect to σ_0 component additive games.

Finally, a necessary and sufficient condition on a sequencing situation is given such that the EGS rule and the nucleolus of the corresponding sequencing game coincide.

Theorem 5 *Let (N, p, α) be a sequencing situation and (N, v) be the corresponding sequencing game. Then $EGS(N, p, \alpha) = \eta(N, v)$ if and only if*

$$\begin{aligned} \sum_{a=1}^{p-1} \sum_{b=i+1}^n g_{ab} &\leq \sum_{a=1}^{p-1} \sum_{b=p}^i g_{ab} \quad \text{for all } 1 \leq p \leq i < n \\ \sum_{a=1}^{p-1} \sum_{b=i+1}^n g_{ab} &\leq \sum_{a=p}^i \sum_{b=i+1}^n g_{ab} \quad \text{for all } 1 < p \leq i \leq n. \end{aligned}$$

PROOF: From *Potters and Reijnierse (1992)* it follows that the nucleolus of a sequencing game is the unique point x satisfying efficiency and $\bar{s}_{ii+1}(x) = \bar{s}_{i+1i}(x)$ for all $i \in N \setminus \{n\}$.

Here $\bar{s}_{ij}$ is defined as follows

$$\bar{s}_{ij}(x) := \max\{v(T) - x(T) \mid i \in T \subset N \setminus \{j\}, T \text{ is connected}\}.$$

From this and the fact that the EGS-rule is the unique rule that satisfies efficiency and the fair head-tail property, it follows that the nucleolus is equal to the allocation given by the EGS-rule when for all $i \in N \setminus \{n\}$ the maximum in the definition of $\bar{s}_{ii+1}$ is achieved at $T = \{1, \dots, i\}$ and the maximum in the definition of $\bar{s}_{i+1i}$ is achieved at $T = \{i+1, \dots, n\}$ when we take x to be equal to the allocation given by the EGS-rule. Since only connected coalitions have to be considered for these maxima we obtain the following inequalities.

$$F_{\{1, \dots, i\}}(x) \geq F_{\{p, \dots, i\}}(x) \text{ for all } 1 \leq p \leq i < n$$

$$F_{\{i+1, \dots, n\}}(x) \geq F_{\{i+1, \dots, q\}}(x) \text{ for all } i+1 \leq q \leq n.$$

where F is the excess function. This set of inequalities is equivalent to

$$F_{\{1, \dots, i\}}(x) \geq F_{\{p, \dots, i\}}(x) \text{ for all } 1 \leq p \leq i < n$$

$$F_{\{p, \dots, n\}}(x) \geq F_{\{p, \dots, i\}}(x) \text{ for all } 1 < p \leq i \leq n. \quad (7)$$

Since

$$v(\{p, \dots, i\}) = \sum_{a=p}^{i-1} \sum_{b=a+1}^i g_{ab} = \frac{1}{2} \sum_{a=p}^{i-1} \sum_{b=a+1}^i g_{ab} + \frac{1}{2} \sum_{b=p+1}^i \sum_{a=p}^{b-1} g_{ab}$$

and

$$x(\{p, \dots, i\}) = \frac{1}{2} \sum_{a=p}^i \sum_{b=a+1}^n g_{ab} + \frac{1}{2} \sum_{b=p}^i \sum_{a=1}^{b-1} g_{ab}$$

we have for any $p, i \in N, p < i$ that

$$\begin{aligned} v(\{p, \dots, i\}) - x(\{p, \dots, i\}) &= -\frac{1}{2} \left(\sum_{a=p}^{i-1} \sum_{b=i+1}^n g_{ab} + \sum_{b=i+1}^n g_{ib} + \sum_{b=p+1}^i \sum_{a=1}^{p-1} g_{ab} + \sum_{a=1}^{p-1} g_{ap} \right) \\ &= -\frac{1}{2} \left(\sum_{a=p}^i \sum_{b=i+1}^n g_{ab} + \sum_{b=p}^i \sum_{a=1}^{p-1} g_{ab} \right). \end{aligned}$$

Define $G_{I \times J} = \sum_{a \in I} \sum_{b \in J} g_{ab}$ for any connected $I, J \subset N$ with $g_{ab} = 0$ if $a \geq b$. Then

(7) is equivalent to

$$G_{\{1, \dots, i\} \times \{i+1, \dots, n\}} \leq G_{\{p, \dots, i\} \times \{i+1, \dots, n\}} + G_{\{1, \dots, p-1\} \times \{p, \dots, i\}} \text{ for all } 1 \leq p \leq i < n$$

$$G_{\{1, \dots, p-1\} \times \{p, \dots, n\}} \leq G_{\{p, \dots, i\} \times \{i+1, \dots, n\}} + G_{\{1, \dots, p-1\} \times \{p, \dots, i\}} \text{ for all } 1 < p \leq i \leq n.$$

These reduce to

$$G_{\{1, \dots, p-1\} \times \{i+1, \dots, n\}} \leq G_{\{1, \dots, p-1\} \times \{p, \dots, i\}} \text{ for all } 1 \leq p \leq i < n$$

$$G_{\{1, \dots, p-1\} \times \{i+1, \dots, n\}} \leq G_{\{p, \dots, i\} \times \{i+1, \dots, n\}} \quad \text{for all } 1 < p \leq i \leq n.$$

which proves the if part.

In the only if part we have that $EGS(N, p, \alpha) = \eta(N, v)$. The efficiency and the head-tail fairness implies that the maximum in the definition of s_{ii+1} is achieved at $T' = \{1, \dots, i\}$ and the maximum in the definition of $\bar{s}_{i+1i}$ is achieved at $T' = \{i+1, \dots, n\}$. From the if part follows the result immediately. $\square$

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