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On Expectations, Information and Dynamic Game Equilibria.

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Abstract

This paper deals with information and policy announcements in non-cooperative dynamic games. It fits in the discussion on time inconsistency of optimal policy under forward looking expectations. For some simple examples results are derived in the field of memory strategies and in the field of consistent, credible and cheating strategies.

1. Introduction

Optimization techniques developed for physical and engineering systems are often applied to the control of economic systems. The objective functional of the controller (e.g. the government) is minimized given a passive economic system. However, as Lucas [1976] has stressed, an economy is not a passive system. In an economy there are, in general, several independently acting controllers. The actions of these controllers depend on their expectations with respect to the actions of the other decision makers. Forward looking expectations ruin the standard non-anticipation property of a system. As Kydland and Prescott [1977] have noticed the optimal policy is time inconsistent.

Kydland and Prescott [1977] came more or less to the conclusion that the optimal control approach as such was under reconsideration. Later on, however, it became the common idea that the optimal control approach had to be placed in a game theory framework. The revival of dynamic game theory in economic literature was a fact. The famous result of Simaan and Cruz [1973] that Bellman's principle of optimality does not generalize to the Stackelberg solution became known as time inconsistency.

What is actually the problem? The answer to this question has a technical side and a conceptual side. Technically speaking the problem is that dynamic programming can not be used to find the closed loop no memory Stackelberg solution. Conceptually speaking the problem is that the global Stackelberg decision model yields policies which become sub-optimal in the course of the game when reoptimizing is allowed for: in the future there can be an incentive to change the policy which was originally established. Furthermore, it is hard to defend a decision model in which the follower believes such a time inconsistent announcement of the leader.

The Stackelberg solution concept is a sequential concept: the players act one after another. The follower knows the action of the leader when he acts himself. However, the follower does not know the leader's future actions. He has to decide on the basis of the leader's announcement. Incentives to cheat arise for the leader because of the time inconsistency. Two solutions can be distinguished. Firstly, the follower believes the announcement and the leader does not cheat or reoptimize. This situ-

ation results in the global Stackelberg outcome. Secondly, a reoptimization in the future is to be expected. Oudiz and Sachs [1984] give a nice interpretation: a change in government. This outcome has to be consistent. That is to say, a future reoptimization will not lead to a change in policy. Otherwise the announcement will not be believed. The feedback stagewise Stackelberg solution, which is found by means of dynamic programming, is consistent by construction. It is also possible to formulate a consistent open-loop Stackelberg solution. The reputation of the government plays an important role here. It is an interesting idea to try to formalize the concept of reputation (see e.g. Kreps and Wilson [1982] and Barro and Gordon [1983]). In dynamic finite horizon games the loss of reputation in the course of the game can be formalized by an end-penalty.

It is important to note that these problematic aspects of the Stackelberg concept can also occur in a game where the players have to perform their actions at the same time. One of the players (e.g. the government) can try to become a Stackelberg leader by announcing his policy before it is actually played. In this type of game additional incentives to cheat arise! Generally, the leader can gain by cheating on his announcement at the time of action. It is reasonable to assume that the effect of cheating will be that the follower will not believe the announcements anymore. Three solutions can be distinguished. Firstly, the follower believes the announcement and the leader does not cheat or reoptimize. This situation results again in the Stackelberg outcome. Secondly, a reoptimization in the future is to be expected. This outcome has to be consistent. Finally, cheating is expected. In this case the announcement has to be "cheating-proof". Therefore it is necessary, but not sufficient, that the announcement is consistent. It will be shown that the Nash concept seems to be the only reasonable concept for this game. For this reason the Nash announcement can be called a credible announcement. When cheating is not expected, but occurs anyhow, several outcomes are possible. For example, the follower believes the announcement, consistent or not, but the leader cheats on his announcement. It is reasonable to assume that after this has happened the follower stops believing. All possible outcomes have to be evaluated by the leader against his expected loss of reputation.

