Neoclassical input-output analysis

ten Raa, Thijs; Mohnen, P.

Published in:
Regional Science and Urban Economics

Publication date:
1994

Regional Science and Urban Economics 24 (1994) 135–158. North-Holland

Neoclassical input–output analysis

Thijs ten Raa

Tilburg University, P.O. Box 90153, 5000 LE Tilburg, Netherlands

Pierre Mohnen*

Université du Québec à Montréal, Montréal, Canada

Received August 1991, final version received September 1992

The Canadian comparative advantage is determined by maximization of foreign earnings subject to input–output relations between 29 industries and 92 commodities. Free trade would boost the mining, quarrying & oil wells, tobacco, and machinery sectors. The structure of the economy is not self-sufficient, as a necessary and sufficient price condition shows. When commodities are aggregated to the 29 sectors, the shadow prices to the programs fulfill the value equations of input–output analysis and admit a decomposition of Canadian inefficiency in 5% X-inefficiency, 15% allocative inefficiency, and 80% international specialization mismatch.

JEL classification: F11; C67

1. Introduction

Neoclassical input–output analysis?! If neoclassical economists and input–output economists share a view at all, it is the agreement to disagree. The two schools differ in terms of subject as well as method. Neoclassical economists address the question of value (including allocation) and relate it to the endowments and technology of an economy by the concept of marginal productivity. Input–output economists address the question of transmission of effects (due to shocks, for example) and relate it to the structure of an economy by the concept of technical coefficient. Marginal analysis of value seems particularly relevant in the short to medium run, while structural analysis of transmissions seems relevant in the medium to

Correspondence to: Thijs ten Raa, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, Netherlands.

*A Royal Netherlands Academy of Sciences senior fellowship, a Canadian Government research award and ACSN conference participation support provided to the first author, fellowships from the Fonds pour la Formation de Chercheurs et l'Aide à la Recherche, Québec and from the Centre National de la Recherche Scientifique, France to the second author, and observations by Chiang Dang, Peter Hanel, Pieter Kop Jansen, Franz Palm, Pierre Pestieau, anonymous referees and editor Konrad Stahl are gratefully acknowledged.

This paper was refereed under the editorial control of Konrad Stahl and John Quigley.

SSDI 0166-0462(93)02030-7

long run. When the scopes differ, it is perfectly all right to have differences of method.

A certain degree of complementarity can be considered a source of synthesis. In this paper we instill a neoclassical ingredient in the input–output framework, such that prices and quantities can be determined simultaneously. The ingredient is the concept of profit maximization at the aggregate level. Intersectoral substitution of activity provides the economy with neoclassical features at the macro level, such as the pricing of labor and capital according to their marginal productivities. At the sectoral level, production functions remain of the standard input–output type.

In a classical input–output study, Leontief (1953) assessed the factor position of an economy. He concluded that the exports of the U.S. economy were labor intensive relative to the imports. Factor contents of exports and imports were calculated with the aid of U.S. input–output coefficients. This so-called Leontief paradox casted doubt on the Heckscher–Ohlin theorem of international trade which predicts that exports are relatively factor intensive in the abundant endowment. (Abundance is taken relative to the endowments in the rest of the world.) But, Leontief did not explain the pattern of trade. To detect the sectors of comparative advantage or disadvantage of sectors, one needs a criterion. The criterion we take is profit or, in the context of international trade, foreign earnings. We follow Williams (1978), but endogenize the direction of trade.

In this paper we make three contributions. First, the quantity and value equations of input–output analysis are unified in a neoclassical model of profit maximization. Second, we perform the analysis in a rectangular use make framework and relate it to traditional input–output analysis. Third, we identify the comparative advantage of an economy given only its factor endowments and technology, and reveal the inefficiencies present in the data.

Sectors will be characterized by the observed input and output proportions and will be hypothetically scaled down or up in accordance with profitability. The number of commodities may exceed the number of sectors. As far as we know, this is the first empirical application of von Neumann's (1945) activity model. The optimum levels of activity signal comparative advantages and will be compared with the observed ones and the increase of profits will be decomposed into three parts. The parts are associated with full capacities utilization, reallocations that increase all components of net exports, and respecialization, respectively. In this way we make operational the neoclassical notions of X-efficiency, allocative efficiency, and gains to trade.

The paper is organized as follows. The rectangular commodity-sector model is presented in section 2. How the model can detect the comparative advantages is explained in section 3. The relationship with traditional input–output analysis follows in section 4. Section 5 explains the efficiency decomposition. The results of the traditional and the rectangular analysis are presented in sections 6 and 7, respectively.

2. The model

We study the Canadian economy of 1980. The data comprise material inputs, U_0, outputs, V_0, labor employment by sector, L_0, capital stocks by sector, K_0, capacity utilization rates by sector, c, and a labor force, N. The economy is divided into 29 sectors and broken down further into 92 commodities. Therefore, U_0 and V_0 are rectangular matrices of dimension sources 92×29 and 29×92, respectively. L_0, K_0 and c are row vectors of dimension 1×29. N is a scalar. The total capital stock is obtained by summing the components of K_0: K_0e, where e is the vector with all entries equal to unity. We also need world prices p for the tradable commodities. p is a commodity row vector with non-tradable components set equal to zero. These prices will be considered parametrically given to the Canadian economy. In view of the small size of the Canadian economy, this assumption seems reasonable. Note that $y_0=(V_0^{\mathrm{T}}-U_0)e=f_0+g_0$ is observed final demand, consisting of net exports, g_0, and all other components (consumption and investment) which may be referred to as domestic final demand, f_0. Net exports will be varied given the terms of trade and, therefore, the analysis is relevant for small, open economies.

We wish to investigate the optimum pattern of trade. All other components of final demand, collected in commodity vector f_0, are considered exogenous. We thus determine trade improvements upon the status quo. Indirect improvements, through consumption and investment adjustments, are ignored. Variables are obtained by dropping subscripts. Since net exports are the only varying component, we may just as well optimize the entire final demand vector, $y=(V^{\mathrm{T}}-U)e$. Introducing industry activity levels (or scales) by the vector s, the maximization of foreign earnings subject to the input–output structure of the economy is [see ten Raa (1994)]

$$\max_{s\geqq 0} py$$

subject to

$$y=(V^{\mathrm{T}}-U)e\geqq z,\quad Le\leqq N,\qquad Ke\leqq K_0e,$$

$$(U,V,L,K)=(U_0\hat{s},\hat{s}V_0,L_0\hat{s},K_0\hat{c}\hat{s}).$$

Final demand, y, and hence net exports, is subject to alternative restrictions, z, reflecting different trade programs. Under *free trade*, net exports and hence y_i, are free for tradable commodities i. For non-tradable commodities j, final demand, y_j, consists of domestic final demand and may not drop below the observed level: In short, the free trade program is specified by

$$z_i = -\infty \ (i \text{ tradable}), \qquad z_j = y_{0j} \ (j \text{ non-tradable}).$$

This restriction may also be written as follows. Let J be the 0 1 matrix which selects the non-tradable commodities. If commodity i is non-tradable, J has one 92-dimensional row with the ith entry one and all others zero. The number of rows of J equals the number of non-tradable commodities. The free trade constraint becomes

$$Jy \geqq Jy_0.$$

Under an *export promotion* program, net exports exceed prevailing levels, obtained by specifying

$$z = y_0.$$

Under an *import substitution* program, autarky is imposed by the self-sufficiency constraint that final demand exceeds domestic final demand, $y \geqq f_0$, where the lower bound can also be written

$$z = y_0 - g_0.$$

The free trade constraint is wider than either the export promotion or the import substitution constraint. The latter are not comparable, since some components of y_0 exceed those of $y_0 - g_0$, while others fall short, depending on the sign of observed net exports, as indicated by the components of g_0.

