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**EXCLUSION RESTRICTIONS IN INSTRUMENTAL
VARIABLES EQUATIONS**

Theo E. Nijman
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FEW 327

EXCLUSION RESTRICTIONS IN INSTRUMENTAL VARIABLES EQUATIONS

Theo E. Nijman^{*)}
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April 1988

Abstract

In this paper we consider two stage estimators of parameters of a structural equation in a model with exclusion restrictions on the instrumental variables equations. The estimators considered are simple OLS and GLS estimators after substitution of estimates of the systematic part of the IV equations for the endogenous variables. It is known in the literature that neither imposing the restrictions in the first stage nor ignoring them will in general be more efficient than the alternative. We introduce a class of mixed instrumental variables estimators (MIV) with these two possibilities as special cases which yields an estimator which is not only more efficient than the two stage estimators considered in the literature but as efficient as an efficient system estimator like 3SLS.

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1.Introduction

A very common phenomenon in econometrics is the necessity to estimate far more parameters than the model user is ultimately interested in. The reasons for attaching a particular importance to a small subset of the parameters, the so-called parameters of interest, may be multiple. Their meaning may derive from economic theory and inference on them may constitute evidence in favour of or against certain theories. An alternative source of interest might be their relative stability in changing environments, leading to models which can be used for policy simulations.

Two examples in which the problem of the dimensionality of the vector of nuisance parameters is often particularly important are incomplete simultaneous equations models and models containing unobserved rational expectations. As explained in detail in Richard (1979) and Richard (1984) formulating statistical assumptions in terms of the observable endogenous variables instead of adding random error terms to an already existing set of deterministic equations will often lead to incomplete simultaneous equation models. As such an incomplete system allows an infinity of solutions, auxiliary equations have to be added, which usually express the endogenous variables as linear functions of a set of instrumental variables and introduce a number of nuisance parameters in the model. In case of a model containing unobserved rational expectations, the model has to be completed by equations that describe how these expectations are generated. The parameters in these equations will also typically be nuisance parameters only.

In this paper we assume that the parameters of interest are the coefficients of a structural equation in a simultaneous equation model. We restrict ourselves to a case of limited information, i.e. the model is completed with reduced form equations, but we assume that a block of zero restrictions on the reduced form coefficients holds. These zero restrictions can originate e.g. from economic theory, from assumptions of strong exogeneity in the sense of Engle et.al. (1983), etc. Efficient parameter estimates can of course be obtained using a three stage least squares procedure, but this approach is computationally not very attractive if the number of nuisance parameters is large. In applications one will typically use a two stage least squares procedure either imposing the zero restrictions on

the reduced form or not. It is well known in the literature (see e.g. Turkington (1985)) that imposing the restrictions in the first stage does not necessarily yield an efficiency gain over standard (unrestricted) two stage least squares. In this paper we show how to construct simple two stage estimators which are more efficient than 2SLS. Moreover we derive a two stage estimator which is as efficient as 3SLS.

The plan of this paper is as follows. In section two we introduce the model, while section three describes the class of mixed IV estimators to be considered. The optimal mixed IV estimator and its asymptotic efficiency are derived in section four. In section five we extend the results to a more general model. Finally, section six contains some concluding remarks.

2. The model

Consider the following model:

$$y = Y_1 \beta_1 + Y_2 \beta_2 + W \beta_3 + e \quad (1)$$

$$Y_1 = Z_1 \Pi_{11} + Z_2 \Pi_{21} + V_1 \quad (2)$$

$$Y_2 = Z_1 \Pi_{12} + \quad + V_2 \quad (3)$$

where Y_1 , Y_2 , W , Z_1 and Z_2 are matrices of dimension $T \times k_1$, $T \times k_2$, $T \times k_3$, $T \times l_1$, $T \times l_2$ respectively, and assume that

$$V(e \mid V_1 \mid V_2) = \Sigma \otimes I_T, \quad (4)$$

where $\Sigma \in C^{k+1}$, the space of all real PDS matrices of dimension $(k_1+k_2+1) \times (k_1+k_2+1)$. The matrix Σ will be partitioned corresponding to $(e \mid V_1 \mid V_2)$ as

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

The variables in Z_1 , Z_2 , and W are assumed to be weakly exogenous for the parameters in (1), (2) and (3). Note that the exogenous variables in the structural equation (1) are not necessarily included in the reduced form equations (2) and (3).