The paper is organized as follows. Section 2 elaborates on information. Some papers (e.g. Backus and Driffill [1985]) jump very quickly from an open-loop Stackelberg framework to a feedback stagewise or dynamic programming framework in order to achieve consistency. It is true that the feedback stagewise solution concept has very nice properties (like subgame perfectness). It presupposes, however, information on the state of the system. A change in information structure should not be justified alone on the strive for time consistency. Section 2 shows the impact of the information structure on the outcome with the help of some simple examples. Section 3 elaborates on announcements, consistency and cheating. A simple linear quadratic example is used to illustrate some possible outcomes which were described in this introduction. Section 4 is a conclusion. Definitions and propositions are brought together in an appendix.

2. On information

This section discusses the impact of the information structure on the outcome of Nash and Stackelberg games with the help of some simple examples. Papers by Starr and Ho [1969a], [1969b], Simaan and Cruz [1973a], [1973b] and Basar [1976] have particularly initiated understanding in this field. The section is organized as follows. Firstly, the terminology is briefly summarized. Secondly, some results are called to mind. Thirdly, a few examples illustrate these results and show some of the typical problems and facts outside these results.

For the moment, two types of information structures are distinguished:

- the open-loop information structure, where the players have no information on the actual state of the system; controls are a function of time and initial state.
- the closed-loop no memory or feedback information structure, where the players have perfect information on the actual state of the system; controls are a function of time and actual state.

For a fixed initial state an open-loop control is in fact only a function of time. In this case it belongs to the set of feedback controls.

Two modes of play are distinguished. In the global mode the players lay down their strategies at the beginning of the planning period. This mode is compatible with both the open-loop and the feedback information structure. In the stagewise mode the players decide at each point of time. For each state which can be reached at that point of time a strategy is formulated such that the expected loss for the remainder of the planning period is minimized. Stagewise solutions are sub-game perfect (see Selten [1975]) where a sub-game is a game over the remainder of the planning period. This mode is only compatible with the feedback information structure.

Pontryagin's minimum principle solves global open loop problems and dynamic programming solves feedback stagewise problems (see e.g. de Zeeuw [1984]). The other problems are mostly difficult to solve (see examples).

Two more rules of a non-cooperative game are distinguished:

- the players act at the same time (simultaneously); they do not know each others actions in the current and future periods when they act.
- the players act one after another (sequentially); in contrast to the

simultaneous concept one of the players (the follower) knows the action of the other player (the leader) in the current period.

So far as a player does not know the actions of the other player he has to decide on the basis of expectations with respect to these actions. When these expectations turn out to be right a solution (an equilibrium) for the game results. In the simultaneous case this equilibrium is called the Nash equilibrium. In the sequential case this equilibrium is called the Stackelberg equilibrium.

In the case of two players the Stackelberg outcome is better for the leader than the Nash outcome. For the follower the outcome can be worse, but does not have to be worse.

In a strict convex linear quadratic framework the (unique) linear feedback global Nash equilibrium coincides with the feedback stagewise Nash equilibrium (see de Zeeuw [1984]). However, for a fixed initial state the open-loop Nash solution is also a feedback global solution! As will be seen below the feedback global Stackelberg concept causes problems. Three examples will enlighten this material.

Example 1

Starr and Ho [1969b] constructed the game represented by figure 1. There are two players and two stages. At each stage each player has two possible controls: 0 and 1. Figure 1 shows the possible transitions and the associated costs.

Figure 1

Starr and Ho end up with the following results:

a) open-loop Nash solution:

$$u_1(0) = 1, \quad u_2(0) = 0,$$

$$u_1(1) = 1, \quad u_2(1) = 0,$$

with costs $(3,2)$,

where $u_i(t)$ is the action of player i at stage t .

b) feedback stagewise Nash solution:

$$u_1(0,1) = 0, \quad u_2(0,1) = 1,$$

$$u_1(1,2) = 0, \quad u_2(1,2) = 0,$$

$$u_1(1,1) = 1, \quad u_2(1,1) = 1,$$

$$u_1(1,0) = 1, \quad u_2(1,0) = 0,$$

with costs $(4,4)$,

where $u_i(t,x)$ is the action of player i at time t in state x .

c1) feedback global Nash solution:

$$u_1(0,1) = 0, u_2(0,1) = 1,$$

$$u_1(1,2) = 0, u_2(1,2) = 0,$$

$$u_1(1,1) = 1, u_2(1,1) = 1,$$

with costs (4,4).