Substitution simplifies the canonical model to

$$\max_{s \geqq 0} p(V_0^{\mathrm{T}} - U_0)s$$

subject to

$$(V_0^{\mathrm{T}} - U_0)s \geqq z, \quad L_0 s \leqq N, \quad K_0 \hat{e} s \leqq K_0 e.$$

By the first constraint, production must meet a prescribed level of net demand, z. For example, in the export promotion program production must meet the levels called forth by the requirements of the prevailing levels of final demand. This, however, constitutes no more than a lower bound on the effective sales, since the latter also include additions to net exports. Through variations of the latter, the whole pattern of net output may change. This liberty is neoclassical in spirit and constitutes a departure from traditional input output analysis and the closely related linear program of Dorfman et al. (1958, p. 228) who choose gross outputs to minimize total labor costs of a *specified* bill of final goods.

3. Prices and comparative advantages

In neoclassical economics, factor and material inputs are priced according to their marginal productivities. In input–output analysis, proportions are assumed fixed and an increase in a single input, however, marginal, does not contribute to output or profit. When marginal productivity analysis is not applicable at the sectoral level, it may be relevant economy wide. A marginal increase in a single factor input contributes to foreign earnings, provided the economy accommodates it by a shift towards sectors that are relatively intensive in the factor considered. Intersectoral substitution in the input–output model facilitates a marginal productivity analysis of value. Formally, the wage and rental rates are the Lagrange multipliers associated with the labor and capital constraints. The determination of commodity prices is analogous.

The Lagrange multipliers to the three constraints of the above generic model, also called shadow prices, can be denoted by tariffs, t, wage rate, w, and rental rate, r, respectively. They solve the so-called dual program [Schrijver (1986, p. 90)], which in the present context reads

$$\min_{t,\,w,\,r \geqq 0} \quad wN + rK_0 e - tz$$

subject to

$$(p+t)(V_0^{\mathrm{T}} - U_0) \leqq wL_0 + rK_0 \hat{c}.$$

Like the wage and rental rates, the tariffs are not observed ones, but purely theoretical constructs. Their meaning will transpire after the presentation of duality theory. The neoclassical primal objective of profit maximization naturally yields the above neoclassical dual of cost minimization. In Dorfman et al. (1958, p. 228) quantities were chosen to minimize the costs of a specified bill of final goods. The dual of this problem involves the maximization of the value of net output by choice of prices, with quantities fixed. This combination of objective and instruments is not neoclassical.

I now return to the above linear program. By the so-called phenomenon of complementary slackness [Schrijver (1986, p. 95)], a primal (commodity) constraint has slack only if the dual price (tariff) is zero:

$$t[(V_0^{\mathrm{T}} - U_0)s - z] = 0.$$

Commodities whose production exceeds minimum requirements contribute to the objective function of the primal program. They are signaled by a competitive domestic price (world price cum tariff) which is equal to just the world price. These are the comparative advantage commodities which can compete on the world market. However, not all commodities with a zero tariff are truly commodities of comparative advantage in a rectangular model. Some commodity production is unavoidable given the fixed net output proportions.

Another application of the phenomenon of complementary slackness yields that a sectoral activity level is positive only if the dual constraint is binding.

Such sectors are active and break even at shadow prices $p+t$. In other words, the revenues of net outputs match the costs of the factor inputs. Since the shadow prices render sectors that are active in the solution just profitable, they constitute a competitive price system through which the optimum may be attained in a decentralized fashion. The competitive price system reflects the second welfare theorem of neoclassical economics. A warning is in order: positive activity need not signal a comparative advantage. It may merely be required to fulfill intermediate demand of other sectors through the trade regime constraints.

By the main theorem of linear programming [Schrijver (1986, p. 90)], the solution values of the primal and dual programs match:

$$p(V_0^{\mathrm{T}}-U_0)s=wN+rK_0e-tz.$$

Substituting the last term out by the complementary slackness equation, we obtain

$$wN+rK_0e=(p+t)(V_0^{\mathrm{T}}-U_0)s=(p+t)y.$$

This is the macro identity between national income and product. Note that domestic final demand is valued at competitive domestic prices. The role of the tariffs is to fill the gap between factor costs and world prices of the commodities that must be produced due to restrictions on net exports. Note also that national income entails a valuation of fully employed resources at flexible prices. The solutions to the programs involve full employment indeed. We do not claim free trade, export promotion or import substitution as simple recipes for full employment, but merely use the programs as analytical devices to associate hypothetical, competitive outcomes, featuring comparative advantages, with the input–output structure of the Canadian economy. In other words, competitive outcomes are an analytical device to link concepts as comparative advantages with the structure of an economy.

4. Traditional input–output analysis

When commodities are aggregated up to the same classification as industries (see the columns of table 1 for the correspondence), the use- and-make tables become square and the latter may be inverted to define one-to-one changes of variables between sectoral activity levels, s, industry outputs, $q=Ve=\hat{s}V_0e$, and commodity outputs, $x=V^{\mathrm{T}}e=(\hat{s}V_0)^{\mathrm{T}}e=V_0^{\mathrm{T}}s$. The variables in the canonical trade model concluding section 2 affect the objective function and the constraint through the final demand vector:

$$y=(V_0^{\mathrm{T}}-U_0)s.$$

The change of variables to industry outputs involves rescaling only. More precisely, $q=\hat{s}V_0e=\widehat{V_0e}s$, hence $s=\widehat{V_0e}^{-1}q$. Final demand becomes

Table 1

Sector and commodity aggregations.

Statistics Canada (1990a, 1990b) 29 sectors[a]	Statistics Canada (1987) M-classification 50 sectors	92 commodities
1. Agricultural & related services	1	1–3
2. Fishing & trapping	2	5, 6
3. Logging and forestry	3	4
4. Mining, quarrying & oil wells	4–7	7–12, **13**
5. Food	8	14–22
6. Beverage	9	23, 24
7. Tobacco products	10	25, 26
8. Plastic products	12	29
9. Rubber & leather products	11, 13	27, 28, 30
10. Textile & clothing	14, 15	31–35
11. Wood	16	36–38
12. Furniture and fixtures	17	39
13. Paper & allied products	18	40–42
14. Printing, publishing & allied	19	43, **44**
15. Primary metals	20	45–49
16. Fabricated metal products	21	50–52
17. Machinery	22	53, 54
18. Transportation equipment	23	55–57
19. Electrical and electronic products	24	58, 59
20. Non-metallic mineral products	25	60, 61
21. Refined petroleum & coal	26	62, 63
22. Chemical & chemical products	27	64–67
23. Other manufacturing	28	68, 69
24. Construction	29	**70–72**
25. Transportation & communication	30–33	73–77
26. Electric power and gas	34	78, **79**
27. Wholesale & retail trade	35, 36	80, **81**
28. Finance, insurance and real estate	37–40	**82**, 83
29. Community, business, personal services	41–50	84–87, **88**, 89, 90, **91–92**

