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ten Raa, Thijs; Chakraborty, D.

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A Note on Induced Multiplier

T. T. Raa and D. Chakraborty

The object of the present note is to establish a relationship that exists between the direct and indirect income multiplier and the direct, indirect and induced income multiplier in an input-output model.

A unit change in sales to final demand for industry 'i' generates particular productions and thus, given value added coefficients, a particular income : y_i*. As soon as a consumption function is introduced, a particular second round of direct and indirect requirements will follow. The income y_i* requires directly particular consumption changes and indirectly particular productions and thus income, ad infinitum. In short, a second income change is induced. It is important to note that the second round of requirements is driven by a particular amount of income, independent of its source. To make this point clear, we consider the household sector as an industry, 'n+1', whose output is income and whose input structure is described by the consumption function. The income change induced in the second round is now determined by the income multiplier for industry 'n+1' : Y_{n+1}**. We call this the induced income multiplier. The second round was driven by the amount of income Y_i*. The induced income multiplier blows it up to Y_{n+1}** Y_i*, thus yielding the total direct, indirect and induced income multiplier for industry 'i' (Y_i**) :

$$Y_i^{**} = Y_{n+1}^{**} Y_i^{*} \qquad (i = 1, \ldots, n)$$

This is the constant relationship between the income multipliers. Not surprisingly, the proportionality factor is the induced multiplier (Y_{n+1}**). This factor may depend on consumption determinants, i.e., prices and income. However, it is constant with respect to 'i'.

In the previous analysis it is essential that, given value added coefficients, a particular income change implies particular, i.e., unique consumption changes. In inppt-output there is a one-to-one relationship between value added coefficients and prices. Hence, the crux is that given prices, a particular income implies unique consumptions. Mathematically[1] the consumption vector must be a function of the price vector and income. However, as value added coefficients are fixed throughout the analysis, we may write the consumption vector as a function of income alone.

To compute the induced income multiplier, let :

X = n-dim output vector for all industries but the household vector

Y = income

$d_{n+1}(Y)$ = n-dim consumption vector

t = n-dim vector of final demand (excluding consumption) for all industries but the household sector

t_{n+1} = final demand for income

A = Technical-coefficient matrix for all industries but the household vector

d_o = n-dim row of value added coefficients

Then the physical and monetary balances are :

$$X = Ax + d_{n+1}(Y) + t$$

$$Y = d_o x + t_{n+1}$$

Now consider the shock $\Delta t_{n+1} = 1$. Then the balances become

$$X + \Delta X = A(X + \Delta X) + d_{n+1}(Y + Y_{n+1}^{**}) + t$$

$$Y + Y_{n+1}^{**} = d_o(X + \Delta X) + t_{n+1} + 1$$

Subtracting the initial balances from the last ones, linearizing d_{n+1} (which should be done as formally an infinitesimal shock determines a multiplier), and denoting differentiation by a prime,

$$\Delta X = A\Delta X + d_{n+1}(Y)\, Y_{n+1}^{**}$$

$$Y_{n+1}^{**} = d_o \Delta X + 1$$

To summarize this, define $B(Y)$ as the matrix A augmented by row d_o and column $d_{n+1}(Y)$. With our B, we get

$$\begin{bmatrix} \Delta X \\ \\ Y_{n+1}^{**} \end{bmatrix} = B(Y) \begin{bmatrix} \Delta X \\ \\ Y_{n+1}^{**} \end{bmatrix} + \begin{bmatrix} \underline{0} \\ \\ 1 \end{bmatrix}$$

Where $\underline{0}$ denotes null vector.

Hence,

$$Y_{n+1}^{**} = \frac{|I - A|}{|I - B\ (Y)|}, \quad \text{a constant}$$

One may question if, conversely, the constant relationship between the multipliers implies that consumption is a *function* of prices and income. The answer is in the negative. One may add up any value of the consumption function a consumption which is perpendicular to the vector of value added coefficients. It would induce no income.

We conclude that the relation between consumption on the one hand and prices and income on the other may be a correspondence not necessarily a function, and yet the relationship between the direct and indirect multipliers and the direct, indirect and induced multipliers is constant.[2]

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Foot-notes

1. $C(p,Y) = C(d_o(I-A)^{-1},Y) = d_{n+1}(Y)$

where, C = consumption, P = price, and Y, d_o, d_{n+1}, A

are defined later on.

2. Sandoval (1967) has also computed this relationship assuming a linear and homogenous income-consumption functions. Our results are obtained more directly and hold for any consumption functions which has been suggested in a foot-note to a paper by Moore and Peterson (1955).

References

Sandoval, A. David, (1967), 'Constant Relationship between Input-Output Income Multipliers'. *The Review of Economics and Statistics*, Vol. XLIX. November No. 4, pp. 599-600.

Moore, F.T. and Peterson, J.W. (1955), 'Regional Analysis : An Inter-industry Model of Utah'. *The Review of Economics and Statistics*, Vol. XXXVII, November No. 4, pp. 368-383.