The last result is not completely correct. Properly speaking there are two more feedback global Nash solutions:

c2)	$u_1(0,1) = 1, u_2(0,1) = 0,$	c3)	$u_1(0,1) = 0, u_2(0,1) = 1,$
	$u_1(1,1) = 1, u_2(1,1) = 0,$		$u_1(1,2) = 0, u_2(1,2) = 0,$
	$u_1(1,0) = 1, u_2(1,0) = 0,$		$u_1(1,1) = 1, u_2(1,1) = 0.$

The solution (c2) is in fact the open-loop solution (a). Because the initial state is fixed, the open-loop strategies are only a function of time! This implies that they can be conceived as feedback strategies.

The solution (c3) differs only outside the equilibrium trajectory from (c1). However, if for some reason the trajectory passes through state 1 at stage 1, only the solution (c1) will yield a Nash equilibrium for the remainder of the game. If this argument is extended to all possible states at stage 1, the feedback stagewise solution (b) results. This is the only solution which remains Nash after all possible mistakes. That is to say, this is the only solution which is sub-game perfect (see Selten [1975]) where the sub-game is the game over the remainder of the planning period.

Finally, an interesting aspect of this example is that the open-loop solution dominates the feedback solution. That is to say, the outcome is better for both players.

Example 2 (see de Zeeuw [1984]).

Consider the following two-stage linear quadratic game:

$$J_1 := \frac{1}{2} (u_1^2(0) + x^2(2)),$$

$$J_2 := \frac{1}{2} (x^2(1) + u_2^2(1)),$$

$$x(2) = x(1) + u_2(1),$$

$$x(1) = x(0) + u_1(0),$$

$$x(0) = x_0$$

Player 2 is the leader and player 1 is the follower.

Open-loop Stackelberg solution:

$$u_1(0) = -\frac{3}{5} x_0, \quad u_2(1) = \frac{1}{5} x_0,$$

with costs $(0.36 x_0^2, 0.1 x_0^2)$.

Feedback stagewise Stackelberg solution:

$$u_1(0, x) = -\frac{1}{2} x, \quad u_2(1, x) = 0,$$

with costs $(0.25 x_0^2, 0.125 x_0^2)$.

The open-loop Stackelberg solution is time inconsistent.

Linear feedback global Stackelberg game:

suppose $u_2(1, x) = ax$;

rational reaction of the follower:

$$u_1(0, x) = -\frac{(1+a)^2}{1 + (1+a)^2} x;$$

the leader faces the problem:

$$\text{minimize } \frac{1}{2} \left(\frac{1 + a^2}{(1 + (1+a)^2)^2} x_0^2 \right);$$

it follows that the leader can enforce as low costs as he wants by choosing "a" big enough; a big "a" can be considered as an incentive for the follower to steer $x(1)$ close to zero.

There are more information structures than the two mentioned above. For example, there is the closed-loop-memory information structure, where controls are a function of time and the state of the system in present and past. Even in a strict convex linear quadratic framework this information structure leads to infinitely many Nash solutions (see Basar and Olsder [1982]). As will be seen in example 3 below there are closed loop memory solutions which dominate both the open-loop and feedback solution! That is to say, the outcome is better for both players. A closed loop memory Stackelberg solution can be found in Basar and Selbuz [1979].

Example 3 (see de Zeeuw [1984]).

Consider the following two-stage strict convex linear quadratic game:

$$J_1 := \frac{1}{2} (x^2(0) + u_1^2(0) + x^2(1) + u_1^2(1) + x^2(2)),$$

$$J_2 := \frac{1}{2} (2x^2(0) + u_2^2(0) + 2x^2(1) + u_2^2(1) + 2x^2(2)),$$

$$x(2) = x(1) + u_1(1) + u_2(1),$$

$$x(1) = x(0) + u_1(0) + u_2(0),$$

$$x(0) = x_0.$$

Open-loop Nash solution:

$$u_1(0) = -\frac{5}{19} x_0, \quad u_2(0) = -\frac{10}{19} x_0,$$

$$u_1(1) = -\frac{1}{19} x_0, \quad u_2(1) = -\frac{2}{19} x_0,$$

with costs $(0.56 x_0^2, 1.191 x_0^2)$.

Feedback stagewise Nash solution:

$$u_1(0,x) = -\frac{1}{4}x, \quad u_2(0,x) = -\frac{19}{36}x,$$

$$u_1(1,x) = -\frac{1}{4}x, \quad u_2(1,x) = -\frac{1}{2}x,$$

with costs $(0.559 x_0^2, 1.198 x_0^2)$.