The model outlined above is sufficiently general to illustrate the argument and to be of relevance in applications but not too general to cloud the essentials by tedious mathematical details. The model introduces some flexibility in the choice of instruments by avoiding that the same set of instruments is attributed to every Y variable. This flexibility can be important on three accounts: the suitability of instruments might vary a lot over the variables in Y , one might end up with more parsimonious models if the flexibility is incorporated and non-causality (strong exogeneity) assumptions can be imposed. For a detailed discussion of the merits of flexible IV equations see Richard (1984), Lubrano et. al. (1986), Steel (1987) and Richard and Steel (1988).

A special case of (1)-(3) which has been extensively discussed in the literature (see e.g. Pagan (1984), Turkington (1985) and Pesaran (1987)) is the model

$$y = Y_2^e \beta_1 + W\beta_3 + e \quad (5)$$

$$Y_2 = Z_1 \pi_{12} + V_2 \quad (6)$$

$$Y_2^e = Z_1 \pi_{12} \quad (7)$$

where Y_2^e denotes the rational expectation of Y_2 conditional on the variables in Z_1 and W . Substitution of the realization Y_2 for the unobserved expectations Y_2^e yields (1)-(3) with $k_1 = 0$. Moreover it is well known that the results for (5)-(7) carry over to the static rational expectations model with unanticipated components (see e.g. Pesaran (1987), section 7.3).

In the sequel two cases will be distinguished, referred to as model A and model B. In sections 3 and 4 we restrict ourselves to model A in which all exogenous variables in the structural equation (1) appear in the instrumental variables equations (2) and (3). In section 5 we generalize the results to model B in which not all exogenous variables in (1) appear in the IV equations, implying that zero restrictions common to all IV equations are imposed. Model A is referred to as the conventional simultaneous equations model (SEM) by Richard (1984) who stresses that model B has much greater empirical relevance.

Model A can be written in short as

$$y = X \beta + e \quad (8)$$

$$X = Z \Pi + V \quad (9)$$

where

$$X = (Y_1 \mid Y_2 \mid W),$$

$$Z = (Z_1 \mid Z_2),$$

$$V = (V_1 \mid V_2 \mid 0),$$

$$\beta' = (\beta'_1 \mid \beta'_2 \mid \beta'_3),$$

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & S_{13} \\ \pi_{21} & 0 & S_{23} \end{bmatrix},$$

and S_{13} and S_{23} are selection matrices.

If the submatrix of W consisting of variables which do not appear in (2) and (3) is denoted by W_2 , model B can be written as

$$y = X \beta + e \quad (10)$$

$$X = Z_* \Pi_* + V \quad (11)$$

where

$$Z_* = (Z_1 \mid Z_2 \mid W_2),$$

$$\Pi_* = \begin{bmatrix} \Pi & 0 \\ 0 & I \end{bmatrix}.$$

Throughout we assume that $\text{plim } T^{-1}Z_*'Z_*$ is finite and non-singular.

3. The class of estimators considered

As already noted in the introduction, the parameters of interest of our analysis are assumed to be confined to the structural equation as this is the part of the system that originated from economic theory conveying relevance and interpretability to its parameters. Therefore our parameters of interest are β and σ_{11} whereas the parameters in Π and the other elements of Σ will be treated as nuisance parameters. As has often been stressed in the literature there is no need to estimate all parameters of such a system jointly. This has prompted the wide-spread use of two stage estimators, the general theory of which is discussed in Pagan (1986).

Of course one could also use a full system method like 3SLS to obtain efficient estimates of β . However 3SLS will require matrix inversions of a dimension equal to the total amount of free parameters in β and Π . The dimension of the matrices to be inverted in two stage approaches will usually be far smaller. Admittedly, the specific structure of model A allows a considerable simplification of the computation of the 3SLS estimator (see e.g. Schmidt (1976, p.216)) but even in this case it will still be computationally more demanding than two stage procedures. In addition the computational differences become much clearer if the model gains in generality as in model B.