[a]The industry codes adopted here are slightly different from those in Statistics Canada (1990a, 1990b), where sector 26 is missing for reasons of confidentiality so that the last sector is indexed by no. 30. Non-tradable commodities and the sectors declared non-trable are set bold face.

$$y=(V_0^{\mathrm{T}}\widehat{V_0e}^{-1}-U_0\widehat{V_0e}^{-1})q$$

and features the commodity-by-industry input–output matrix, $U_0\widehat{V_0e}^{-1}$. Industry outputs are not obtained by the Leontief inverse of the latter, since $V_0^{\mathrm{T}}\widehat{V_0e}^{-1}\neq I$. The change of variables to commodity outputs involves full inversion, $s=V_0{}^{\mathrm{T}}x$. Final demand becomes

$$y=(I-U_0V_0^{-\mathrm{T}})x$$

and features the commodity-by-commodity input–output matrix, $U_0V_0^{-\mathrm{T}}$. No change in the sectoral activity variables can generate an industry-by-industry variant, because the left-hand side of the final demand equation has the commodity dimension. For sectoral analysis it is more advisable to stick to the sectoral activity levels variables. Then there is no need to identify commodities and sectors. Such an analysis can just as well be performed in the rectangular framework (section 7 below).

We confine the discussion of traditional input–output to the commodity-by-commodity input–output model, $A = U_0 V_0^{-\mathrm{T}}$, which was obtained by the change in variables, $x = V_0^{\mathrm{T}} s$. Any positive element of s yields a multitude of positive elements of x, due to the off-diagonal elements of V_0. In other words, the domain $s \geqq 0$ corresponds not to the entire non-negative orthant, $x \geqq 0$, but to only a subset, in fact a cone. Conversely, the non-negative orthant, $x \geqq 0$, corresponds to a larger subset of sectoral activity space than $s \geqq 0$. Hence, by admitting all $x \geqq 0$, traditional input–output economists implicitly extend the analysis to sectoral activity sectors with negative components. Some input output coefficients are negative for this reason. On the suggestion of an anonymous referee, we have considered adjusting the negatives and the observed output vector to preserve feasibility, but it did not affect the results. In other words, the extension of the domain implied by the traditional input–output instead of the sectoral activity analysis is not pertinent to the solution of the trade programs.

Substitution of the change of variables, $s = V_0^{-\mathrm{T}} x$, transforms the canonical trade program of section 2 to

$$\max p(I-A)x$$

subject to

$$(I-A)x \geqq z, \quad lx \leqq N, \quad kx \leqq K_0 e,$$

where technical coefficients are defined by the so-called commodity technology model:

$$A = U_0 V_0^{-\mathrm{T}}, \quad l = L_0 V_0^{-\mathrm{T}} \quad \text{and} \quad k = K_0 \hat{c} V_0^{-\mathrm{T}}.$$

Maximizing over $x \geqq 0$, the dual program becomes

$$\min_{t,\, w,\, r \geqq 0} \; wN + rK_0 e - tz$$

subject to

$$(p+t)(I-A) \leqq wl + rk.$$

Slack in the (primal) commodity constraint or a zero (dual) tariff detects a comparative advantage. Strictly speaking, it belongs to commodities, but in this traditional input–output framework they are identified with sectors. It is not difficult to see that in the export promotion and import substitution programs, the material balance constraints yield positive gross outputs. By the phenomenon of complementary slackness, the dual constraints are binding. Postmultiplication by the Leontief inverse yields

$$p+t=(wl+rk)(I-A)^{-1},$$

the traditional value equations of input output analysis. Here, however, the value equations are constraints to the dual program which determines all shadow prices. It is interesting to note that in the traditional input–output framework, factor costs, $wl+rk$, cannot be equated with net revenues, $p(I-A)$, for exogenous prices p. The number of degrees of freedom (two, for w and r) is too low. The resolution is possible, however, if prices include tariffs.

In traditional input–output analysis [Leontief (1979)], prices are determined by the value system and outputs by the quantity system. Although the systems are similar mathematically, prices and outputs are determined independently of each other. The introduction of the neoclassical principle of profit maximization pairs the systems and allows a simultaneous determination of value and output. The value system emerges as the dual to the quantity system in the sense of linear programming. It should be mentioned that it has been attempted before to consolidate the equations of input–output analysis in this manner, but the attempt [Dorfman et al. (1958)] has failed to explain quantities, by unfortunate combinations of objective functions and instruments. When profit is the criterion and activity levels are the instruments, the dual program can be used to calculate the tariffs necessary to sustain economic programs, such as export promotion or import substitution. The primal program can be used to compute the required activity levels which can be attained by pure competition under the shadow prices. In short, a neoclassical specification unifies the elements of input–output analysis.

5. Efficiency analysis

Consider the free trade program, the export promotion program, and a constrained export promotion program. The constraint defining the latter rules out reallocations of labor and capital and will be specified below. Let the solutions be attained by y_{ft}, y_{ep}, and y_{cep}. Let us compare them with observed final demand, y_0, y_{ep} and y_{cep} are bigger, but not ordered among each other. y_{ft} is not ordered relative to any of the other vectors. In terms of value, py, the picture is clearer. Since the free trade program is least constrained, it yields the greatest value. The constrained export promotion program can generate no more value than the export promotion program.

Since the observed vector is feasible in all programs, its value constitutes a lower bound. In short,

$$py_0 \leqq py_{\mathrm{cep}} \leqq py_{\mathrm{ep}} \leqq py_{\mathrm{ft}}.$$

The total potential efficiency gain, $py_{\mathrm{ft}} - py_0$, can be decomposed into three terms, associated with the above inequalities. The first term, $py_{\mathrm{cep}} - py_0$, is the efficiency gain that can be attained without labor or capital reallocation, and is called the X-inefficiency of the economy. It represents a distance towards the production possibility frontier. We do not allow for temporary location in the interior of the production possibility set as a means to overcome a recession, while maximizing output in a boom, because we neglect adjustment costs of capital and labor. In this respect, our estimate of X-inefficiency will be an overstatement. The sum of X-inefficiency and allocative inefficiency amounts to $py_{\mathrm{ep}} - py_0$ and measures the gain that can be made without any reduction in the net output vector; it may be called the domestic inefficiency. The third term, $py_{\mathrm{ft}} - py_{\mathrm{ep}}$, is the efficiency gain that can be obtained by reductions of the net output vector through imports. It constitutes the pure potential gain to trade and is called the international specialization mismatch. In sum, total inefficiency, $py_{\mathrm{ft}} - py_0$, consists of X-inefficiency, allocative inefficiency, and international specialization mismatch.