The open-loop Nash solution is time consistent in the sense that the actions $u_1(1)$ and $u_2(1)$ relate to the state $x(1)$ in the way as prescribed by the results of dynamic programming.

Consider the following class of linear closed-loop memory strategies:

$$u_1(0,x) = cx, \quad u_2(0,x) = fx,$$

$$u_1(1,x,y) = ay + bx, \quad u_2(1,x,y) = dy + ex,$$

where x is the observed state at stage 0 and y is the observed state at stage 1.

The Nash solutions are represented by tuples (a,b,c,d,e,f) which fit the following set of equations:

$$4q = 1 + c + f,$$

$$2qa = -10q - f,$$

$$qd = -5q - c,$$

$$b = 19q + 2f,$$

$$e = 18q + 4c.$$

For all (c,f) such that $1 + c + f \neq 0$ this set of equations yields one closed loop memory Nash solution.

For $b = e = 0$ the linear feedback global Nash solution results. It coin-

cides with the feedback stagewise solution.

Some closed-loop memory solutions are dominated by others. The sub-set of undominated solutions is given by:

$$(c, f) = \frac{-10\mu + 19}{8\mu^2 + 2\mu - 19} (1-\mu, \mu), \quad \mu \in (0, 1).$$

For example, for $\mu = \frac{2}{3}$, the result is

$$\begin{aligned} u_1(0, x) &= -\frac{37}{127} x, & u_2(0, x) &= -\frac{74}{127} x, \\ u_1(1, x, y) &= \frac{17}{4} y - \frac{72}{127} x, & u_2(1, x, y) &= \frac{17}{4} y - \frac{76}{127} x, \end{aligned}$$

with costs $(0.551 x_0^2, 1.189 x_0^2)$.

This closed-loop memory Nash solution dominates both the open-loop Nash solution and the feedback stagewise Nash solution!

Whatever nice properties the feedback stagewise solution may have, the fact that it can be dominated by solutions with a comparable type of information structure should at least be a matter of concern. A reconciling approach could lie in the definition of the state of the system (see e.g. Tenney [1981]).

3. On announcements, consistency and cheating.

Two earlier conclusions motivate this section. Firstly, in the Stackelberg solution concept the leader is better off than in the Nash concept. Secondly, the Stackelberg solution is time inconsistent (except for the feedback stagewise case which is time consistent by construction).

From the first conclusion the idea was born that in the case of simultaneous acting one of the players (e.g. the government) can try to become a Stackelberg leader by making an announcement before the actual play. When the other player is "naive" and believes the announcement, the Stackelberg solution may result. However, when the other player does not give in to the attempt of the first player, he can try to force the first player back to the Nash solution by playing Nash himself.

The introduction of an announcement implies the introduction of two possibilities to cheat on the announcement. The first possibility emanates from time inconsistency. The optimal announcement generally becomes sub-optimal after some actions are performed. An incentive arises to change the original plan in the course of the game. Only a time consistent strategy is "cheating-proof" in this respect. A time consistent strategy is a strategy with the property that future reoptimizations will not lead to a change in strategy. The second possibility occurs even in a static context. The announcement generally differs from "the optimal reaction to the optimal reaction to the announcement". Only the Nash announcement is "cheating-proof" in this respect. Moreover, the Nash announcement is consistent. Therefore it will be called credible.

When cheating is possible the question rises whether cheating is expected. Suppose that the second player expects the first player to cheat and suppose the first player expects this, etc. A process results of optimal reactions to and fro starting from the initial considerations of the first player. The credible solution concept is the only reasonable concept for this situation.

Suppose that the second player does not expect the first player to cheat. If the first player cheats anyhow, several outcomes are possible depending on at what point of time he cheats. It is reasonable to assume that after cheating has occurred the game falls back in the Nash mode. The player who can cheat on his announcement has a choice. If he cheats, from then on the Nash mode which is "cheating-proof" is the result. If

he does not cheat, he can benefit from the Stackelberg mode and from future possibilities to cheat. After all this player has to compare a cheating gain with a loss in reputation. The loss in reputation can be conceived as a penalty on cheating. This aspect should lead to a change in the concept of credibility. Furthermore, the expectation of the other player with respect to this reputation should play a role here.