As stated before we restrict ourselves in sections 3 and 4 to model A, i.e. each variable in W appears either in Z_1 or in Z_2 . Extensions to model B

will be treated in section 5. It is easily verified that a regression of y on $Z\bar{\Pi}$ with $\bar{\Pi}$ a consistent estimator of Π yielding the estimator

$$\hat{\beta}_{OLS} = (\bar{\Pi}'Z'Z\bar{\Pi})^{-1} \bar{\Pi}'Z'y \quad (12)$$

will be consistent for β . If $\bar{\Pi}$ is chosen to be the unrestricted estimator $\hat{\Pi}$ of Π ,

$$\hat{\Pi} = (Z'Z)^{-1} Z'Y \quad (13)$$

with $Z = (Z_1 \mid Z_2)$ and $Y = (Y_1 \mid Y_2)$ the estimator $\hat{\beta}_{OLS}$ coincides with 2SLS. Because every Y variable is explained by the same set of instruments this procedure will be denoted by "common blocks of instrumental variables" or CIV following Richard (1987). In the literature on rational expectations the CIV or 2SLS estimator is usually referred to as the "errors in variables method", following Wickens (1982), because Y_2^e in (5) is replaced by Y_2 and the problem is subsequently treated as a standard errors in variables model.

A choice of $\bar{\Pi}$ in (12) which imposes the zero restrictions in the IV equations is

$$\tilde{\Pi} = \left[\begin{array}{c|c|c} \tilde{\Pi}_1 & \tilde{\Pi}_{12} & \begin{matrix} S_{13} \\ S_{23} \end{matrix} \end{array} \right], \quad (14)$$

where

$$\tilde{\Pi}_1 = (Z_1'Z)^{-1} Z_1'Y_1,$$

$$\tilde{\Pi}_{12} = (Z_1'Z_1)^{-1} Z_1'Y_2.$$

This procedure will be labelled "individual blocks of instrumental variables" (IIV). In the rational expectations literature this procedure is,

somewhat misleadingly perhaps, referred to as the substitution approach although of course every procedure based on (12) is based on substitution of an estimate $\bar{\Pi}$ for Π .

It has been established in the literature (see e.g. Pagan (1984) and Turkington (1985)) that neither CIV nor IIV will in general be more efficient than the other. This is easily checked for the special case considered in Nijman (1985) with only two endogenous variables in the right hand side of the structural equation in which case $k_1 = k_2 = 1$. Let us first of all reconsider this case. If we define $P_R = R(R'R)^{-1}R'$ and $M_R = I - P_R$ for arbitrary matrix R of full column rank, $\hat{\beta}_{OLS}$ is the OLS estimator of β in

$$y = Z \bar{\Pi} \beta + w \quad (15)$$

with

$$w = \epsilon + M_Z (V_1 \beta_1 + V_2 \beta_2) \text{ in case of CIV}$$

and

$$w = \epsilon + M_Z V_1 \beta_1 + M_{Z_1} V_2 \beta_2 \text{ in case of IIV.}$$

The large sample variance of $\sqrt{T} \hat{\beta}_{OLS}$ can therefore be written as

$$\text{Avar}(\hat{\beta}_{OLS}) = B^{-1} \{ \sigma_{11} B + a (B-C) \} B^{-1} \quad (16)$$

with $B = \text{plim} T^{-1} \Pi' Z' Z \Pi$, $C = \text{plim} T^{-1} \Pi' Z' P_{Z_1} Z \Pi$, and $a = 0$ for CIV while $a = \beta_2^2 \sigma_{33} + 2\beta_2 \sigma_{13}$ for IIV. Note that as $B-C$ is positive semi-definite the sign of the coefficient in case of IIV determines the relative efficiency of CIV and IIV, i.e. of ignoring the restrictions versus incorporating them exactly in the first stage. In particular if β_2 and σ_{13} have similar signs CIV will be more efficient than IIV which implies that we lose efficiency by taking the true exclusion restrictions into account in the first stage in this manner.

A natural question to ask next is whether we can find a way in between the two extremes considered here which will dominate both in terms of the asymptotic covariance matrix. For this simple example such an estimator was derived by Nijman (1985) who proposed to put

$$\bar{\pi} = \gamma \tilde{\pi} + (1-\gamma) \hat{\pi} \quad (17)$$

with $\gamma = -\beta_2^{-1} \sigma_{33}^{-1} \sigma_{13}$. The normalized large sample variance of the resulting estimator of β is again given by (16), now with $a = -\sigma_{13}^2 \sigma_{33}^{-1}$. Therefore this estimator is more efficient than CIV as well as IIV. In applications the weights γ and $1-\gamma$ in (17) will have to be replaced by consistent estimates but this does not affect the large sample variance of the resulting estimator (see appendix).

In this paper we generalize the proposition in Nijman (1985) and propose to use in model A in general the OLS estimator defined in (12) with

$$\bar{\pi} = \tilde{\pi} \Gamma + \hat{\pi} (I - \Gamma) \quad (18)$$

where Γ is a $(k_1 + k_2) \times (k_1 + k_2)$ weighting matrix. This approach will be denoted by mixed instrumental variables (MIV) estimation. Note that it reduces to CIV for $\Gamma = 0$ and to IIV for $\Gamma = I$.