The X-efficiency gain in the export promotion program is isolated by ruling out reallocations of labor and capital between sectors. In the use make framework, the constraints

$$s \leqq c^{-1} \quad \text{and} \quad L_0 \max\{s, e\} \leqq N$$

(where c^{-1} is the column vector of inverse sectoral capital utilization rates and max operates on each component) limit activities to full capacity levels and confine labor recruits to the pool of the unemployed, without decreasing the employment in other sectors. In the traditional framework, X-efficiency is isolated by the imposition of

$$k_i x_i \leqq K_i^c \text{ (all } i) \quad \text{and} \quad l \max\{x, x_0\} \leqq N,$$

where capital and labor are associated with commodities rather than sectors. K_i^c is the stock of capital available for the production of *commodity* i. It is inaccurate to substitute K_i, the stock of capital in sector i, since that is also used for the production of commodities other than i. In doing traditional input–output analysis, not only intermediate flows U_0 have to be purified in the construction of $A = U_0 V_0^{-\mathrm{T}}$, but also the stocks. The construction of the capital stock vector, K^c, is explained in the appendix.

The import substitution program is included for the sake of theoretical comparison. The shadow prices associated with the commodity constraints

Table 2

Net exports (millions of dollars[a]).

Sector	Actual	X-efficiency	Export promotion	Free trade	Import substitution
1.	3,645.4	3,645.4	3,645.4	−258,870.0	0.0
2	51.8	**220.8**	51.8	10.8	0.0
3.	10.1	10.1	10.1	−126.9	0.0
4.	−2,929.1	**1,343.0**	**16,340.7**	**26,732.8**	**32,499.0**
5.	−847.4	−847.4	−847.4	−16,040.0	0.0
6.	12.1	12.1	12.1	−2,703.7	0.0
7.	10.3	10.3	10.3	**1,013,304.5**	0.0
8.	−435.6	−435.6	−435.6	−5,061.2	0.0
9.	−818.0	−818.0	−818.0	−1,758.6	0.0
10.	−2,231.4	−2,231.4	−2,231.4	−6.554.7	0.0
11.	3.568.0	3.568.0	3,568.0	−2,873.1	0.0
12.	−90.5	−90.5	−90.5	−2,363.5	0.0
13.	7,218.4	7,218.4	7,218.4	116.050.8	0.0
14.	−583.5	−583.5	−583.5	−583.5	0.0
15.	2,934.2	2,934.2	2,934.2	−2,945.3	0.0
16.	−1,554.5	−1,554.5	−1.554.5	−10,482.0	0.0
17.	−6,743.5	6,743.5	**31,807.9**	−10,496.3	**30,661.0**
18.	−2,781.9	−2,781.9	−2,781.9	−12,800.1	0.0
19.	−3,158.6	−3,158.6	−3,158.6	−8,536.8	0.0
20.	543.2	−543.2	−543.2	−3,271.0	0.0
21.	1,597.2	1,597.2	1,597.2	−8,253.3	0.0
22.	−3,561.0	−3,561.0	−3,561.0	−14,195.0	0.0
23.	2,102.3	−2,102.3	−2,102.3	−4,389.5	0.0
24.	0.0	0.0	0.0	0.0	0.0
25.	719.8	719.8	719.8	−15,514.5	0.0
26.	807.5	**1,764.4**	807.5	−9,483.4	0.0
27.	2,170.6	**10,876.2**	2,170.6	−49,245.3	0.0
28.	−753.9	−753.9	−753.9	−753.9	0.0
29.	1,982.8	1,982.8	1,982.8	186,026.5	0.0
Increase as % of GDP	0.0	6.0	23.7	121.0	27.7

[a]Figures in bold indicate improvements on actual levels, i.e. sectors with comparative advantages.

$z = y_0 - g_0$ are essentially autarky prices. The Ricardian theory of trade uses them to predict the pattern of free trade.

6. Traditional input–output analysis of the Canadian economy

We report the traditional input–output results, obtained by aggregating commodities up to the sectoral classification according to table 1 and by maximization with respect to gross output levels subject to commodity technology constraints. See tables 2 and 3 for net trades and prices, respectively, for all trade programs.

The comparative advantages are detected in sectors 4 (mining, quarrying

Table 3
Tariffs.

Sector	X-efficiency	Export promotion	Free trade	Import substitution
1.	0.15	1.33	0.00	1.33
2.	0.00	0.83	0.00	0.83
3.	0.74	0.23	0.00	0.23
4.	0.00	0.00	0.00	0.00
5.	1.09	0.72	0.00	0.72
6.	2.41	0.31	0.00	0.31
7.	1.42	0.35	0.00	0.35
8	1.65	0.49	0.00	0.49
9.	0.98	0.41	0.00	0.41
10.	0.57	0.36	0.00	0.36
11.	0.80	0.32	0.00	0.32
12.	0.51	0.30	0.00	0.30
13.	0.82	0.53	0.00	0.53
14.	153.25	1.19	1.35	1.19
15.	0.59	0.38	0.00	0.38
16.	0.28	0.15	0.00	0.15
17.	0.12	0.00	0.00	0.00
18.	1.23	0.26	0.00	0.26
19.	0.27	0.06	0.00	0.06
20.	0.99	0.42	0.00	0.42
21.	0.45	0.39	0.00	0.39
22.	1.70	0.62	0.00	0.62
23.	1.31	0.31	0.00	0.31
24.	1.56	1.02	1.12	1.02
25.	0.58	0.88	0.00	0.88
26.	0.00	2.25	0.00	2.56
27.	0.00	0.25	0.00	0.25
28.	0.87	0.57	0.66	0.57
29.	8.35	0.46	0.00	0.46
Wage rate ($/hour)	0.00	10.8	21.0	10.8
Rental rate	147.3%	33.1%	31.4%	33.1%

and oil wells) and 7 (tobacco products) under free trade. Sector 4 persists under the export promotion and import substitution programs, but is then accompanied by sector 17 (machinery), in either case. In fact, table 3, reveals that the shadow prices under export promotion and import substitution are equal. Woodland (1982) has shown that comparative advantages are locally constant with respect to endowment changes, even in the presence of substitution. Apparently, the difference between the constraints characterizing export promotion and import substitution constitutes a small change in terms of factor intensities relative to the final demand vector, i.e. GDP. In other words, Canadian endowments are balanced with respect to domestic final demand.

The slack in the Canadian economy consists of 5% X-inefficiency, 15% allocative inefficiency, and 80% international specialization mismatch. These figures are obtained by taking the increments in the bottom line of table 2 as percentages of the total figure of the free trade scenario (121.0). The procedure has been explain in section 5. The main problem is the international misdirection of the Canadian economy. The patterns of optimum and actual commodity net exports are very different.

Domestic production or *autarky* prices are obtained by adding the import substitution tariffs to the world prices (recall the derivation of the traditional value equations from the dual program). Table 3 shows that the lowest autarky prices are for sectors 4 and 17. The Ricardian theorem predicts that they signal the net exports under free trade. Table 2 confirms this result for sector 4, but not sector 17, in agreement with recent theoretical falsifications [Drabicki and Takayama (1979) and Woodland (1982)]. A more detailed analysis, undertaken in the next section, will include sector 17 as an exporter in the free trade scenario and thus resurrect the Ricardian theorem.