In the case of sequential acting the only cheating possibility emanates from time inconsistency in the global mode. The credible announcement is the consistent Stackelberg announcement.

The feedback stagewise Stackelberg solution (FBS) is consistent by construction. A reoptimization or a reconsideration of strategy in the course of the game will not lead to a change in strategy. In the case of simultaneous acting the claim for credibility yields the feedback stage-wise Nash (FBN) solution.

In an open-loop information structure the claim for consistency tends to a contradictio in terminis. A reoptimization in the course of the game seems to require information on the state of the system in the course of the game which contradicts open-loop information. However, in this case the state of the system has to be seen as solely a function of the initial state and the performed actions. A consistent open-loop Stackelberg (COLS) solution can be defined along the lines of the model in Kydland and Prescott [1977]. It will be clear that stochastic modelling will find difficulties here. In the case of sequential acting the claim for consistency or credibility yields the consistent open-loop Stackelberg (COLS) solution. In the case of simultaneous acting the claim for consistency gives the same solution. However, the claim for credibility leads to the open-loop Nash (OLN) solution.

It is time to schematize these considerations and to give an example.

Table 1 shows the classical point of view. Two rules of the dynamic game are emphasized:

- what information is available to the players?
- do the players act simultaneously or sequentially?

Table 2 introduces where necessary the claim for consistency.

Table 3 extends the columns of table 1 to several possibilities that emanate from considering one player to announce his strategy before the actual play. In addition, where possible, outcomes can be considered in which somewhere cheating has occurred.

	simultaneous	sequential
open-loop	OLN	OLS
feedback/stagewise	FBN	FBS

Table 1. Classical

	simultaneous	sequential
open-loop	OLN	COLS
feedback/stagewise	FBN	FBS

Table 2. Consistency

sequential	reputation	consistent	credible
open-loop	OLS	COLS	COLS
feedback/stagewise	FBS	FBS	FBS
	cheating possible		
simultaneous	reputation	consistent	credible
open-loop	OLS	COLS	OLN
feedback/stagewise	FBS	FBS	FBN
	cheating possible	cheating possible	

Table 3. Announcements

OLN : open-loop Nash;
 OLS : open-loop Stackelberg;
 COLS: consistent open-loop Stackelberg;
 FBN : feedback/stagewise Nash;
 FBS : feedback/stagewise Stackelberg.

Example 4

The two-stage strict convex linear quadratic game from example 3 in section 2 is considered again. Player 2 is the player who makes the announcement.

Table 4 lists the possible solutions.

	$u_2(0)$	$u_2(1)$	$u_1(0)$	$u_1(1)$	$x(1)$	$x(2)$	J_2	J_1
OLS	-0.286	0	-0.429	-0.143	0.286	0.143	1.143	0.653
COLS	-0.286	-0.102	-0.408	-0.102	0.306	0.102	1.150	0.641
OLN	-0.526	-0.105	-0.263	-0.053	0.211	0.053	1.191	0.560
FBS	-0.321	-0.102	-0.374	-0.102	0.306	0.102	1.160	0.627
FBN	-0.528	-0.111	-0.25	-0.056	0.222	0.056	1.198	0.559

Table 4

It is always true in two player games that the Stackelberg outcomes are better for the leader than the Nash outcomes. In this example the follower is better off in the Nash concept.

When the information structures are compared, player 2 has lower costs in the open-loop structure and player 1 has lower costs in the feedback/stagewise structure in this example.

Suppose that the players act sequentially and player 2 announces the open-loop Stackelberg solution. In this case there is only one cheating possibility due to time inconsistency. The outcome is:

$$u_2(0) = -0.286, u_1(0) = -0.429 \quad x(1) = 0.286 \quad J_2 = 1.136$$

$$u_2(1) = -0.095, u_1(1) = -0.095 \quad x(2) = 0.095 \quad J_1 = 0.642$$

The cheating gain for player 2 is 0.007. This gain has to be compared with his loss in reputation. Note that cheating is also better for player 1!

Suppose that the players act simultaneously. In this case there are several cheating possibilities, when cheating is not expected. Table 5

lists the outcomes. The numbers I and II refer to cheating in the first and second period, respectively.