Another estimator that we will consider in this paper instead of $\hat{\beta}_{OLS}$ in (12) is the GLS estimator

$$\hat{\beta}_{GLS} = (\bar{\pi}' Z' \hat{\Omega}^{-1} Z \bar{\pi})^{-1} \bar{\pi}' Z' \hat{\Omega}^{-1} y \quad (19)$$

where $\bar{\pi}$ is again generated by (18), Ω is the $T \times T$ covariance matrix of the structural disturbance term after substituting for the endogenous variables and the hat ($\hat{}$) denotes that unknown parameters are replaced by consistent estimates. A similar GLS estimator has recently been proposed by Hoffman

(1987), who however considers IIV only. Of course the GLS estimator will be at least as efficient as the OLS estimator asymptotically. Expressions for Ω and its inverse will be given in the next section.

4. The choice of the weighting matrix

In this section we will derive the weighting matrix Γ which minimizes the asymptotic variance covariance matrix of the OLS and GLS estimators introduced in the previous section.

The first estimator to be considered is the OLS estimator of β in

$$y = Z \bar{\Pi} \beta + w, \quad (20)$$

with

$$w = e + V\beta + Z(\Pi - \bar{\Pi})\beta + Z(\hat{\Pi} - \bar{\Pi})\Gamma\beta. \quad (21)$$

It is evident from (21) that if we partition Γ as

$$\Gamma = \begin{bmatrix} \Gamma_{1.} \\ \Gamma_{2.} \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix},$$

corresponding to the partitioning in $(Y_1 | Y_2)$ and $(Z_1 | Z_2)$, the choice of $\Gamma_{1.}$ will be irrelevant because the first k_1 columns of $\hat{\Pi}$ and $\bar{\Pi}$ coincide. Now assume for simplicity that e and V are independent of Z . The variance covariance matrix of w conditional on Z can be shown (see appendix) to equal

$$E[ww' | Z] = \Omega = (\sigma_{11} + b_1) P_Z + b_2 M_Z - b_1 P_{Z_1}, \quad (22)$$

with

$$b_1 = (\Gamma_{2.}\beta + \Sigma_{33}^{-1}\sigma_{31})' \Sigma_{33} (\Gamma_{2.}\beta + \Sigma_{33}^{-1}\sigma_{31}) - \sigma_{13} \Sigma_{33}^{-1} \sigma_{31} \quad (23)$$

and

$$b_2 = (1 \mid \beta') \Sigma \begin{bmatrix} 1 \\ \beta \end{bmatrix}. \quad (24)$$

Generalizing equation (16) the large sample variance of $\sqrt{T}\hat{\beta}_{OLS}$ can be expressed as

$$\text{Avar}(\hat{\beta}_{OLS}) = B^{-1} \{ \sigma_{11}B + b_1 (B - C) \} B^{-1}, \quad (25)$$

with B and C defined below (16). From (23) and (25) it is evident that the weights of the optimal MIV estimator should satisfy

$$\Gamma_{2,\beta} = -\Sigma_{33}^{-1} \sigma_{31}, \quad (26)$$

in which case the large sample variance is given by (25) with b_1 replaced by b_1^{opt} defined as

$$b_1^{\text{opt}} = -\sigma_{13} \Sigma_{33}^{-1} \sigma_{31}. \quad (27)$$

As $\Gamma_{2,\cdot}$ contains $k_2 \times (k_1 + k_2)$ free elements and (26) only defines k_2 equations we have (unless $k_1 = 0$ and $k_2 = 1$) a continuum of solutions for the weighting matrix Γ that will yield an efficient MIV estimator. The restrictions on Π are imposed on $\tilde{\Pi}$ if $\Gamma_{22} = I$ only. One can e.g. impose the restrictions and choose the efficient MIV estimator as

$$\Gamma_{22} = I; \quad \Gamma_{21} = -(\beta_1' \beta_1)^{-1} (\Sigma_{33}^{-1} \sigma_{31} + \beta_2') \beta_1', \quad (28)$$

but one can just as well choose e.g.