We have also calculated the optimum activity levels under the various trade regimes by maximizing with respect to the activity vector, s, rather than the gross output vector, x. This model is in between traditional input–output and activity analysis, as commodities are aggregated into sectors, but sectors are not purified by change of variables (from s to x). Within the class of square input–output models [Kop Jansen and ten Raa (1990)], the traditional input–output model is essentially the commodity technology model, while the intermediate model with its fixed output proportions is essentially the by-product model. The results of the intermediate model are qualitatively the same as the traditional model and quantitatively very close. We have decided, therefore, not to report them.

7. Rectangular input–output analysis of the Canadian economy

Returning to the full use–make framework, we maximize surplus with respect to activity levels and subject to observed sectoral input and output proportions. When we use the observed or zero values, $s=e$ or 0, as initial points, the program got stuck. One reason for this might be that any increase in the activity levels sparks off a flurry of commodity net input increases and that fulfilment of the detailed commodity constraints cannot be controlled by the relatively few activity variables. If so, the commodity constraints would imply that the value of the objective function cannot be increased in the admittedly rigid activity model with its fixed input and output proportions.

To investigate this possibility, we have utilized a result of Rockafellar (1970) on inequalities implied by a system of inequalities. His theorem 22.3 shows that if and only if the coefficients of a 'new' inequality are non-negative combinations of the coefficients of a system of 'old' inequalities and the right-hand side of the new inequality is a relaxation of the non-negative combination of the right-hand sides of the old inequalities, any solution to the system of old inequalities also fulfills the new inequality. Now our model comprises the system of inequalities

$$\begin{bmatrix} U_0 - V_0^{\mathrm{T}} \\ L_0 \\ K_0\hat{c} \\ -I \end{bmatrix} s \leqq \begin{bmatrix} -z \\ N \\ K_0 e \\ 0 \end{bmatrix}.$$

The model can provide no better than $s=e$ if and only if the system implies

$$p(V_0^{\mathrm{T}} - U_0)s \leqq p(V_0^{\mathrm{T}} - U_0)e.$$

By Rockafeller's theorem the latter inequality is implied if and only if there exists $(t\ w\ r\ \sigma) \geqq 0$ such that

$$p(V_0^{\mathrm{T}} - U_0) = (t\ w\ r\ \sigma) \begin{bmatrix} U_0 - V_0^{\mathrm{T}} \\ L_0 \\ K_0\hat{c} \\ -I \end{bmatrix}$$

and

$$p(V_0^{\mathrm{T}} - U_0)e \geqq (t\ w\ r\ \sigma) \begin{bmatrix} -z \\ N \\ K_0 e \\ 0 \end{bmatrix}.$$

An alternative derivation of this result is by application of the main theorem of linear programming [Schrijver (1986, p. 90)]. So we are stuck at $s=e$ if and only if there exists $(t\ w\ r) \geqq 0$ such that

$$(p+t)(V_0^{\mathrm{T}} - U_0) \leqq wL_0 + rK_0\hat{c}$$

and

$$p(V_0^{\mathrm{T}} - U_0)e + tz \geqq wN + rK_0 e.$$

We investigate the possibilities of being stuck at $s=e$ for the three scenarios: the export promotion program, the import substitution program, and the free trade program, respectively. The scenarios differ only by specification of the constraints vector, z.

In the *export promotion* program,

$$z = y_0 = (V_0^{\mathrm{T}} - U_0)e.$$

Multiplying the first inequality by e and combining with the second inequality through $L_0 e < N$ and $K_0\hat{c}e < K_0 e$, we obtain a string of inequalities with equal extreme left- and right-hand sides. Hence the middle sides are also equal:

$$wL_0e+rK_0\hat{c}e=wN+rK_0e,$$

which is equivalent to $w=r=0$. Thus, $s=e$ is optimum if and only if there exists $t\geqq 0$ such that

$$(p+t)(V_0^{\mathrm{T}}-U_0)\leqq 0$$

and

$$(p+t)(V_0^{\mathrm{T}}-U_0)e\geqq 0.$$

Since the first inquality is equivalent to the statement that for all non-negative s, $(p+t)(V_0^{\mathrm{T}}-U_0)s\leqq 0$, the observed levels of activities are optimum if and only if there exist competitive domestic prices under which profits are non-negative and any other combination of activities would yield non-positive profits. (This connection between optimality and competitive prices reflects the welfare theorems of neoclassical economics.) The pair of inequalities is equivalent to

$$(p+t)(V_0^{\mathrm{T}}-U_0)=0$$

for some $t\geqq 0$. By homogeneity it suffices to find $\pi\geqq\varepsilon\geqq 0$, with $\varepsilon_i>0$ for tradables and $\pi e=1$ (constituting a closed set), such that

$$\pi(V_0^{\mathrm{T}}-U_0)=0.$$

For this purpose, consider the linear program,

$$\min_{\pi,\mu}\mu$$

subject to

$$\pi(V_0^{\mathrm{T}}-U_0)=\mu e^{\mathrm{T}}(V_0^{\mathrm{T}}-U_0).$$

where the scalar μ is non-negative. Then $\pi=e^{\mathrm{T}}/n$, $\mu=1/n$, where n is the number of commodities, is feasible. If the solution is (π^*,μ^*) and $\mu^*=0$, then π^* is as desired and $s=e$ is optimum in the export promotion program.

The analysis of the *import substitution* program is a corollary to the investigation of the export promotion program. In the import substitution program net outputs may not decrease below domestic final demand. Since the latter is non-negative, net outputs must certainly be non-negative:

$$(V_0^{\mathrm{T}}-U_0)s\geqq 0.$$

Consider any $s \geqq 0$ consistent with this autarky constraint. Then $e+\varepsilon s$ is consistent with the export promotion constraints. [The labor and capital constraints are fulfilled for ε small enough. The commodity constraint,

$$(V_0^{\mathrm{T}}-U_0)(e+\varepsilon s) \geqq (V_0^{\mathrm{T}}-U_0)e = y_0 = z \text{ (export promotion)},$$

is equivalent to the above autarky constraint.] If e is optimum in the export promotion program, then

$$p(V_0^{\mathrm{T}}-U_0)(e+\varepsilon s) \leqq p(V_0^{\mathrm{T}}-U_0)e,$$

and therefore

$$p(V_0^{\mathrm{T}}-U_0)s \leqq 0,$$

meaning that the autarky constraints admit no generation of surplus either.

A slight strengthening of the analysis shows that s, the underlying activity vector, may be stuck at the observed value. Recall that prices, π, fulfilling

$$\pi(V_0^{\mathrm{T}}-U_0)=0,$$

were found by minimizing $\mu \geqq 0$ subject to $\pi(V_0^{\mathrm{T}}-U_0)=\mu e^{\mathrm{T}}(V_0^{\mathrm{T}}-U_0)$. Note that $e^{\mathrm{T}}(V_0^{\mathrm{T}}-U_0)$ is the value added vector, and hence is positive. If we allow μ to go into the negatives, and suppose it will do so, we then find prices π fulfilling $\pi(V_0^{\mathrm{T}}-U_0)<0$. By homogeneity there exists $t \geqq 0$ such that the negativity becomes as strong as you like, e.g. $t(V_0^{\mathrm{T}}-U_0) \leqq -e^{\mathrm{T}}$. Multiply through by any $s>0$ fulfilling $(V^{\mathrm{T}}-U_0)s \geqq 0$ (obtained under autarky):

$$-e^{\mathrm{T}}s \geqq t(V_0^{\mathrm{T}}-U_0)s \geqq 0.$$

That is, the sum of components of s is negative or zero. Since $s \geqq 0$, it must be zero. In the context of the export promotion program, replacement of s by $s-e$ yields that not only is the solution value stuck at the observed level, but also the underlying activities ($s=e$), when μ goes into the negatives.