	$u_2(0)$	$u_2(1)$	$u_1(0)$	$u_1(1)$	$x(1)$	$x(2)$	J_2	J_1
OLS-I	-0.402	-0.085	-0.429	-0.085	0.169	0.042	1.115	0.608
OLS-II	-0.286	-0.095	-0.429	-0.142	0.286	0.048	1.129	0.644
COLS-I	-0.416	-0.088	-0.408	-0.044	0.175	0.044	1.123	0.601
COLS-II	-0.286	-0.136	-0.408	-0.102	0.306	0.068	1.148	0.638
FBS-I	-0.441	-0.093	-0.374	-0.046	0.186	0.046	1.138	0.589
FBS-II	-0.321	-0.136	-0.374	-0.102	0.306	0.068	1.159	0.624

Tabel 5.

In the open-loop information structure the best possibility for player 2 is the open-loop Stackelberg announcement with cheating in the first period. The cheating gain is 0.028. He should not cheat, when the penalty on cheating or the loss in reputation is equal or higher than 0.028. Note that player 1 has the second to lowest costs, when player 2 announces the consistent open-loop Stackelberg solution and cheats in the first period. He achieves the lowest costs when Nash is announced (and played).

In the feedback/stagewise information structure cheating in the first period is also the best possibility. The gain is 0.022. The same remarks for player 1 apply.

4. Conclusion

This paper deals primarily with the possible role of information and announcements in non-cooperative dynamic games where players act simultaneously.

Open-loop and feedback/stagewise information structures, which are directly linked to the minimum principle and dynamic programming, are well understood. It is felt, however, that feedback/global and memory information structures should still be subject of concern. For example, by means of a simple two stage game it is shown that there exist many equilibria in these information structures which dominate both the classical solutions.

The analysis of announcements fits very well in the present discussion in the literature with respect to the interaction between a policy declaration by the government and the actual play by both the private sector and the government. It is tried to conceptualize when and to what extent the classical sequential solution concepts reappear. Furthermore, for a simple example the several cheating possibilities for the leader are evaluated against his loss of reputation.

All things considered the feedback/stagewise Nash solution concept seems still the most appropriate concept for applications. It is a consistent, credible or cheating-proof and sub-game perfect equilibrium. However, it presupposes state information. Moreover, when the concept of reputation can be formalized further, the leader role of the government may reappear.

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Appendix

Let:

- (i) $T := \{0, 1, 2, \dots, t_f - 1\}$ be the time axis.
- (ii) $x(t) \in X$ be the state of the system at time t ,
 $t \in \{0, 1, \dots, t_f\}$
- (iii) Ω be the information set (note that $\Omega = X$ for the open-loop and the feedback information structure)
- (iv) V_i be the set of input values for player i ($i=1, 2$)
- (v) $[u_i : T \times \Omega \rightarrow V_i] \in U_i$ be the control law for player i ($i=1, 2$)
- (vi) $J_i : X \times U_1 \times U_2 \rightarrow \mathbb{R}$ be the function that determines the outcome of the game for player i ($i=1, 2$)
- (vii) $[u_{i,t} : \Omega \rightarrow V_i] \in U_{i,t}$ be the restriction of the control law u_i to the time point t ($i=1, 2, t \in T$)
- (viii) $\underline{u}_{i,t} := (u_{i,0}, u_{i,1}, \dots, u_{i,t-2}, u_{i,t-1}) \in \underline{U}_{i,t}$ be the restriction of the control law u_i to the time interval $[0, t-1]$
 $(i=1, 2, t \in T)$
- (ix) $\bar{u}_{i,t} := (u_{i,t}, u_{i,t+1}, \dots, u_{i,t_f-2}, u_{i,t_f-1}) \in \bar{U}_{i,t}$ be the restriction of the control law u_i to the time interval $[t, t_f-1]$
 $(i=1, 2, t \in T)$

Note that:

$$(u_{i,t}, \bar{u}_{i,t+1}) = \bar{u}_{i,t}$$

$$(\underline{u}_{i,t}, u_{i,t}) = \underline{u}_{i,t+1}$$

$$(\underline{u}_{i,t}, \bar{u}_{i,t}) = u_i$$

Definition 1 (see Basar and Olsder [1982] p. 94)