$$\Gamma_{21} = 0 \quad ; \quad \Gamma_{22} = -(\beta_2' \beta_2)^{-1} \Sigma_{33}^{-1} \sigma_{31} \beta_2', \quad (29)$$

which generalizes the result below (17). It can be checked that both Zellner (1962) 's SURE estimator and Richard (1984) 's restricted maximum likelihood estimator of Π from (2) and (3) coincide with $\bar{\Pi}$ in (18) if Γ is chosen as

$$\Gamma_{\text{SUR}} = \Gamma_{\text{RML}} = \begin{bmatrix} 0 & 0 \\ \hat{\Sigma}_{33}^{-1} \hat{\Sigma}_{32} & I \end{bmatrix}, \quad (30)$$

where $\hat{\Sigma}_{ij} = T^{-1} Y_{i-1}' M_Z Y_{j-1}$. This choice of the weighting matrix Γ does not satisfy (26) and therefore the use of such an efficient estimator of the parameters in the IV equations in order to obtain the OLS estimator in (12) leads to an efficiency loss for the parameters of interest.

Let us now consider the GLS estimator defined in (19). The structure of Ω in (22) gives rise to a simple expression for its inverse

$$\Omega^{-1} = b_2^{-1} M_Z + (\sigma_{11} + b_1)^{-1} \{P_Z + b_1 \sigma_{11}^{-1} P_{Z_1}\}, \quad (31)$$

which avoids the need to invert a $T \times T$ matrix numerically. Using this result the large sample variance of $\sqrt{T} \hat{\beta}_{\text{GLS}}$ can be shown to be

$$\text{Avar}(\hat{\beta}_{\text{GLS}}) = (\sigma_{11} + b_1) \{B + b_1 \sigma_{11}^{-1} C\}^{-1}, \quad (32)$$

where $C = \text{plim} T^{-1} \Pi' Z' P_{Z_1} Z \Pi$ and $B = \text{plim} T^{-1} \Pi' Z' Z \Pi$ as before. The most efficient estimator is obtained again if b_1 is minimal, that is if Γ satisfies equation (26). Finally note that the assumption of independence between e and V on the one hand and Z on the other is only required to interpret Ω as the variance covariance matrix of w conditional on Z . The results in (25) and (32) remain valid if this assumption is not met.

Let us now consider whether the proposed two stage estimators will be as efficient as an efficient full system estimator. The asymptotic variance covariance matrix of the latter can easily be obtained using the result (see e.g. Schmidt (1976, p. 216)) that because equation (2) is exactly identified it can be ignored in performing 3SLS on the rest of the system, except for the fact that this equation might supply us with some variables in Z_2 in so far these are not already present in the structural equation. The expression obtained along these lines in the appendix for the large sample variance of $\sqrt{T} \hat{\beta}_{3SLS}$ is

$$\text{Avar}(\hat{\beta}_{3SLS}) = \{ \tilde{\sigma}^{11} B - \tilde{\sigma}^{13} (\tilde{\Sigma}^{33})^{-1} \tilde{\sigma}^{31} C \}^{-1} \quad (33)$$

where superscripts refer to the corresponding blocks in the inverse and a tilde ($\tilde{}$) indicates that a block is derived from the "shrunk" Σ matrix

$$\tilde{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{31} & \Sigma_{33} \end{bmatrix}. \quad (34)$$

If the optimal weighting matrix Γ which satisfies (26) is used the variance covariance matrix of $\sqrt{T} \hat{\beta}_{GLS}$ is given in (32) with b_1 replaced by b_1^{opt} in (27). It is easily seen that this expression coincides with (33) so that in case of optimal weighting $\hat{\beta}_{GLS}$ is as efficient as $\hat{\beta}_{3SLS}$.

5. Extension to exogenous variables excluded from the IV equations.

The results obtained in the previous sections can be extended to a number of more general models. The only extension we consider in this paper is to the case where exogenous variables appearing in the structural equation are excluded from the IV equations, referred to as model B in section 2 where the model and the notation were introduced.

Analogously to earlier results, define $\bar{\Pi}_*$ to be some consistent estimator of the matrix Π_* in (11) and consider the estimator

$$\hat{\beta}_{OLS} = (\bar{\pi}_* ' Z_* ' Z_* \bar{\pi}_*)^{-1} \bar{\pi}_* ' Z_* ' y \quad (35)$$

which is consistent for the parameter β in (10) under the assumptions made.