The investigation of the optimality of $s=e$ in the *free trade* program is similar to the export promotion program analysis. The commodity constraints are restricted to non-tradables. The system continues to imply the inequality, $p(V_0^{\mathrm{T}}-U_0)s \leqq p(V_0^{\mathrm{T}}-U_0)e$, if and only if there exists $(t\ w\ r\ \sigma) \geqq 0$ as above, with t restricted, however, to non-tradables (for $t_i=0$ for i tradable). By the same derivation, the question is whether there exist tariffs $t_j \geqq 0$, j non-tradable, such that

$$(p+t)(V_0^{\mathrm{T}}-U_0)=0.$$

This is a system of equalities, one for each sector. One can hope to find a price solution only if the number of degrees of freedom (the dimension of t

or the number of non-tradables) is at least the number of sectors. The number of non-tradables (eleven, see table 1) is too small for this purpose. Since the existence of prices fulfilling the equality was shown to be necessary and sufficient for the optimality of the observed levels of activities, it follows that a free trade improvement always is feasible.

To which sectors the comparative advantages of the economy, in terms of commodities, can be ascribed is an open issue. A natural guess is to pick the primary producers of the commodities with comparative advantages. However, a number of complications arise. What if there is no clear-cut primary producer? If it exists, what if its other outputs perform badly in the sense of having a high competitive domestic price? A more direct investigation of the issue would be to compare the solution of the primal program with the observed levels of activity, e. Thus, a high activity level would signal a comparative advantage. This approach is also troublesome because, as we have noted before, levels of activity may be driven by intermediate demand of other sectors through the trade regime constraints, rather than contributions to the objective function.

Unambiguous ascription of comparative advantages to sectors seems possible only if the trade constraints apply to sectors instead of commodities. For example, if sectors are permitted to compensate some commodity imports by exports of commodities belonging to the same sector, then the trade constraints would be $S(V_0^{\mathrm{T}}-U_0)s \geqq Sz$, where S is an aggregation matrix of dimension: no. of sectors $\times$ no. of commodities. Although this assumption is implicit in traditional input–output analysis, it ignores the non-tradability of certain commodities, be they within or across sectors.

We now turn to the results. Recall that $s=e$ solves the export promotion program if $\mu^*=0$ solves the linear program associated with $\pi(V_0^{\mathrm{T}}-U_0)=0$. This happens to be the case for the Canadian use-and-make tables, (U_0, V_0). We can therefore conclude that the observed levels of activity solve the export promotion program. Any increase in activity would violate a commodity import constraint. Thus, the 1980 Canadian economy cannot boost or maintain its net exports in all commodities simultaneously. In this sense the economy is truly open. As a corollary, the import substitution program for the 1980 Canadian economy admits no generation of surplus.

Recall also that if μ goes into the negatives when allowed, then $s=e$ and $s=0$ are the only solutions to the export promotion and import substitution programs, respectively. Also this happens to be the case for the Canadian use-and-make tables. (U_0, V_0). In essence, we have shown that the 1980 Canadian economy is incapable of supporting non-negative final demand. In other words, it is not self-reliant. The demonstration was through our competitive price test. It should be mentioned again that this result is obtained in the rigid context of an activity model with fixed input and output coefficients.

The results of the free trade program are reported in tables 4 and 5. Recall from section 2 that $(V_0^T - U_0)e = f_0 + g_0$ is observed final demand, comprising net exports, g_0, and domestic final demand, f_0. In the solution final demand becomes $(V_0^T \; U_0)s = f_0 + g$, with g the optimum net exports obtained at activity levels s. Net exports (g_0 and g, respectively) are reported in table 4 and the activity levels (vector s) in table 5. The activity levels of three sectors are significantly boosted, with the remaining activity levels suppressed or slightly increased (particularly services) to meet intermediate demand requirements of non-tradable commodities. Likewise, table 4 shows that some net exports are boosted and these items correspond to the three very active sectors. The comparative advantages are thus considered to reside in *mining, quarrying & oil wells*, *tobacco*, and *machinery*. The contributions to optimum net exports are 154,073 by mining, quarrying & oil wells (or 54%), 27,598 by tobacco (or 10%), and 105,858 by machinery (or 37%). (The figures are millions of dollars. The percentages do not add up precisely due to rounding.) From the view point of factor endowments and technology, the Canadian economy is resource oriented. The mining, quarrying & oil wells sector is extremely capital intensive and the residual labor-intensive mix of factor endowments is fully employed by two more sectors. Qualitatively, the outcome confirms the aggregated version of the model. In the traditional input–output model (section 6), the comparative advantages were in mining, quarrying & oil wells (all trade regimes), plus tobacco (free trade regime) or machinery (export promotion and import substitution regimes). Machinery is now also an exporting sector in the free trade scenario, resurrecting the Ricardian theorem (see the last section). Note also that the surplus of some non-tradables (commodities 13, 70, 71 and 88) are increased, even though they are not valued in the objective function. This is because they are by-products of some sectoral activities. Excluding these increases, net exports increase by 41.5% of GDP, comprising ten commodities (table 4). Of these optimum net exports, only four commodities show net exports in actuality (table 4), suggesting serious international misspecialization of the Canadian economy. The other two components of inefficiency, namely X-inefficiency and allocative inefficiency, are degenerate in the rectangular model, since the constraints needed to identify them would make the linear program get stuck at the observed levels of activities and net outputs, as we have analyzed above.

Tariffs are ascribed to non-tradable commodities only (of which there are seven), but not the ones that are sufficiently produced as by-products (commodities 13, 70, 71 and 88). There is a dual relationship between tariffs (table 4) and activities (table 5). By the theory of linear programming, the number of active variables is essentially equal to the number of binding constraints where the latter are signaled by positive shadow prices. If more variables are active, they are collinear in terms of the utilization rates of resources and other constrained entities. Now from table 4 we see that the

Table 4

Free trade and commodities.