(u_1^*, u_2^*) is a global Nash equilibrium for $x(0) = x_0$ if

$$a) J_1(x_0, u_1^*, u_2^*) \leq J_1(x_0, u_1, u_2^*) \quad \forall u_1 \in U_1$$

$$b) J_2(x_0, u_1^*, u_2^*) \leq J_2(x_0, u_1^*, u_2) \quad \forall u_2 \in U_2$$

Definition 2 (see Basar and Olsder [1982] p. 127-128)

(u_1^*, u_2^*) is a global Stackelberg equilibrium for $x(0) = x_0$ if

$$a) \max_{u_1 \in F(x_0, u_2^*)} J_2(x_0, u_1, u_2^*) \leq$$

$$\max_{u_1 \in F(x_0, u_2)} J_2(x_0, u_1, u_2) \quad \forall u_2 \in U_2$$

$$b) u_1^* \in F(x_0, u_2^*)$$

$$\text{where } F(x_0, u_2) := \{u_1 \in U_1 \mid J_1(x_0, u_1, u_2) \leq J_1(x_0, \hat{u}_1, u_2) \quad \forall \hat{u}_1 \in U_1\}$$

Definition 3 (see Basar and Olsder [1982] p. 116-117)

(u_1^*, u_2^*) is a stagewise feedback Nash equilibrium if

$$a) J_1(x(0), (u_{1,t}, \bar{u}_{1,t}^*), (u_{2,t}, \bar{u}_{2,t}^*)) \leq$$

$$J_1(x(0), (u_{1,t}, u_{1,t}, \bar{u}_{1,t+1}^*), (u_{2,t}, \bar{u}_{2,t}^*)) \quad \forall u_{1,t} \in U_{1,t}$$

$$b) J_2(x(0), (u_{1,t}, \bar{u}_{1,t}^*), (u_{2,t}, \bar{u}_{2,t}^*)) \leq$$

$$J_2(x(0), (u_{1,t}, \bar{u}_{1,t}^*), (u_{2,t}, u_{2,t}, \bar{u}_{2,t+1}^*)) \quad \forall u_{2,t} \in U_{2,t}$$

$$\forall t \in T, \forall x(0) \in X, \forall u_{1,t} \in U_{1,t}, \forall u_{2,t} \in U_{2,t}$$

Note that the last line of this definition can be interpreted as "at any time, at any reachable state of the system".

Definition 4 (see Basar and Olsder [1982] p. 131)

(u_1^*, u_2^*) is a stagewise feedback Stackelberg equilibrium if:

$$a) \quad \max_{u_{1,t} \in \hat{F}_t(\bar{u}_{1,t+1}^*, \bar{u}_{2,t}^*)} J_2(x(0), (u_{1,t+1}, \bar{u}_{1,t+1}^*), (u_{2,t}, \bar{u}_{2,t}^*)) <$$

$$\max_{u_{1,t} \in \hat{F}_t(\bar{u}_{1,t+1}^*, (u_{2,t}, \bar{u}_{2,t+1}^*))} J_2(x(0), (u_{1,t+1}, \bar{u}_{1,t+1}^*),$$

$$(u_{2,t}, u_{2,t}, \bar{u}_{2,t+1}^*)) \quad \forall u_{2,t} \in U_{2,t}$$

$$b) \quad u_{1,t}^* \in \hat{F}_t(\bar{u}_{1,t+1}^*, \bar{u}_{2,t}^*)$$

$$\forall t \in T, \quad \forall x(0) \in X, \quad \forall u_{1,t} \in U_{1,t}, \quad \forall u_{2,t} \in U_{2,t}$$

$$\text{where } \hat{F}_t(\bar{u}_{1,t+1}, \bar{u}_{2,t}) := \{u_{1,t} \in U_{1,t} \mid J_1(x(0), u_{1,t}, u_{2,t}) <$$

$$J_1(x(0), (u_{1,t}, \hat{u}_{1,t}, \bar{u}_{1,t+1}), (u_{2,t}, \bar{u}_{2,t})) \quad \forall \hat{u}_{1,t} \in U_{1,t}\}$$

Proposition 1

(u_1^*, u_2^*) is a global feedback Nash equilibrium (for $x(0)=x_0$) if it is a stagewise Nash equilibrium.