Many choices of $\bar{\pi}_*$ can be thought of, e.g.

$$\hat{\pi}_* = \begin{bmatrix} \hat{\pi} & 0 \\ 0 & I \end{bmatrix} \quad (36)$$

or

$$\tilde{\pi}_* = \begin{bmatrix} \tilde{\pi} & 0 \\ 0 & I \end{bmatrix} \quad (37)$$

or

$$\pi_*^* = (Z_* ' Z_*)^{-1} Z_* ' x \quad (38)$$

where $\hat{\pi}$ and $\tilde{\pi}$ have been defined in (13) and (14). Note that the restrictions that some exogenous variables do not appear in the IV equation are imposed in (36) and (37) but not in (38). We now restrict ourselves to estimators $\bar{\pi}_*$ satisfying

$$\bar{\pi}_* = \tilde{\pi}_* \Delta + \hat{\pi}_* (\Lambda - \Delta) + \pi_*^* (I - \Lambda). \quad (39)$$

The OLS estimator in (35) is the OLS estimator of β in

$$y = Z_* \bar{\pi}_* \beta + w \quad (40)$$

with

$$w = e + (I - P_{Z_*}) V\beta + (P_{Z_*} - P_Z)(V_1 \wedge_{1.} \beta + V_2 \wedge_{2.} \beta) + (P_Z - P_{Z_1}) V_2 \Delta_{2.} \beta \quad (41)$$

so that (22) can be generalized to

$$E[ww' | Z] = \Omega = \sigma_{11} I + c_1 (I - P_{Z_*}) + c_2 (P_{Z_*} - P_Z) + c_3 (P_Z - P_{Z_1}) \quad (42)$$

with

$$c_1 = (1 \quad \beta') \Sigma \begin{bmatrix} 1 \\ \beta \end{bmatrix} - \sigma_{11},$$

$$c_2 = (1 \quad \beta' \wedge_{1.}' \quad \beta' \wedge_{2.}') \Sigma \begin{bmatrix} 1 \\ \wedge_{1.} \beta \\ \wedge_{2.} \beta \end{bmatrix} - \sigma_{11},$$

$$c_3 = (1 \quad \beta' \Delta_{2.}') \tilde{\Sigma} \begin{bmatrix} 1 \\ \Delta_{2.} \beta \end{bmatrix} - \sigma_{11},$$

where $\wedge_{1.}$ and $\wedge_{2.}$ are the first and second block of rows of $\wedge$ respectively, $\Delta_{2.}$ is the second block of rows of Δ and $\tilde{\Sigma}$ is defined in (34). The optimal values of Δ and $\wedge$ should satisfy

$$\Delta_{2.} \beta = - \Sigma_{33}^{-1} \sigma_{31} \quad (43)$$

and

$$\begin{bmatrix} \wedge_{1.} \beta \\ \wedge_{2.} \beta \end{bmatrix} = - \begin{bmatrix} \Sigma_{22} & \Sigma_{23} \\ \Sigma_{32} & \Sigma_{33} \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{21} \\ \sigma_{31} \end{bmatrix} \quad (44)$$

in which case we obtain (42) with c_2 and c_3 replaced by

$$c_2^{\text{opt}} = - (\sigma_{12} \ \sigma_{13}) \begin{bmatrix} \Sigma_{22} & \Sigma_{23} \\ \Sigma_{32} & \Sigma_{33} \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{21} \\ \sigma_{31} \end{bmatrix} = (\sigma^{11})^{-1} - \sigma_{11} \quad (45)$$

and

$$c_3^{\text{opt}} = - \sigma_{13} \Sigma_{33}^{-1} \sigma_{31} = (\tilde{\sigma}^{11})^{-1} - \sigma_{11} \quad (46)$$

respectively.

In general the OLS estimator will not be as efficient as a system estimator like 3SLS. However one can use the above results to define the GLS estimator

$$\hat{\beta}_{\text{GLS}} = (\bar{\Pi}_* ' Z_* ' \hat{\Omega}^{-1} Z_* \bar{\Pi}_*)^{-1} \bar{\Pi}_* ' Z_* ' \hat{\Omega}^{-1} y \quad (47)$$

where Ω is the variance covariance matrix of w in (40) and the hat denotes that unknown parameters have been replaced by consistent estimates. The properties of projection matrices can be used to show that if (43) and (44) hold the inverse of Ω is given by

$$\Omega^{-1} = (\sigma_{11} + c_1)^{-1} (I - P_{Z_*}) + \sigma^{11} P_{Z_*} + (\tilde{\sigma}^{11} - \sigma^{11}) P_Z + (\sigma_{11}^{-1} - \tilde{\sigma}^{11}) P_{Z_1} \quad (48)$$