Commodity	Actual net exports	Optimum net exports[a]	Tariffs
1. Grains	3,764.2	445.8	0.00
2. Live animals	169.0	−688.7	0.00
3. Other agricultural products	−287.8	−9,708.0	0.00
4. Forestry products	10.1	−147.7	0.00
5. Fish landings	55.0	−47.1	0.00
6. Hunting & trapping products	−3.2	−0.1	0.00
7. Iron ores & concentrates	879.3	**9,967.3**	0.00
8. Other metal. ores & concentrates	−3,014.7	**34,073.9**	0.00
9. Coal	−328.4	**4,507.0**	0.00
10. Crude mineral oils	−4,974.2	**60,483.0**	0.00
11. Natural gas	3,775.6	**34,922.4**	0.00
12. Non-metallic minerals	733.3	**10,119.0**	0.00
13. Services incidental to mining	0.0	11,867.3	0.00
14. Meat products	292.5	−6,413.3	0.00
15. Dairy products	73.8	−3,611.6	0.00
16. Fish products	−320.3	−1,530.1	0.00
17. Fruits & vegetables preparations	−401.6	−2,023.3	0.00
18. Feeds	42.1	−290.5	0.00
19. Flour, wheat, meal & other cereals	−29.7	−340.8	0.00
20. Breakfast cereal & bakery prod.	4.7	−1,949.3	0.00
21. Sugar	3.3	−314.2	0.00
22. Misc. food products	−512.2	−2,857.7	0.00
23. Soft drinks	−10.7	−972.0	0.00
24. Alcohol beverages	22.8	−1,989.3	0.00
25. Tobacco processed unmanufactured	26.0	**2,161.8**	0.00
26. Cigarettes & tobacco mfg.	−15.7	**25,436.4**	0.00
27. Tires & tubes	−170.0	−170.0	0.00
28. Other rubber products	−199.0	−3,795.7	0.00
29. Plastic fabricated products	−435.6	−1,850.9	0.00
30. Leather & leather products	−449.0	−1,164.5	0.00
31. Yarns & man made fibres	−329.9	−40.9	0.00
32. Fabrics	−781.7	−346.9	0.00
33. Other textile products	−316.0	−1,744.4	0.00
34. Hosiery & knitted wear	−347.7	−1,275.4	0.00
35. Clothing & accessories	−456.1	−3,844.1	0.00
36. Lumber & timber	3,090.7	−1,082.9	0.00
37. Veneer & plywood	109.6	−607.2	0.00
38. Other wood fabricated materials	367.7	−2,173.1	0.00
39. Furniture & fixtures	−90.5	−2,379.7	0.00
40. Pulp	3,570.9	−94.7	0.00
41. Newspaper & other paper stock	3,975.9	−2,250.0	0.00
42. Paper products	−328.4	−5,710.0	0.00
43. Printing & publishing	−583.5	−91.3	0.00
44. Advertising, print media	0.0	0.0	1.35
45. Iron & steel products	417.0	−22,216.3	0.00
46. Aluminum products	−424.4	−3,477.6	0.00
47. Copper & copper alloy products	903.4	−1,027.4	0.00
48. Nickel products	1,038.9	−417.3	0.00
49. Other non-ferrous metal products	999.3	−133.7	0.00
50. Boilers, tanks & plates	−24.1	−944.4	0.00
51. Fabricated structural metal products	147.6	−3,355.7	0.00
52. Other metal fabricated products	−1,678.0	−7,535.1	0.00

Table 4 (Continued)

Commodity	Actual net exports	Optimum net exports[a]	Tariffs
53. Agricultural machinery	−1,208.5	**29,392.5**	0.00
54. Other industrial machinery	−5,535.0	**76,465.1**	0.00
55. Motor vehicles	923.9	−4,653.4	0.00
56. Motor vehicle parts	3,795.4	−4,966.7	0.00
57. Other transport equipment	89.6	−2,650.5	0.00
58. Appliances & receivers, household	−1,465.9	706.8	0.00
59. Other electrical products	−1,692.7	4,830.2	0.00
60. Cement & concrete products	94.7	−2,276.3	0.00
61. Other non-metallic mineral products	637.9	−3,094.6	0.00
62. Gasoline & fuel oil	326.2	−10,903.6	0.00
63. Other petroleum & coal products	1,271.0	5,701.4	0.00
64. Industrial chemicals	−2,038.5	3,299.5	0.00
65. Fertilizers	−64.1	5,367.9	0.00
66. Pharmaceuticals	−300.5	−1,128.3	0.00
67. Other chemical products	−1,157.9	−5,170.0	0.00
68. Scientific equipment	−1,806.6	−3,215.7	0.00
69. Other manufactured products	−295.7	2,718.9	0.00
70. Residential construction	0.0	6,035.2	0.00
71. Non-residential construction	0.0	12,278.2	0.00
72. Repair construction	0.0	0.0	6.25
73. Pipeline transportation	153.6	−758.8	0.00
74. Transportation & storage	610.2	23,732.9	0.00
75. Radio & television broadcasting	−10.1	−1,717.6	0.00
76. Telephone & telegraph	−48.7	−6,729.1	0.00
77. Postal services	14.8	−1,405.3	0.00
78. Electric power	807.5	−1,145.5	0.00
79. Other utilities	0.0	0.0	8.36
80. Wholesale margins	2,170.6	780.0	0.00
81. Retail margins	0.0	0.0	1.91
82. Inputed rent owner-occupied dwelling	0.0	0.0	0.36
83. Other finance, insurances real estate	−753.9	−29,065.6	0.00
84. Business services	1,205.1	−2,298.0	0.00
85. Education services	32.6	299.9	0.00
86. Health services	−16.5	2,144.8	0.00
87. Amusement & recreation services	150.3	827.8	0.00
88. Accomodation & food services	0.0	4,014.8	0.00
89. Other personal & misc. services	−90.9	3,053.7	0.00
90. Transportation margins	3,413.0	6,271.0	0.00
91. Supplies for office, lab. & cafeteria	0.0	0.0	1.84
92. Travel, advertising & promotion	0.0	0.0	2.57
Increase as % of GDP	0.0	55.6	
Wage rate ($/hour)			13.7
Rental rate			14.2%

[a]Exports are in millions of dollars. Bold figures indicate comparative advantages. Bold indexes indicate non-tradable commodities.

Table 5

Free trade and sectors.

Sector	Activity level[a] (actual = 1)
1. Agricultural & related services	0.00
2. Fishing & trapping	0.00
3. Logging and forestry	0.00
4. Mining, quarrying & oil wells	**6.28**
5. Food	0.00
6. Beverage	0.00
7. Tobacco products	**29.10**
8. Plastic products	0.00
9. Rubber & leather products	0.00
10. Textile & clothing	0.00
11. Wood	0.00
12. Furniture and fixtures	0.00
13. Paper & allied products	0.00
14. Printing, publishing & allied	1.34
15. Primary metals	0.00
16. Fabricated metal products	0.00
17. Machinery	**27.58**
18. Transportation equipment	0.00
19. Electrical and electronic products	0.00
20. Non-metallic mineral products	0.00
21. Refined petroleum & coal	0.00
22. Chemical & chemical products	0.00
23. Other manufacturing	0.00
24. Construction	1.42
25. Transportation & communication	0.00
26. Electric power and gas	0.90
27. Wholesale & retail trade	1.01
28. Finance, insurance and real estate	1.00
29. Community, business, personal services	1.34

[a]Bold figures are explained in table 4.

number of positive shadow prices is nine. In fact, binding are seven commodity non-tradability constraints and both factor input constraints. Table 5 shows that nine sectors are active indeed. The low activity levels fulfill final demand for non-tradables. Three sectors operate at a high activity level: mining, quarrying & oil wells, tobacco, and machinery. These three sectors exhaust the factor inputs and contribute heavily to net exports (table 4). Mining is capital intensive, while the other two, tobacco and machinery, are labor intensive. There are two labor intensive sectors active, as they also take care of non-tradability constraints, particularly on travel, advertising & promotion (commodity 92).