Proof: The system player i ($i=1,2$) controls given u_j^* ($j=1,2, i \neq j$) is causal. Bellman's principle of optimality is valid so the (global) optimal value of the objective functional can be found by dynamic programming (see Basar and Olsder [1982], p. 117)

Proposition 2

In a strict convex linear quadratic framework the global linear feedback Nash equilibrium $\forall x(0) \in X$ exists and is unique.

This is also the unique stagewise feedback Nash equilibrium.

Proof: see de Zeeuw [1984] p. 100.

Definition 5 (see Kydland and Prescott [1977])

A control law u_i is consistent if $\forall t \in T$, $\bar{u}_{i,t}$ minimizes the objective functional (in the class of admissible control laws) given $x_0 \in X$, $\bar{u}_{i,t}$, $\bar{u}_{j,t}$ and player j 's (re)action $\bar{u}_{j,t}$.

In the stagewise mode, that is based on the idea of optimization at any moment in time, every equilibrium control law is consistent by construction. A global Stackelberg-leader control law is, in general, inconsistent. A consistent global Stackelberg equilibrium can be defined as follows:

let (for $t \in T$) :

$$(i) \quad G_{1,t}(x(0), \bar{u}_{1,t}, u_2) :=$$

$$\{\bar{u}_{1,t} \in \bar{U}_{1,t} \mid \bar{u}_{1,t+1} \in H_{1,t+1}(x(0), (\bar{u}_{1,t}, u_{1,t}), u_2)\} \quad \text{if } t < t_{f-1}$$

$$\bar{U}_1 \quad \text{if } t = t_{f-1}$$

$$(ii) \quad H_{1,t}(x(0), \bar{u}_{1,t}, u_2) :=$$

$$\{\bar{u}_{1,t} \in G_{1,t}(x(0), \bar{u}_{1,t}, u_2) \mid J_1(x(0), (\bar{u}_{1,t}, \bar{u}_{1,t}), u_2) <$$

$$J_1(x(0), (\bar{u}_{1,t}, \hat{\bar{u}}_{1,t}), u_2) \forall \hat{\bar{u}}_{1,t} \in G_{1,t}(x(0), \bar{u}_{1,t}, u_2)\}$$

$$(iii) \quad M_t(x(0), \underline{u}_{1,t}, \underline{u}_{2,t}) :=$$

$$\{\bar{u}_{2,t} \in \bar{U}_{2,t} \mid \bar{u}_{2,t+1} \in N_{t+1}(x(0), (\underline{u}_{1,t}, \underline{u}_{1,t}), (\underline{u}_{2,t}, \underline{u}_{2,t}))\}$$

$$\bigvee_{\bar{u}_{1,t} \in H_{1,t}(x(0), \underline{u}_{1,t}, \underline{u}_{2,t})} \quad \text{if } t < t_{f-1}$$

$$\underline{u}_2 \quad \text{if } t = t_{f-1}$$

$$(iv) \quad N_t(x(0), \underline{u}_{1,t}, \underline{u}_{2,t}) :=$$

$$\{\bar{u}_{2,t} \in M_t(x(0), \underline{u}_{1,t}, \underline{u}_{2,t}) \mid$$

$$\max_{\bar{u}_{1,t} \in H_{1,t}(x(0), \underline{u}_{1,t}, \underline{u}_{2,t})} J_2(x(0), (\underline{u}_{1,t}, \bar{u}_{1,t}), \underline{u}_2) <$$

$$\max_{\bar{u}_{1,t} \in H_{1,t}(x(0), \underline{u}_{1,t}, (\underline{u}_{2,t}, \hat{\underline{u}}_{2,t}))} J_2(x(0), (\underline{u}_{1,t}, \bar{u}_{1,t}), (\underline{u}_{2,t}, \bar{u}_{2,t}))$$

$$\bigvee_{\bar{u}_{2,t} \in M_t(x(0), \underline{u}_{1,t}, \underline{u}_{2,t})} \}$$

Definition 6

$(\underline{u}_1^*, \underline{u}_2^*)$ is a consistent Stackelberg equilibrium for $x(0) = x_0$ if

$$a) \quad \underline{u}_2^* \in N_0(x_0)$$

$$b) \quad \underline{u}_1^* \in H_{1,0}(x_0, \underline{u}_2^*)$$

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