so that the large sample variance of the optimal GLS estimator will be

$$\text{Avar}(\hat{\beta}_{\text{GLS}}) = \{ \sigma^{11} A_* + (\tilde{\sigma}^{11} - \sigma^{11}) B_* + (\sigma_{11}^{-1} - \tilde{\sigma}^{11}) C_* \}^{-1} \quad (49)$$

where

$$A_* = \text{plim} T^{-1} \bar{\Pi}_* ' Z_* ' Z_* \bar{\Pi}_* , \quad (50)$$

$$B_* = \text{plim} T^{-1} \bar{\Pi}_* ' Z_* ' P_Z Z_* \bar{\Pi}_* , \quad (51)$$

$$C_* = \text{plim} T^{-1} \bar{\Pi}_* ' Z_* ' P_{Z_1} Z_* \bar{\Pi}_* . \quad (52)$$

Finally the theory of partitioned matrices can be used to show that the large sample variance of an efficient estimator of β such as 3SLS coincides with (49) (see appendix). As in the previous sections a correctly weighted MIV estimator will be fully efficient.

6. Concluding remarks

In this paper we considered two stage estimators of parameters of a structural equation in a model with exclusion restrictions on the instrumental variables equations. Neither imposing these restrictions in the first stage regression (IIV) nor ignoring them (CIV) will in general be more efficient than the alternative even if e.g. the efficient SUR estimator of the reduced form coefficients is used. We introduced a class of mixed instrumental variables estimators (MIV) with IIV and CIV as special cases which yields a two stage estimator which is more efficient than IIV and CIV and as efficient as an efficient system estimator such as 3SLS. The estimator of the reduced form coefficients to be substituted in the structural equation is a weighted average of the standard restricted and unrestricted estimators. The resulting estimator of the structural parameters is computationally much more attractive than other efficient estimators such as 3SLS.

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Appendix Details on the technicalities

In this appendix we will derive the result in (22), we show that the substitution of consistent estimates for unknown coefficients in the weighting matrices in equations like (17), (18) and (39) does not affect the large sample variance of the two-stage estimators and finally we show how the expression for the asymptotic variance covariance matrix of the 3SLS estimator can be obtained.

In order to prove the result in (22) we first rewrite w in (21) as

$$w = e + (I - P_Z) V\beta + (P_Z - P_{Z_1}) V_2 \Gamma_2' \beta. \quad (A1)$$

Using the well known fact that for matrices and vectors of appropriate dimensions it holds true that $\text{vec}(NUn) = (n' \otimes I_N) \text{vec } U$ one can establish that if

$$V(U_1' : U_2') = S \otimes I_T \quad (A2)$$

where U_1 and U_2 are $T \times r_1$ and $T \times r_2$ matrices of random variables it will also be true that

$$E \text{vec}(N_1 U_1 n_1) \text{vec}'(N_2 U_2 n_2) = n_1' S_{12} n_2 N_1 N_2' \quad (A3)$$

if N_1 and N_2 are $T \times T$ matrices and n_1 and n_2 are $r_1 \times 1$ and $r_2 \times 1$ vectors respectively, and S_{12} is the upper-right block of the matrix S defined in (A2). The evaluation of $\Omega = E[ww' | Z]$ involves a large number of expressions of the type (A3), yielding

$$\Omega = b_2 I + (b_1 - b_2 + \sigma_{11}) P_Z - b_1 P_{Z_1} \quad (A4)$$

which can be rearranged to (22).

As far as the estimation of the weighting matrices is concerned, reconsider e.g. equations (20) and (21). If the weighting matrix Γ is estimated (20) and (21) have to be replaced by

$$y = Z \tilde{\Pi} \beta + \hat{w} \quad (20')$$

with

$$\hat{w} = e + V\beta + Z(\Pi - \tilde{\Pi})\beta + Z(\tilde{\Pi} - \hat{\Pi})\hat{\Gamma}\beta. \quad (21')$$

Note however that

$$(1/\sqrt{T}) \Pi' Z' \hat{w} = (1/\sqrt{T}) \Pi' Z' w + \Pi' T^{-1} Z' Z \sqrt{T}(\tilde{\Pi} - \hat{\Pi}) (\hat{\Gamma} - \Gamma)\beta \quad (A5)$$

and that the second term in the right hand side of (A5) converges to zero in probability, so that the large sample variance of $\hat{\beta}_{OLS}$ is not affected by the estimation of Γ . Similar arguments hold true for the models in (17) and (39) and for GLS estimators.