8. Conclusion

The maximization of foreign earnings subject to material balance and factor input constraints constitutes a linear program. The variables in the program are sectoral output levels, both gross and net. If the latter are positive in the solution, they indicate sectors that contribute to net exports under conditions of free trade. Thus, the primal program detects the comparative advantage of the economy. As is well known, the Lagrange multipliers of the constraints can be considered shadow prices and are interrelated by the dual program. The constraints of the dual program are essentially the value equations of input–output analysis. The neoclassical ingredient of profit maximization thus embeds the determination of value in the quantity system. Prices and quantities are determined simultaneously, yielding marginal productivities of factor inputs and comparative advantages of sectors.

The Canadian economy is not self-reliant. It is not possible to increase the net export of any commodity without calling forth some additional import requirements. This result does not hinge on import coefficients. In fact, all imports are endogenous to the model. The only distinction is between tradable and non-tradable commodities. Although fixed commodity proportions are properly specified in a commodity-by-sector framework, it turns out that this hypothesis is so restrictive that it admits no efficiency decomposition of gains to free trade. Input–output analysis is no different from other methodologies. When the assumptions are pushed to the limit, input–output nips in the bud.

Traditional input–output analysis circumvents these complications. Commodities are aggregated and sectoral outputs are purified in the construction of the matrix. Detailed commodity constraints are no longer binding and sectors can freely neutralize each others' net outputs. From a methodological view point, the latter two aspects can be considered sources of substitution which free the use make model from its being stuck at observed or even zero levels of activities. However, the underlying hypotheses are extreme. Aggregation implicitly assumes perfect substitution, albeit within classes of commodities. Purification assumes, also implicitly, the possibility of negative sectoral activity levels. The difference between the use–make and the traditional models can be ascribed to aggregation. Purification does not alter the results further. The choice between the by-product and commodity technology models [in the square case of equal commodities and sectors, Kop Jansen and ten Raa (1990)] is immaterial for the Canadian economy.

The Lagrange multipliers associated with the material balance constraints are shadow prices that include tariffs. Zero values of the latter signal comparative advantages. In a model with 29 sectors and 92 commodities, we have located the comparative advantage of the 1980 Canadian economy in mining, quarrying & oil wells, tobacco, and machinery. Optimum exploitation of the Canadiam resources would boost these sectors and increase GDP by 41.5%. A traditional version of the model with the commodities aggregated into the sectors permits a decomposition analysis and a verification of the Ricardian theorem, notwithstanding theoretical rejections. The main problem is the international misdirection of the Canadian economy. The patterns of optimum and actual commodity net exports are very different. No wonder severe adjustment problems emerge in the face of the free trade agreement with the United States.

Appendix

We present the data base in this appendix. The use-and-make tables are directly available from Statistics Canada (1987). For the sources and constructions of the sectoral labor flows, the total labor force, the capital stocks and the capacity utilization rates, we refer to ten Raa and Mohnen (1991). Non-business activities, mostly government services, are treated as exogenous. The labor pertaining to those activities are netted out from the employment and total labor force figures. The use-and-make tables and capital stock data relate to business activities only. The total labor force figure has been converted from persons to person-hours using the average number of person-hours a year per person for the entire economy. The final demand vector is obtained residually by subtraction of the new totals of the use-and-make tables to neutralize errors of measurement. Domestic final demand is obtained by subtracting from final demand the domestic exports plus re-exports minus imports, contained in the final demand table of Statistics Canada (1987). All data are expressed in millions of 1980 Canadian dollars or in thousands of person-hours. For the traditional model, we put the capital coefficient for sector 8 equal to 0, in lieu of a small negative number. The sector and commodity aggregations are presented in table 1. We are constrained by a 29 sectoral classification because of the capital statistics.

Those commodities, printed in bold in table 1, for which neither imports nor exports were reported in the 1980 final demand table, are declared as non-tradables. The number of non-tradable commodities is eleven. The only sectors that are declared as non-tradable are sector 24 (construction), all commities of which are non-tradable as, well as sectors 14 (printing, publishing & allied) and 28 (finance, insurance & real estate), each of which comprises a non-traded commodity and a non-exported affiliate.

We have to allocate sectoral stock K_i to products v_{ij} and to aggregate over i to get the stock available for commodity j. The vector of sectoral stocks may be divided into utilized stocks and excess stocks,

$$K_0 = K_0\hat{c} + K_0(I-\hat{c}).$$

Utilized stocks are allocated to commodities by applying capital coefficients, $k = K_0\hat{c}V_0^{-\mathrm{T}}$, to gross commodity outputs, one at a time, $\widehat{V_0^{\mathrm{T}}e}$:

$$K_0\hat{c}V_0^{-\mathrm{T}}\widehat{V_0^{\mathrm{T}}e}.$$

Under-utilized stocks per dollar of sectoral outputs (obtained by division by $\widehat{Ve}$) are allocated to commodities in proportion to outputs (V),

$$K_0(I-\hat{c})\widehat{Ve}^{-1}V.$$

In sum, the row vector of capital stocks per commodity is defined by the following expression,

$$K^{\mathrm{c}}=K_0[\hat{c}V_0{}^{\mathrm{T}}\widehat{V_0^{\mathrm{T}}e}+(I-\hat{c})\widehat{Ve}^{-1}V].$$

As a check, note that the total stock is preserved:

$$K^{\mathrm{c}}e=K_0[\hat{c}V_0^{-\mathrm{T}}V_0^{\mathrm{T}}e+(I-\hat{c})\widehat{Ve}^{-1}Ve]=K_0[\hat{c}e+(I-\hat{c})e]=K_0e.$$

References

Dorfman, R., P.A. Samuelson and R.M. Solow, 1958, Linear programming and economic analysis (McGraw-Hill, New York).

Drabicki, J.Z. and A. Takayama, 1979, An antinomy in the theory of comparative advantage, Journal of International Economics 9, 211–223.

Kop Jansen, P. and T. ten Raa, 1990, The choice of model in the construction of input–output coefficients matrices, International Economic Review 31, no. 1, 213–227.

Leontief, W., 1953, Domestic production and foreign trade: The American capital position re-examined, Proceedings of the American Philosophical Society 97, no. 4, 332–349.

Leontief, W., 1979, Input–output economics (Oxford University Press, New York).

von Neumann, J., 1945, A model of general economic equilibrium, Review of Economic Studies 13, 1–9.

ten Raa, Th., 1994, On the methodology of input–output analysis, Regional Science and Urban Economics 24, 3–25.

ten Raa, Th. and P. Mohnen, 1991, Dometsic efficiency and bilateral trade gains, with an application to Canada and Europe, CERPE Cahier de recherche 70.

Rockafellar, T., 1970, Convex analysis (Princeton University Press, Princeton, NJ).

Schrijver, A., 1986, Theory of linear and integer programming (John Wiley & Sons, Chichester).

Statistics Canada, 1987, System of national accounts the input output structure of the Canadian economy 1961–1981 (Minister of supply and services Canada, Ottawa).

Statistics Canada, 1990a, Input–output division, Person-hours 1961–1989, unpublished.

Statistics Canada, 1990b, Input output division, Current and constant price capital stock for 1980, unpublished.

Williams, J.R., 1978, The Canadian–United States tariff and Canadian industry: A multisectoral analysis (University of Toronto Press, Toronto).

Woodland, A.D., 1982, International trade and resource allocation (North-Holland, Amsterdam).