Finally we consider the expressions for the large sample variance covariance matrix of the 3SLS estimator. First of all write the model in (1)-(3) in vec form as

$$y^* = H \delta + u \quad (A6)$$

with

$$y^* = \text{vec} (y \mid Y_1 \mid Y_2) = \begin{bmatrix} y \\ \text{vec } Y_1 \\ \text{vec } Y_2 \end{bmatrix}, \quad (A7)$$

$$H = \begin{bmatrix} (Y_1 \mid Y_2 \mid w) & 0 & 0 \\ 0 & (I_{k_1} \otimes Z) & 0 \\ 0 & 0 & (I_{k_2} \otimes Z_1) \end{bmatrix}, \quad (A8)$$

$$\delta' = (\beta_1' \mid \beta_2' \mid \beta_3' \mid \text{vec}'\Pi_1 \mid \text{vec}'\Pi_{12}) , \quad (\text{A9})$$

and

$$u' = (e' \mid \text{vec}'V_1 \mid \text{vec}'V_2) . \quad (\text{A10})$$

In equation (4) we have already assumed that $V(u) = \Sigma \otimes I_T$. It is well known (see e.g. Schmidt (1976), p.207) that the asymptotic variance covariance matrix of the 3SLS estimator $\sqrt{T}\hat{\delta}_{3\text{SLS}}$ of δ can be expressed as

$$\text{Avar}(\hat{\delta}_{3\text{SLS}}) = \text{plimT} \{ H' (\Sigma^{-1} \otimes P_{Z_*}) H \}^{-1} \quad (\text{A11})$$

with P_{Z_*} defined in the main text. From (A11) we then deduce the asymptotic covariance matrix of the first $k_1+k_2+k_3$ elements of $\hat{\beta}_{3\text{SLS}}$ using the theory of partitioned matrices and projection matrices.

Rewrite (A11) in the notation of (10) and (11), denoting blocks of Σ^{-1} by superscripts, as $\text{Avar}(\hat{\delta}_{3\text{SLS}}) =$

$$T \begin{bmatrix} \sigma^{11} \Pi_*' Z_*' Z_* \Pi_* & \sigma^{12} \otimes \Pi_*' Z_*' Z & \sigma^{13} \otimes \Pi_*' Z_*' Z_1 \\ \sigma^{21} \otimes Z' Z_* \Pi_* & \Sigma^{22} \otimes Z' Z & \Sigma^{23} \otimes Z' Z_1 \\ \sigma^{31} \otimes Z_1' Z_* \Pi_* & \Sigma^{32} \otimes Z_1' Z & \Sigma^{33} \otimes Z_1' Z_1 \end{bmatrix}^{-1} . \quad (\text{A12})$$

The upper-left block of this inverse corresponds to β and can be written as

$$\text{Avar}(\hat{\beta}_{3\text{SLS}}) = \{ \sigma^{11} A_* - \sigma^{12} (\Sigma^{22})^{-1} \sigma^{21} B_* - c_* C_* \}^{-1} \quad (\text{A13})$$

where

$$c_* = (\sigma^{13} - \sigma^{12}(\Sigma^{22})^{-1}\Sigma^{23}) (\Sigma^{33} - \Sigma^{32}(\Sigma^{22})^{-1}\Sigma^{23})^{-1} (\sigma^{31} - \Sigma^{32}(\Sigma^{22})^{-1}\sigma^{21}),$$

and A_* , B_* and C_* are defined in (50)-(52). Using the fact that the inverse of the "shrunk" Σ matrix in (34) can be written as

$$\tilde{\Sigma}^{-1} = \begin{bmatrix} \sigma^{11} - \sigma^{12}(\Sigma^{22})^{-1}\sigma^{21} & \sigma^{13} - \sigma^{12}(\Sigma^{22})^{-1}\Sigma^{23} \\ \sigma^{31} - \Sigma^{32}(\Sigma^{22})^{-1}\sigma^{21} & \Sigma^{33} - \Sigma^{32}(\Sigma^{22})^{-1}\Sigma^{23} \end{bmatrix} \quad (A14)$$

and realizing that

$$\sigma_{11}^{-1} = \tilde{\sigma}^{11} - \tilde{\sigma}^{13} (\tilde{\Sigma}^{33})^{-1} \tilde{\sigma}^{31} \quad (A15)$$

we obtain that $\text{Avar}(\hat{\beta}_{3\text{SLS}})$ coincides with $\text{Avar}(\hat{\beta}_{\text{GLS}})$ in (49). Clearly the efficiency of GLS in model A is a special case of this result.

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